

# Inventory Model with Inflation under Credit Period and Shortage



Anjali Harit, Anurag Sharma, S.R. Singh

**Abstract:** In this study, an deterministic inventory model based on the concept of permissible delay in payments is discussed. Demand is assumed to be price dependent, and a constant price function represents it. Shortages are allowed and partially backlogged. In the realistic environment, it is observed that there are several items like dry fruits, vegetables, grocery, and fruits, etc. which deteriorate after a time gap. So this model is also based on non-instantaneous deterioration. This study aims to optimize the optimal order level and selling price to maximize the retailer's total profit. Finally, numerical examples are solved by using a proposed algorithm to show the validity of the model and sensitivity analysis is done on parameters..

**Keywords:** Inventory, Price-dependent Demand, Non-instantaneous deterioration, Inflation, Trade credit, Shortage Backordering.

## I. INTRODUCTION

For many years, a lot of work has been published based on controlling inventory of deteriorating items. Deteriorating are the products which are spoiled with some period. Based on the decaying item first inventory model was published by [1]. They developed a simple inventory model based on economic order quantity with a constant rate of decaying items. In the realistic environment concept of deterioration does not work because there are some items like furniture, grocery, clothes, and vice versa are not getting decayed instantaneously. This phenomenon is known as non-instantaneous deterioration. Considering this concept [2] developed the first inventory model on stock dependent demand and shortage with non-instantaneous deterioration items, which are partially backlogged. An inventory model especially for a manufacturer where the manufactured product not only deteriorates but also has a maximum lifetime is developed by [3]. Inflation is the increase in the general price level of goods and services. The idea of inflation is first introduced by [4]. Inflation is an essential aspect of the market. In this paper, we also consider inflation to find total cost so that the value of money will not affect optimal profit. In business practice, it is assumed that the customer pays for the purchased items as they are received from the vendor. In practice, the vendor allows the customer to settle the account with some delay period. This contract of delay in giving payment is known as trade credit. This phenomenon provides a very fruitful result to the merchants. Such an arrangement

has attracted the interest of so many researchers and practitioners in recent years. The first paper using this concept was published by [5]. [6] discussed an inventory model on the EOQ model with permissible delay in time. The concept of discounted cash flow (DCF) approach was first introduced by [7]. They presented a DCF approach for trade credit with an economic ordering policy for deteriorating items. A generalization of [7] model done by [8] by introducing the concept of deteriorating item with trade credit linked to order quantity. [9] discussed the impact of trade credit on vendor and buyer considering and also the effect of items of imperfect quality. [3] discussed the effect on inflation on trade credit policy under a fuzzy environment. [10] proposed an inventory model based on price-dependent demand. [11] developed a model in which they considered trade credit and replenishment decisions on default risk. [12] proposed a model on trade credit they choose asymmetric credit default risk which is divided into three parts: they are screening, checking, or insurance. By considering existing literature on trade credit, this study is based on an inventory model using price-dependent demand, non-instantaneous deteriorating items, and credit policy under the effect of inflation. Shortages are allowed and partially backlogged. The objective of this study is to determine the retailer's optimal replenishment policies that maximize total optimal profit/time (unit). The paper is organized as follows: section 2 defines the assumptions and notation used throughout the article. Section 3 describes the inventory model and then establishes the retailer's annual total profit function of the system. Part 4 discusses the proposed inventory model with numerical examples, and a sensitivity analysis is presented in part 5. Finally, Section 6 discusses the conclusion of the paper.

## II. ASSUMPTIONS AND NOTATIONS

The assumptions used to develop the mathematical modelling:

- Replenishment rate is instantaneous
- Lead-time is negligible.
- The planning horizon of the inventory system is infinite.
- The demand rate is price dependent i.e.  $D(p) = a - bp$ .
- Rate of production is greater than the demand rate.
- Unsatisfied demand/ shortages are allowed.

Revised Manuscript Received on June 22, 2020.

