

# Some Probability Distributions and L-Moment in Hydrological Engineering



Khurshid Ahmad, Bashir Ahmad, Akhtar Alam

**Abstract:** *Extreme hydrological situations constantly disturb the earth activities and life, to envisage such extreme activities we need a system that alarms well on time and recognized the expected danger; to prepare such systems one must have knowledge of the significant factors that are actively responsible for such extreme situations and we should have a reliable statistical technique that helps to prepare a useful model for such systems. In this paper we investigate the historical data of peak flood from several gauging stations of river Jhelum in Kashmir, India. A reliable estimation technique (L-moment) is applied for parametric estimation of the probability distributions and a reliable testing techniques are used to check the accuracy of fitting of the distribution, in addition to that L-moment ratio diagram (LMRD) is used to impart information about fitting of distribution. Log Pearson-III distribution shows better results and satisfies tests of distribution fitting, same probability distribution is globally accepted for flood forecasting.*

**Key Words:** *L-Moments, Jhelum, P-P plot, L-moment ratio diagram, Return period.*

Parametric estimation in these distributions are mostly done by using method of moments, in this method assessing to higher moments becomes very difficult and this method is considered as less accurate comparative to other methods, to overcome such shortcomings L-moment method (LM) provides better results [4], Kumar, Saf and Yue used the L-moment methods for estimations of parameters of probability distributions [11,13,16].

In the valley of Kashmir river Jhelum is the main source of hydrological extremes, flows through the region from south to north and entered into Pakistan (figure-I), we have investigate the instrumental data of peak flood of 58 years of this region from the different gauging stations, Sangum in south Kashmir, Ram-Munshibagh in central Kashmir and Asham in north Kashmir.

## I. INTRODUCTION

Extreme hydrologic situations cause miseries to human beings and earth activities. To identify the trend and the movements of such situations we should have good knowledge of the significant factors involving in system and the reliable statistical techniques, in order to save the human and economic losses, we should build such a system that not only provides the warning before the extreme situation but also support and remain effective in those conditions. As far as the hydrologic forecast and allied constructional/managerial scrutiny is considered one must have the adequate knowledge about the concerning factors that plays vital role and latest statistical techniques that is used to frame better models. Some methodologies are used to make decisions about flood warning response, flood warning zones [8,10], optimal decisions are taken for hydrological forecasting based on the adequate information and efficient models [2,9,12].

Different researchers suggest that a good number of probability distributions are used for flood frequency analysis [5,7]. Mostly the probability distributions that are used in flood frequency distributions are extreme value Type –I, Generalized Pareto distribution, Log Pearson-III, Generalized extreme value distribution and normal distribution [3].

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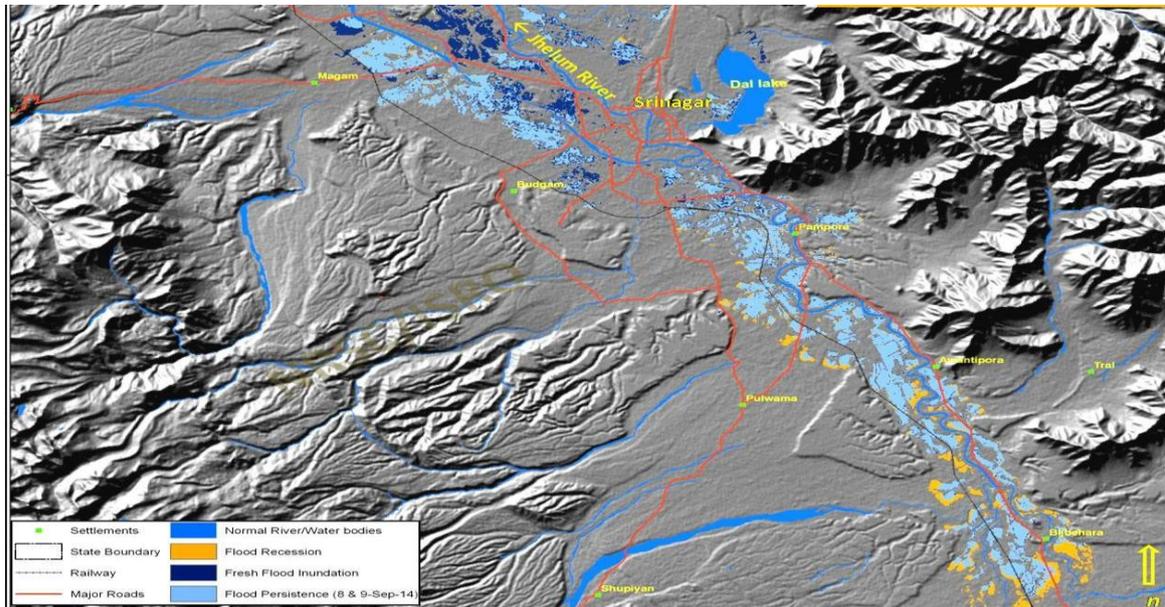


Fig. I. Ariel view of River Jhelum

II. METHOD

L-moments(LM) are linear functions of ordered statistics having less biasing, less effected by sampling variability and are more robust than conventional moments [4,15]. The L-moments for the order statistics is:

$$\lambda_r = I/r \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} E(X_{r-k:r})$$

Three probability distributions are used, two of them are three parametric (GEV and LP3) and one is two parametric (Gumbel). The aptness of the distributions is investigated by various tests like Kolmogorov-Smirnov (KS), Anderson-Darling (AD), and the Chi-Squared.

