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Power-3 Heronian Mean Labeling of Graphs

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Abstract: *Let $G = (V, E)$ be an undirected graph having a vertices and b edges. Now, defining a function say, $\beta: V(G) \longrightarrow \{1, 2, 3, \dots, b+1\}$ is called Power-3 Heronian Mean Labeling of a graph G if we could able to label the vertices $x \in V$ with dissimilar elements from $1, 2, \dots, b+1$ such that it induces an edge labeling $\beta^*: E(G) \longrightarrow \{1, 2, 3, \dots, b\}$ defined as,*

$$\beta^*(e = uv) = \left\lceil \sqrt[3]{\frac{\beta(u)^3 + (\beta(u)\beta(v))^{\frac{3}{2}} + \beta(v)^3}{3}} \right\rceil,$$

is dissimilar for all the edges $e = uv \in E$. (i,e.) It intimates that the dissimilar vertex labeling induces a dissimilar edge labeling on the graph. The graph which owns Power-3 Heronian Mean Labeling is called an Power-3 Heronian Mean Graph. In this, we have advocated the Power-3 Heronian Mean Labeling of some standard graphs like Path, Comb, Caterpillar, Triangular Snake, Quadrilateral Snake and Ladder.

Keywords : *Power-3 Heronian Mean Labeling, Power-3 Heronian Mean Graph, Path, Comb, Caterpillar, Triangular Snake, Quadrilateral Snake and Ladder.*

I. INTRODUCTION

The graph G we considered here are simple, finite and undirected graphs. $V(G)$ and $E(G)$ represents the vertex set and the edge set of a graph G. For graph theoretic terminology, we refer to Harary.F [2] and Gallian.J.A [1]. The notion of Mean Labeling of graphs was introduced by Somasundaram.S and Ponraj [3] in 2003. Sandhya.S.S, Ebin Raja Merly.E and Deepa.S.D [4] introduced the notion of Heronian Mean Labeling of graphs in 2017. On the same lines we define and study **Power-3 Heronian Mean Labeling of graphs.**

II. BASIC DEFINITIONS

The upcoming basic definitions are needed for the current study.

A. Definition

Generally, **Path** is represented by a walk having dissimilar vertices. A Path is represented by P_n . The Path P_n has n vertices and $n-1$ edges.

B. Definition

Comb is attained by attaching a complete graph K_1 to each vertex of a path. Generally, it gas $2n$ vertices and $2n-1$ edges.

C. Definition

Caterpillar is attained by removing the pendant vertices of a path from the tree. It has $3n$ vertices and $3n-1$ edges.

D. Definition

A **Triangular Snake T_m** is attained by attaching every pair of vertices of a path to another new vertex. (i,e.,) we can replace each edge of a path P_n by a cyclic graph C_3 . Generally, it has $2n+1$ vertices and $3n$ edges.

E. Definition

A **Quadrilateral Snake Q_m** is attained by attaching every pair of vertices of a path to another two new vertices. (i,e.,) we can replace each edge of a path P_n by a cyclic graph C_4 . Generally, it has $3n-2$ vertices and $4n-4$ edges.

F. Definition

The **Ladder L_n** is the product graph $P_2 \times P_n$. L_n has $2n$ vertices and $3n-2$ edges.

III. MAIN RESULTS

Theorem: 1

For every n, Path P_n is said to be a Power-3 Heronian Mean graph.

Proof:

Let us consider a Path P_n having the vertices $u_1, u_2, u_3, \dots, u_n$ of length n. Generally, the gragh P_n have n vertices and $n-1$ edges.

Now, defining a function $\beta: V(P_n) \rightarrow \{1,2,3, \dots, b+1\}$ by

$$\beta(u_i) = i \qquad , where\ \ i = 1,2, \dots, n$$

Then the induced edge labels are given by,

$$\beta^*(u_i u_{i+1}) = i \qquad , where\ \ i = 1,2, \dots, n-1$$

Then we attain a dissimilar value for the edges.

Therefore, P_n is said to be a Power-3 Heronian Mean graph.