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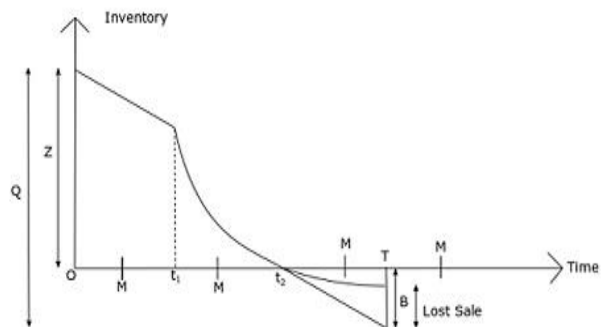
The following notations are used to develop the model is same as used in [13]:

$D(P)$	Price dependent demand
$a$	Constant demand rate, $a > 0$
$p$	Selling price per unit
$b$	Price-dependent demand rate, $b > 0$
$Q$	Order quantity (units)
$Z$	Maximum inventory level(units)
$r$	The discount rate(%)
$i$	Inflation rate, varied by economic situation(%)
$R(r-i)$	Net inflation amount(%)
$c$	Purchasing cost/item
$s$	The backlogging cost/ unit time, Unit opportunity cost due to lost sale,
$C_1$	Fixed cost of placing an order
$A$	Unit holding cost per unit item per unit time
$H$	Retailer's trade credit period provide by supplier(time unit)
$M$	Interest earned by the retailer
$I_e$	Interest paid by retailer(%)
$I_p$	Time period with no deterioration
$t_1$	Time, inventory reaches 0
$t_2$	
<b>Decision variables</b>	
$t_2$	Time at which the inventory reaches 0
$T$	Replenishment cycle length
<b>Functions</b>	
$I(t)$	Inventory level at $t$ time (units)
$B(t)$	Backlogged level
$L(t)$	Number of lost
$TP(t_2, T)$	

	sales,(units)
	Total relevant profit/unittime
<b>Optimal values</b>	
$t_r^*$	Optimal time at which the inventory level reaches 0
$T$	Optimal length of inventory cycle
$TP^*(t_2, T)$	Optimal total profit/time

### III. MATHEMATICAL MODELLING

In this model, we discuss the modelling for non-declining items with shortage which is partially backlogged.  $Q$  units lot size enters in the system. The items in lot assumed as they are not instantaneously deteriorating. Therefore, after time  $t_1$ , the items get deteriorate. In interval  $[0, t_1]$  the inventory decreases due to demand only. During  $[t_1, t_2]$  stock reaches 0 because of demand and declining of items. After that, during  $[t_2, T]$ , shortage start and partially backlogged. From  $Q$  units a portion of  $B$  units utilized to satisfy the shortage in demand from the last cycle, after that  $Z$  units are left for the thenew inventory cycle. The behavior of the model for the cycle in  $[0, T]$  is represented graphically with the help of Fig. 1.



**Fig. 1. Graphical representation for all case of credit period ( $M$ )**

Therefore, the differential equations that describe the inventory level at any time  $t$  over the period  $(0, T)$  are given by

$$\frac{dI(t)}{dt} = -(a - bp), 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a - bp), t_1 \leq t \leq t_2 \quad (2)$$

$$\frac{dB(t)}{dt} = -(a - bp)e^{-v(T-t)}, t_2 \leq t \leq T \quad (3)$$

And the boundary condition are  $I(0) = Z, I(t_2) = 0, B(t_2) = 0$

By solving (1), (2) and (3) using boundary conditions are:

$$I(t) = Z - (a - bp)t, 0 \leq t \leq t_1 \quad (4)$$

$$I(t) = \frac{(a - bp)}{\theta} (e^{\theta(t_2-t)} - 1), t_1 \leq t \leq t_2 \quad (5)$$

$$B(t) = \frac{a - bp}{\theta} (e^{-v(T-t)} - e^{-v(T-t_2)}), t_2 \leq t \leq T \quad (6)$$

The number of lost sales is

$$L(t) = \int_{t_2}^T (a - bp)(1 - e^{-v(T-t)}) dt$$

$$= (a - bp) \left[ (t - t_2) - \frac{1}{v} (e^{-v(T-t)} - e^{-v(T-t_2)}) \right] \quad (7)$$