2.1 Generalized Extreme Values (GEV)

Generalized extreme value distribution is the combination of Weibull, Gumbel and Frechet probability

distributions, it has three parameters: location parameter (z), scale parameter (α) and shape parameter (k). Probability distributions having more number of parameters will be considered as more efficient specially in hydrological engineering [6], even two parametric distributions sometimes have good results but in case of small sample size [1].

2.2 Log Pearson Type- III Distribution (LP3)

Similar to GEV, log pearson Tpe-III also uses three parameters: location parameter (μ), scale parameter (σ) and shape parameter (γ), it belongs to the family of pearson type-III.

2.3 Gumbel Distribution (EV1)

Gumbel distribution is a two parametric distribution, location parameter (z) and scale parameter (α). Standard error of two parametric distributions like (EV1) are smaller but biasing is large as compared to 3 or 4 parametric distributions [1].

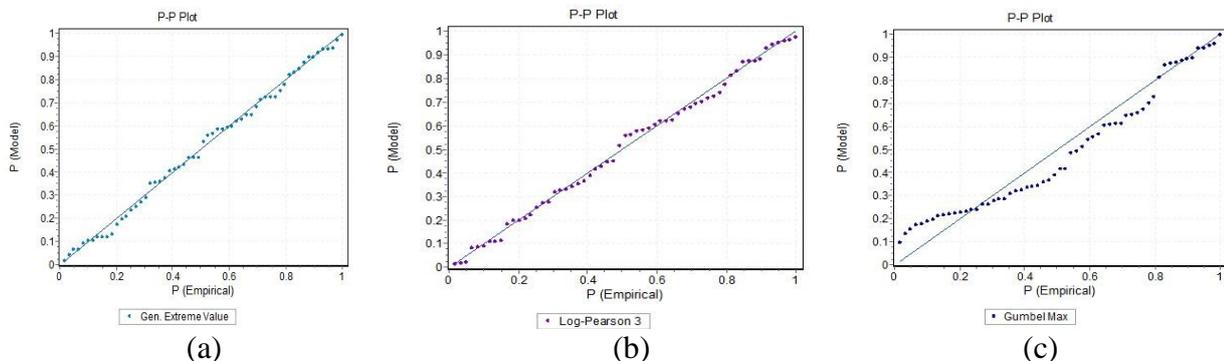


Fig. II. (P-P plot of three stations)

III. GOODNESS OF FIT TESTS

Various tests (parametric and non-parametric) are used to check the efficient fitting of probability distribution to the given data, in these tests, test

statistics are calculated and analyzed. Here we use non-parametric tests like Kolmogorov-Smirnov, Chi-Square and Anderson-Darling [14].



Table I

Station	Distributions	Sample Size	Kolmogorov-Smirnov		Chi-Square		Anderson-Darling
			Statistics	p-Value	Statistics	p-Value	Statistics
I Sangum	Generalized Extreme Value	59	0.06857	0.90131	2.9735	0.70213	0.2228
	Log-Pearson III	59	0.0716	0.9010	2.376	0.795	0.22142
	Gumbel	59	0.1239	0.1239	4.884	0.299	0.42365
II Ram MunshiBagh	Generalized Extreme Value	59	0.0548	0.99023	2.3259	0.80245	0.1625
	Log-Pearson III	59	0.05429	0.99122	1.2423	0.94076	0.1531
	Gumbel	59	0.05516	0.98948	2.3142	0.80419	0.16677
III Asham	Generalized Extreme Value	59	0.05892	0.97913	2.6497	0.75381	0.19416
	Log-Pearson III	59	0.06357	0.95869	0.903	0.97	0.18036
	Gumbel	59	0.07675	0.85148	1.143	0.95022	0.3523

IV. L-MOMENT RATIO DIAGRAM (LMRD)

An additional mode to check the fitting of distributions is the (LMRD), it the diagram which represents the relationship between L-Skewness and L-

Kurtosis ( $\tau_3, \tau_4$ ) of data plotted against points and constant lines. Actually LMR Diagram is the visual check of the necessary fitting of the distribution.