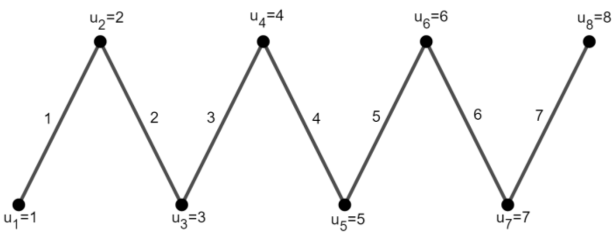

Figure 1: P_8

Theorem: 2

For every n, Comb $P_n \odot K_1$ is said to be a Power-3 Heronian Mean graph.

Proof:

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Let $P_n \odot K_1$ be a comb attained by attaching a complete graph K_1 to each vertex of P_n. Generally, it has $2n$ vertices and $2n-1$ edges.

Now, defining a function $\beta: V(G) \to \{1,2,3,\dots,b+1\}$ by

$$\beta(u_i) = \begin{cases} 2i & ,where\ i = 1 \\ 2i-1 & ,where\ i = 2,3,\dots,n \end{cases}$$

$$\beta(v_i) = \begin{cases} i & ,where\ i = 1 \\ 2i & ,where\ i = 2,3,\dots,n \end{cases}$$

Then the induced edge labels are,

$$\beta^*(u_i u_{i+1}) = 2i \qquad ,where\ i = 1,2,\dots,n-1$$

$$\beta^*(u_i v_i) = 2i-1 \qquad ,where\ i = 1,2,\dots,n$$

Then we attain a dissimilar value for the edges.

Therefore, $P_n \odot K_1$ is said to be a Power-3 Heronian Mean graph.

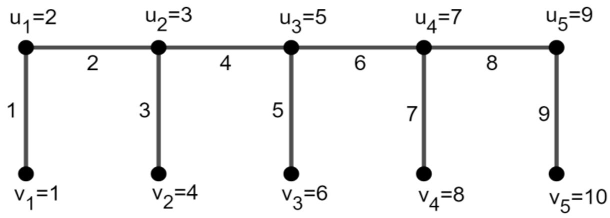

Figure 2: $P_5 \odot K_1$

Theorem: 3

Assume G be a graph attained by joining a single edge to the two sides of each vertex of P_n. Then, G is said to be a Power-3 Heronian Mean graph.

Proof:

Assume G be a graph attained by joining a single edge to the two sides of each vertex of P_n. Let P_n be a path $v_1, v_2, v_3, \dots, v_n$. Let u_i and w_i be the pendant vertices adjacent to v_i. Generally, it has $3n$ vertices and $3n-1$ edges.

Now, defining a function $\beta: V(G) \to \{1,2,3,\dots,b+1\}$ by

$$\beta(u_i) = 3i-2 \qquad ,where\ i = 1,2,\dots,n$$
$$\beta(v_i) = 3i-1 \qquad ,where\ i = 1,2,\dots,n$$
$$\beta(w_i) = 3i \qquad ,where\ i = 1,2,\dots,n$$

Then the induced edge labels are given by,

$$\beta^*(v_i v_{i+1}) = 3i \qquad ,where\ i = 1,2,\dots,(n-1)$$
$$\beta^*(v_i u_i) = 3i-2 \qquad ,where\ i = 1,2,\dots,n$$
$$\beta^*(v_i w_i) = 3i-1 \qquad ,where\ i = 1,2,\dots,n$$

Then we attain a dissimilar value for the edges.

Therefore, G is said to be a Power-3 Heronian Mean graph.

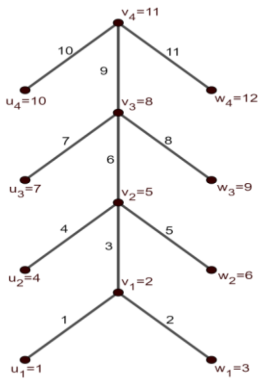

Figure 3: Caterpillar

Theorem: 4

Triangular Snake T_m is said to be a Power-3 Heronian Mean graph.

Proof:

Assume T_m be a Triangular Snake. It is attained by attaching every pair of vertices of a path to another new vertex say v_i . (i,e.,) we can replace each edge of a P_n by a cyclic graph C_3 . Generally, it has $2n+1$ vertices and $3n$ edges.