Considering continuity of I(t) at  $t = t_1$ , it follows from (4) and (5) that

$$(a - bp)t_1 = \frac{(a - bp)}{\theta} (e^{\theta(t_2-t_1)} - 1) \quad (8)$$

Which implies that the maximum inventory level per cycle is

$$Z = (a - bp)t_1 + \frac{(a - bp)}{\theta} (e^{\theta(t_2-t_1)} - 1) \quad (9)$$

Now, replace  $t$  by  $T$  in (6), the maximum amount of backlogged demand is

$$B = \frac{(a - bp)}{v} (1 - e^{-v(T-t_2)}) \quad (10)$$

Order quantity over the replenishment cycle is

$$Q = Z + B(t) = (a - bp)t_1 + \frac{(a - bp)}{\theta} (e^{\theta(t_2-t_1)} - 1) + \frac{(a - bp)}{v} (1 - e^{-v(T-t_2)}) \quad (11)$$

Thus the present worth of the total profit per cycle for the inventory system consists of the following components, which are calculated using DCF approach, as follows:

1) Replenishment cost is  $A$

2) Holding cost

$$= H \left( \int_0^{t_1} I(t) e^{-Rt} dt + \int_{t_1}^{t_2} I(t) e^{-Rt} dt \right)$$

$$= H \left[ \frac{Z}{R} - \frac{1}{R} (Z - (a - bp)t_1 + \frac{(a - bp)}{\theta} (e^{\theta(t_2-t_1)} - 1)t_1) e^{-Rt_1} - e^{-\theta(t_2-t_1)} + \frac{1}{R} (e^{-Rt_2} - e^{-Rt_1}) \right]$$

$$+ \frac{1}{R^2} \left( (a - bp)t_1 + \frac{(a - bp)}{\theta} (e^{\theta(t_2-t_1)} - 1) (e^{-Rt_1} - 1) + \frac{(a - bp)}{\theta} \left( -\frac{1}{\theta} (e^{-Rt_2} - e^{-\theta t_2 - (\theta + R)t_1}) + \frac{1}{R} (e^{-Rt_2} - e^{-Rt_1}) \right) \right) \quad (12)$$

3) Backlogging cost is  $s \int_{t_2}^T B(t) e^{-Rt} dt$

$$= \frac{s(a - bp)}{v} e^{-vT} \left[ \frac{1}{v - R} (e^{(v-R)T} - e^{(v-R)t_2}) + \frac{e^{vt_2}}{R} (e^{-RT} - e^{-Rt_2}) \right] \quad (13)$$

4) Purchasing cost is  $cQe^{-RM}$

$$= ce^{-RM} \left[ (a - bp)t_1 + \frac{(a - bp)}{\theta} (e^{\theta(t_2-t_1)} - 1) + \frac{(a - bp)}{v} (1 - e^{-v(T-t_2)}) \right] \quad (14)$$

5) Revenue amount is

$$p \left\{ \int_0^{t_2} (a - bp) e^{-Rt} dt + B(t) e^{-Rt} \right\}$$

$$= p(a - bp) \left\{ \frac{1}{R} (1 - e^{-Rt_2}) + \frac{e^{-RT}}{v} (1 - e^{v(T-t_2)}) \right\} \quad (15)$$

### A. Interest earned and interest paid

The interest earned, interest paid and profit functions are computed as:

Case1 -  $0 < M \leq t_1 < t_2$

Case2 -  $0 < t_1 < M \leq t_2$

Case3 -  $0 < t_1 < t_2 < M \leq T$

Case4 -  $0 < t_1 < t_2 < T \leq M$

**Case 1:**  $0 < M \leq t_1 < t_2$

In this case the interest earned is calculated in two parts:

**Part 1:** Interest earned by satisfying the shortages is:

$$\text{Interest earned is} = pI_e \int_0^M B(T) e^{-Rt} dt$$

$$= pI_e \frac{(a - bp)}{vR} (1 - e^{-RM}) (1 - e^{-v(T-t_2)})$$

**Part 2:** Interest earned from 0 to M is:

$$\text{Interest earned} = pI_e \int_0^M (a - bp) t e^{-Rt} dt$$

$$= pI_e \frac{(a - bp)}{R} \left[ \frac{1}{R} (1 - e^{-RM}) - M e^{-RM} \right]$$