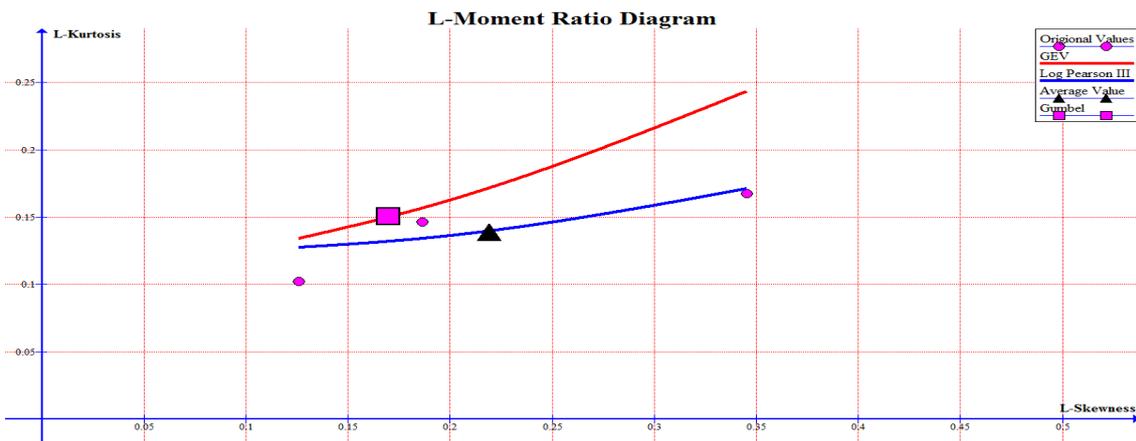


Fig. III. L-Moment Ratio Diagram

V. RETURN PERIOD

The return period (year) of three gauging stations using Log Pearson’s probability distributions (LP3) is given in Table II,

Table II

Return Period (Year)	Station I (Sangum)	Station II (Ram-MunshiBagh)	Station III (Asham)
2	549.54	602.55	732.34
5	1000.00	977.24	1114.36
10	1348.96	1174.89	1344.61
25	1819.70	1479.11	1606.69
50	2137.96	1698.24	1781.77
100	2511.88	1862.08	1940.90
200	2884.03	2041.74	2086.54

VI. CONCLUSION

L-moment method provides better estimation with less biasing of unknown parameters of important probability distributions using in hydrological system, throughout the world it is a main requirement that how to judge the trends of extreme hydrological situation, so that we can reduce economical and human losses. Knowing the key factors involving the system and using the proper statistical techniques we may be able to establish a reliable system that predicts and monitor the situations. Here we see that the Log Pearson-3 distribution shows better results with support of non-parametric tests, L-Moment Ratio Diagrams (LMRD) which is an additional way to measure goodness of fit to supports the claims. Log Pearson-3 distribution are used globally for flood forecasting.

REFERENCES

- Cunnane, C. (1989). "Statistical distributions for flood frequency Analysis". Operational Hydrology Report No. 33, World Meteorological Organization.
- Georgakakos, K.P., (1986). A generalized stochastic hydro meteorological model for flood and flashflood forecasting, Part 1: Formulation. Journal of Water Resources Research 22(13), 2083–2095.
- Haktanir, T., and Horlacher, H. B. (1993). Evaluation of various distributions for flood frequency analysis. Journal of Hydrological Sciences Journal, 38, 15–32.
- Hosking, J. R. M. (1990). L-moments: Analysis and estimation of distributions using linear combinations of order statistics. Journal of Royal Statistical Society (Series B), 52, 105–124.
- Hosking, J. R. M., & Wallis, J. R. (1993). Some statistics useful in regional frequency analysis. Journal of Water Resources Research, 29, 271–281.
- Hosking, J.R.M., Wallis, J.R. (1997). "Regional Frequency Analysis". Cambridge University Press, Cambridge.
- Kjeldsen, T. R., Smithers, J. C., Schulze, R. E. (2002). Regional flood frequency analysis in the KwaZulu-Natal province, South Africa using the index-flood method. Hydrology.
- Krzysztofowicz, R., (1993). A theory of flood warning systems. Journal of Water Resources Research 29(12), 3981–3994.
- Krzysztofowicz, R., (2001). The case for probabilistic forecasting in hydrology. Hydrology 249(1–4), 2–9.
- Krzysztofowicz, R., Davis, D.R., (1983). A methodology for evaluation of flood forecast-response systems, Part 2: Theory. Journal of Water Resources Research 19(6), 1431–1440. 23
- Kumar, R., et. al. (2003). Development of regional flood frequency relationships using L-moments for Middle Ganga Plains Subzone 1(f) of India. Journal of Water Resources Management, 17, 243–257.
- Lardet, P., Obled, Ch., (1994). Real-time flood forecasting using a stochastic rainfall generator. Hydrology 162, 391–408.
- Saf, B. (2009). Regional flood frequency analysis using L-moments for the west Mediterranean region of Turkey. Journal of Water Resources Management, 23, 531–551.
- Solaiman, T.A., (2011). Uncertainty Estimation of Extreme Precipitations Under Climatic Change: A Non-Parametric Approach. PhD Thesis, Department of Civil and Environmental Engineering, The University of Western Ontario.

- Stedinger, J.R., Vogel, R.M., Foufoula-Georgiou, E., (1992). Frequency analysis of extreme events. In: Maidment, D.R. (Ed.), Handbook of Hydrology. McGraw-Hill, New York.
- Yue, S., & Wang, C. Y. (2004). Possible regional probability distribution type of Canadian annual streamflow by L-moments. Journal of Water Resources Management, 18, 425–438.

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