Total interest earned

$$= pI_e \left[ \frac{(a - bp)}{vR} (1 - e^{-RM}) (1 - e^{-v(T-t_2)}) + \frac{(a - bp)}{R} \left( \frac{1}{R} (1 - e^{-RM}) - M e^{-RM} \right) \right] \quad (16)$$

Further, the interest payable from M to

$$t_2 = cI_p \left\{ \int_M^{t_1} Z e^{-Rt} dt + \int_{t_1}^{t_2} I(t) e^{-Rt} dt \right\}$$

$$= cI_p \left\{ \frac{Z}{R} (e^{-RM} - e^{-Rt_1}) + \frac{(a - bp)}{\theta} \left[ -\frac{1}{\theta + R} (e^{-Rt_2} - e^{-\theta(t_2-t_1)}) + \frac{1}{R} (e^{-Rt_2} - e^{-Rt_1}) \right] \right\} \quad (17)$$

**Case 2:**  $0 < t_1 < M \leq t_2 < T$

The interest earned is equivalent to previous case 1.

**Part 1:** Interest payable from M to  $t_2$

$$= cI_p \left\{ \int_M^{t_2} I(t) e^{-Rt} dt \right\}$$

$$= cI_p \frac{(a - bp)}{\theta} \left\{ \frac{1}{\theta + R} [e^{\theta t_2 - (\theta + R)M} - e^{-Rt_2}] + \frac{1}{R} (e^{-Rt_2} - e^{-RM}) \right\} \quad (18)$$

**Case 3:**  $0 < t_1 < t_2 < M \leq T$

The interest earned is calculated as follows:

**Part 1:** Interest earned at the beginning of the cycle i.e.

$$= pI_e \left\{ \int_0^M B(T) e^{-Rt} dt \right\}$$

$$= pI_e \left\{ \frac{(a - bp)}{Rv} (1 - e^{-RM}) (1 - e^{-v(T-t_2)}) \right\}$$

**Part 2:** Interest earned from 0 to  $t_2$ ,

$$= pI_e \left( \int_0^{t_2} D t e^{-Rt} dt \right)$$

$$= pI_e \frac{(a - bp)}{R} \left( \frac{1}{R} (1 - e^{-Rt_2}) - t_2 e^{-Rt_2} \right)$$

**Part 3:** The interest earned on the revenue of sales till  $t_2$ , for the time period  $(t_2, M)$

$$= pI_e \left\{ \int_{t_2}^M \left( \int_0^{t_2} (a - bp) dt \right) e^{-Rt} dt \right\}$$

$$= pI_e \frac{(a - bp)t_2}{R} (e^{-Rt_2} - e^{-RM})$$

Total interest earned



$$= pI_e \left\{ \frac{(a-bp)}{Rv} (1-e^{-RM})(1-e^{-v(T-t_2)}) + \frac{(a-bp)}{R} \left( \frac{1}{R} (1-e^{-Rt_2}) - t_2 e^{-Rt_2} \right) + \frac{(a-bp)t_2}{R} (e^{-Rt_2} - e^{-RM}) \right\} \quad (19)$$

In this case no interest paid.

**Case 4:**  $0 < t_1 < t_2 < T \leq M$

Interest earned and interest paid in this case is similar as in case 3.

**B. Retailer's total profit**

Therefore, the present worth of total profit per unit time during the cycle (0, T) is given by

$$TP(t_1, T) = \frac{1}{T} [\text{Sales revenue} - \text{Ordering cost} - \text{Holding cost} - \text{Purchasing cost} + \text{Interest earned} - \text{Interest paid}]$$

The total profit/ unit time is

$$TP(t_2, T) = \begin{cases} TP_1(t_2, T) & \text{if } 0 < M \leq t_1 < t_2 < T \\ TP_2(t_2, T) & \text{if } 0 < t_1 < M \leq t_2 < T \\ TP_3(t_2, T) & \text{if } 0 < t_1 < t_2 < M \leq T \\ TP_4(t_2, T) & \text{if } 0 < t_1 < t_2 < T \leq M \end{cases} \quad (20)$$

$$\begin{aligned} TP_1(t_2, T) &= \frac{p(a-bp)}{T} \left[ \frac{1}{R} (1-e^{-Rt_2}) + \frac{e^{-RT}}{v} (1-e^{-v(T-t_2)}) \right] \\ &- \frac{A}{T} - \frac{H}{T} \left[ \frac{Z}{R} - \frac{(Z-(a-bp)t_1)}{R} e^{-Rt_1} + \frac{(a-bp)}{R^2} (e^{-Rt_1} - 1) \right] \\ &+ \frac{(a-bp)}{\theta} \left[ -\frac{1}{\theta+R} (e^{-Rt_2} - e^{-\theta t_2 - (\theta+R)t_1}) + \frac{1}{R} (e^{-Rt_2} - e^{-Rt_1}) \right] \\ &- ce^{-RM} \left[ (a-bp)t_1 + \frac{(a-bp)}{\theta} (e^{\theta(t_2-t_1)} - 1) \right] \\ &= \frac{(a-bp)}{v} (1-e^{-v(T-t_2)}) + pI_e \frac{(a-bp)}{R} \left[ \frac{1}{v} (1-e^{-RM}) (1-e^{-v(T-t_2)}) + \frac{1}{R} (1-e^{-RM}) - Me^{-RM} \right] - cI_p \left[ \frac{Z}{R} (e^{-RM} - e^{-Rt_1}) + \frac{(a-bp)}{\theta} \left[ -\frac{1}{\theta+R} (e^{-Rt_2} - e^{-\theta(t_2-t_1)}) + \frac{1}{R} (e^{-Rt_2} - e^{-Rt_1}) \right] \right] \end{aligned}$$

$$\begin{aligned} TP_2(t_2, T) &= \frac{p(a-bp)}{T} \left[ \frac{1}{R} (1-e^{-Rt_2}) + \frac{e^{-RT}}{v} (1-e^{-v(T-t_2)}) \right] \\ &- \frac{A}{T} - \frac{H}{T} \left[ \frac{Z}{R} - \frac{(Z-(a-bp)t_1)}{R} e^{-Rt_1} + \frac{(a-bp)}{R^2} (e^{-Rt_1} - 1) \right] \\ &+ \frac{(a-bp)}{\theta} \left[ -\frac{1}{\theta+R} (e^{-Rt_2} - e^{-\theta t_2 - (\theta+R)t_1}) + \frac{1}{R} (e^{-Rt_2} - e^{-Rt_1}) \right] \\ &- ce^{-RM} \left[ (a-bp)t_1 + \frac{(a-bp)}{\theta} (e^{\theta(t_2-t_1)} - 1) \right] \quad (22) \\ &+ \frac{(a-bp)}{v} (1-e^{-v(T-t_2)}) + pI_e \frac{(a-bp)}{R} \left[ \frac{1}{v} (1-e^{-RM}) (1-e^{-v(T-t_2)}) + \frac{1}{R} (1-e^{-RM}) - Me^{-RM} \right] - cI_p \frac{(a-bp)}{\theta} \\ &\left\{ \frac{1}{\theta+R} [e^{\theta t_2 - (\theta+R)M} - e^{-Rt_2}] + \frac{1}{R} (e^{-Rt_2} - e^{-RM}) \right\} \end{aligned}$$

$$\begin{aligned} TP_3(t_2, T) &= TP_4(t_2, T) = \frac{p(a-bp)}{T} \left[ \frac{1}{R} (1-e^{-Rt_2}) + \frac{e^{-RT}}{v} (1-e^{-v(T-t_2)}) \right] \\ &- \frac{A}{T} - \frac{H}{T} \left[ \frac{Z}{R} - \frac{(Z-(a-bp)t_1)}{R} e^{-Rt_1} + \frac{(a-bp)}{R^2} (e^{-Rt_1} - 1) \right] \\ &+ \frac{(a-bp)}{\theta} \left[ -\frac{1}{\theta+R} (e^{-Rt_2} - e^{-\theta t_2 - (\theta+R)t_1}) + \frac{1}{R} (e^{-Rt_2} - e^{-Rt_1}) \right] \\ &- ce^{-RM} \left[ (a-bp)t_1 + \frac{(a-bp)}{\theta} (e^{\theta(t_2-t_1)} - 1) \right] \\ &+ \frac{(a-bp)}{v} (1-e^{-v(T-t_2)}) + pI_e \frac{(a-bp)}{Rv} (1-e^{-RM}) (1-e^{-v(T-t_2)}) \\ &+ \frac{(a-bp)}{R} \left( \frac{1}{R} (1-e^{-Rt_2}) - t_2 e^{-Rt_2} \right) \\ &+ pI_e \frac{(a-bp)t_2}{R} (e^{-Rt_2} - e^{-RM}) \end{aligned} \quad (23)$$

**C. Optimality of the profit equations function**

Now, discuss the optimal solution of the objective function. The necessary conditions for the total profit to be maximum are

$$\frac{\partial TP_i(t_2, T)}{\partial t_2} = 0 \text{ and } \frac{\partial TP_i(t_2, T)}{\partial T} = 0; \text{ for } i=1, 2, 3 \text{ and } 4 \quad (24)$$

The sufficient conditions for maximizing  $TP_i(t_2, T)$  by using second order partial derivatives are

$$\frac{\partial^2 TP_i(t_2, T)}{\partial t_2^2} > 0, \text{ and } \frac{\partial^2 TP_i(t_2, T)}{\partial T^2} > 0; \text{ for } i=1, 2, 3 \text{ and } 4 \quad (25)$$

The objective of the present model is to obtain the optimal values of  $t_2$  and  $T$  so as to maximize the total profit function. Since it is difficult to solve the optimality condition of the profit function mathematically, the convexity total profit functions has established graphically (Figs. 2-4) by using Mathematica software. Hence, the final optimal solution is unique one.

**D. Special cases**

The proposed model is general inventory model. In this section, we discuss some special cases that present inventory model contains.

- a. When  $t_1 = 0$ ,  $v \rightarrow \infty$  and  $R = 0$ , then proposed model is same as [14].
- b. When  $\theta = 0$ ,  $t_1 = 0$ ,  $v \rightarrow \infty$  and  $R = 0$ , then proposed model same as [15].
- c. When  $t_1 = 0$ , and complete backlogged,  $M = 0$ , and  $I_e = 0$ , then proposed model is same as [16]
- d. When  $t_1 = 0$ ,  $v \rightarrow \infty$ ,  $M = 0$ , and  $I_e = 0$ , then proposed model is same as [1].
- e. When  $\theta = 0$ ,  $t_1 = 0$ ,  $v \rightarrow \infty$ ,  $M = 0$ ,  $I_e = 0$  and  $I_e = 0$  then proposed model is same as traditional economic order quantity inventory model with optimal quantity

$$Q^* = \sqrt{\frac{2A(a-bp)}{H}} \quad (25)$$

**IV. NUMERICAL EXAMPLE**

This section validate the developed inventory model with the numerical examples.

**Example 1:** Consider an inventory system with the following data:



$A = 250/\text{order}$ ,  $C_1 = 5/\text{unit}$ ,  
 $a = 3000\text{unit}/\text{year}$ ,  $R = 0.06$ ,  
 $c = 4/\text{unit}$ ,  $I_p = 0.15/\text{year}$ ,  
 $p = 15/\text{unit}$ ,  $I_e = 0.12/\text{year}$ ,  
 $H = 0.5/\text{unit}/\text{year}$ ,  $t_1 = 0.2$ ,  
 $W = 200\text{units}$ ,  $\theta = 0.05/\text{unit}$   
 $s = 4/\text{unit}/\text{year}$ ,  $M = 0.25$  year.  
 $v = 0.7/\text{unit}$ ,  $t_2 = 0.593\text{year}$ ,  
 $T = 0.875\text{year}$ ,  $Q = 1982.67\text{units}$  and  $TP^* = 4056.45$  with suitable units. Fig. 2 shows convexity function graphically.

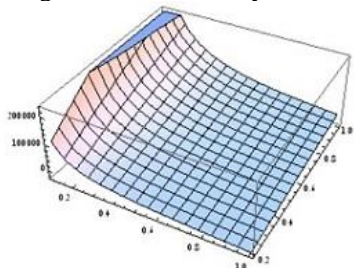


Fig. 2. Convexity of  $TP^*$  versus  $t_2$  and  $T$

**Example2:** Consider an inventory system with the following data:

$A = 250/\text{order}$ ,  $C_1 = 5/\text{unit}$ ,  
 $a = 3000\text{unit}/\text{year}$ ,  $R = 0.06$ ,  
 $c = 4/\text{unit}$ ,  $I_p = 0.15/\text{year}$ ,  
 $p = 15/\text{unit}$ ,  $I_e = 0.12/\text{year}$ ,  
 $H = 0.5/\text{unit}/\text{year}$ ,  $t_1 = 0.2$ ,  
 $W = 200\text{units}$ ,  $\theta = 0.05/\text{unit}$   
 $s = 4/\text{unit}/\text{year}$ ,  $t_2 = 0.613\text{year}$ ,  
 $v = 0.9/\text{unit}$ ,  $M = 0.25$  year.  
 $T = 0.893\text{year}$ ,  $Q = 2039.04$ , and  $TP^* = 4900.63$  with suitable units. Fig. 3 shows convexity function graphically.

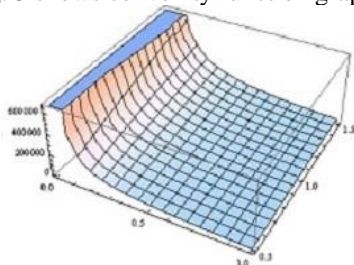


Fig. 3. Convexity of  $TP^*$  versus  $t_2$  and  $T$

**Example3:** Consider an inventory system with the following data:

$A = 250/\text{order}$ ,  $C_1 = 5/\text{unit}$ ,  
 $a = 3000\text{unit}/\text{year}$ ,  $R = 0.06$ ,  
 $c = 4/\text{unit}$ ,  $I_p = 0.15/\text{year}$ ,  
 $p = 15/\text{unit}$ ,  $I_e = 0.12/\text{year}$ ,  
 $H = 0.5/\text{unit}/\text{year}$ ,  $t_1 = 0.2$ ,  
 $W = 200\text{units}$ ,  $\theta = 0.05/\text{unit}$   
 $s = 4/\text{unit}/\text{year}$ ,  $t_2 = 0.561\text{year}$ ,  
 $v = 0.9/\text{unit}$ ,  $M = 0.25$  year.  
 $T = 0.859\text{year}$ ,  $Q = 1928.29$ , and  $TP^* = 6793.42$  with suitable units. Figure 4 shows convexity function graphically.

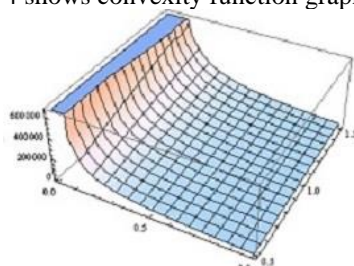


Fig. 4. Convexity of  $TP^*$  versus  $t_2$  and  $T$

V. SENSITIVITY ANALYSIS

In this section, sensitivity analysis has been performed in order to study the impact of non-deteriorating period ( $t_1$ ); credit period ( $M$ ), inflation rate ( $R$ ); interest paid ( $I_p$ ) and interest earned ( $I_e$ ) on the lot size ( $Q^*$ ), cycle length ( $T$ ) and the retailer's total optimal profit ( $TP^*$ ). Sensitivity table is depends on numerical example 1. Results summarized by tables 2-5 given below.

Table 2: Effect of non-deteriorating items and credit period ( $t_1$  and  $M$ ) on optimal replenishment policy

$t_1$	$M$	$t_2$	$T$	$Q$	$TP^*$
0.2	0.15	0.593	0.735	1727.04	11964.6
	0.25	0.613	0.753	1778.12	12665.1
	0.35	0.631	0.771	1828.13	13509.3
0.4	0.15	0.659	0.806	1831.97	13860.1
	0.25	0.677	0.813	1856.49	15086.2
	0.35	0.695	0.829	1899.23	15814.3
0.6	0.15	0.716	0.857	1916.66	11654.1
	0.25	0.747	0.894	2003.01	11773.3
	0.35	0.789	0.934	2101.85	12479.8

Table 3: Effect of  $R$  on the optimal replenishment policy

$R$	$t_1$	$T$	$Q$	$TP^*$
0.11	0.266	0.812	1990.07	10818.3
0.09	0.296	0.853	2057.74	11374.1
0.07	0.329	0.897	2126.71	12579.7
0.05	0.367	0.935	2178.83	15410.2
0.03	0.408	0.974	2230.09	20245.7

Table 4: Effect of  $v$  on optimal replenishment policy

$V$	$t_2$	$T$	$Q$	$TP^*$
0.3	0.349	0.862	1876.07	18290.8
0.6	0.349	0.857	1844.02	18272.4
0.9	0.349	0.852	1816.25	18253.7
1.2	0.349	0.848	1795.37	18234.7

Table 5: Effect of  $I_e$  and  $I_p$  on the optimal replenishment policy

$I_e$	$I_p$	$t_2$	$T$	$Q$	$TP^*$
0.12	0.20	0.583	0.857	1946.16	3369.47
	0.15	0.594	0.868	1976.22	4488.48
	0.10	0.603	0.873	1993.88	5936.58
0.17	0.20	0.584	0.858	1948.88	4022.01
	0.15	0.595	0.869	1978.96	5141.02
	0.10	0.604	0.874	1996.64	6580.46
0.22	0.20	0.585	0.859	1951.61	4674.55
	0.15	0.596	0.870	1981.70	5793.56
	0.10	0.6035	0.875	1999.39	7224.34

Table 2 indicates that  $T$ ,  $Q$  and  $TP$  increase as  $M$  increase. It suggests that permissible delay in payments facilitates the business for each provider and retail merchant. In addition the extended credit amount (without penalty) for payment to the provider, indirectly reduced the prices incurred by the retailer and ultimately ends in higher profit. It specifies the positive influence of non-instantaneous deteriorating items within inventory modeling because the amount for deterioration ( $t_1$ ) will increase; the deterioration value for items decreases that accounts for more significant profits for the corporate.



Table 3 shows when  $R$  decreases (i.e., the inflation rate is increasing),  $Q$  and  $TP$  increase. Since the cost of goods is high in an inflationary market, the retailer should place a large order for an extended interval to maintain the profit. Table 4 clearly shows that as  $\nu$  increases, there is decrease in  $TP$ ,  $T$ , and  $Q$ . As  $Q$  decreases eventually results in less profit. Table 5 exhibits as  $I_e$  increases, the order amount as well as the overall average profit increase. The total earnings increase in this state when considering that growing interest earned rates contribute to additional significant revenue. An assessment of this, while the  $I_p$  prices increased outlets general average income decreases, as in this case, the interest charges get brought to general costs, which outcomes as diminishing profits.

## VI. CONCLUSION

In this study, we developed a model for permissible delay in payment under inflationary conditions and non-instantaneous deteriorating items. In contrast to previous studies, we characterized the concept of trade credit with price dependent demand under the effect of inflation to tackle the real situation in a more effective manner. This study will provide a profitable result for the business situation. Shortages are allowed in this study, which is fulfilled by a partial backlogging rate. This study controls the seller's optimal replenishment policies that maximize total optimal profit/unit time. Sensitivity analysis based on numerical examples for key parameters has summarized to direct the management of the market establishments to take appropriate activity under the prevailing conditions. The study is more effective as it helps the decision-maker to provide a suitable framework to assess profitability.

## ACKNOWLEDGMENT

The author would like to thank the anonymous referees for their helpful suggestions, which improved the presentation of this paper and also like to thank UGC for grant research fellowship.

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