Now, defining a function $\beta: V(G) \to \{1,2,3,\dots,b+1\}$ by

$$\beta(u_i) = 3i-2 \qquad ,where\ i = 1,2,\dots,n$$

$$\beta(v_i) = 3i-1 \qquad ,where\ i = 1,2,\dots,n$$

Then the induced edge labels are given by,

$$\beta^*(u_i u_{i+1}) = 3i-1 \qquad ,where\ i = 1,2,\dots,(n-1)$$
$$\beta^*(u_i v_i) = 3i-2 \qquad ,where\ i = 1,2,\dots,(n-1)$$
$$\beta^*(u_{i+1} v_i) = 3i \qquad ,where\ i = 1,2,\dots,(n-1)$$

Then we attain a dissimilar value for the edges.

Therefore, T_m is said to be a Power-3 Heronian Mean graph.

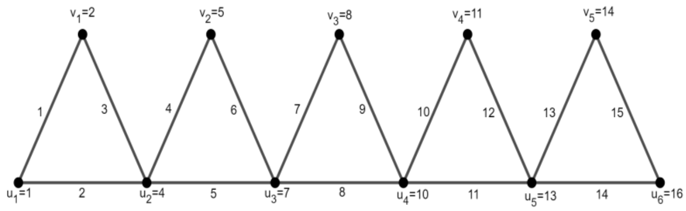

Figure 4: T_5

Theorem: 5

Quadrilateral Snake Q_m is said to be a Power-3 Heronian Mean graph.

Proof:

Assume Q_m be a Quadrilateral Snake. It is attained by attaching every pair of vertices of a path to another two new vertices say v_i and w_i . (i,e.,) we can replace each edge of a P_n by a cyclic graph C_4 . Generally, it has $3n-2$ vertices and $4n-4$ edges.

Now, defining a function $\beta: V(G) \to \{1,2,3,\dots,b+1\}$ by

$$\beta(u_i) = 4i-3 \qquad ,where\ i = 1,2,\dots,n$$
$$\beta(v_i) = 4i-2 \qquad ,where\ i = 1,2,\dots,n$$
$$\beta(w_i) = 4i-1 \qquad ,where\ i = 1,2,\dots,n$$

Then the induced edge labels are given by,

$$\beta^*(u_i u_{i+1}) = 4i-1 \qquad ,where\ i = 1,2,\dots,(n-1)$$
$$\beta^*(u_i v_i) = 4i-3 \qquad ,where\ i = 1,2,\dots,(n-1)$$
$$\beta^*(u_{i+1} v_i) = 4i \qquad ,where\ i = 1,2,\dots,(n-1)$$
$$\beta^*(v_i w_i) = 4i-2 \qquad ,where\ i = 1,2,\dots,(n-1)$$

Then we attain a dissimilar value for the edges.

Therefore, Q_m is said to be a Power-3 Heronian Mean graph.

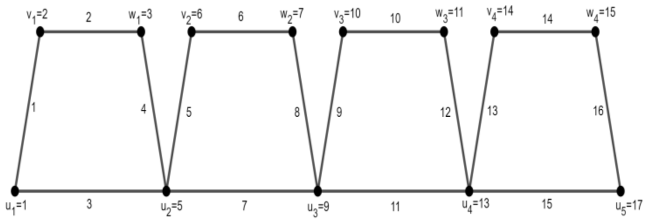

Figure 5: Q_4

Theorem: 6

Ladder L_n is said to be a Power-3 Heronian Mean graph.

Proof:

Assume L_n denote a Ladder graph. Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ be the vertices of two paths having length n in the graph L_n. Join u_i, v_i. Generally, it has $2n$ vertices and $3n-2$ edges.

Now, defining a function $\beta: V(G) \rightarrow \{1,2,3,\dots,b+1\}$ by

$$\beta(u_i) = \begin{cases} 3i-2 & , if \;\; i = 1,3,5,\dots,n \\ 3i-3 & , if \;\; i = 2,4,6,\dots,n \end{cases}$$

$$\beta(v_i) = 3i-1 \qquad , where \;\; i = 1,2,\dots,n$$

Then the induced edge labels are,

$$\beta^*(u_i u_{i+1}) = 3i-1 \qquad , where \;\; i = 1,2,\dots,(n-1)$$

$$\beta^*(u_i v_i) = 3i-2 \qquad , where \;\; i = 1,2,\dots,(n-1)$$

$$\beta^*(v_i v_{i+1}) = 3i \qquad , where \;\; i = 1,2,\dots,(n-1)$$

Then we attain a dissimilar value for the edges.

Therefore, L_m is said to be a Power-3 Heronian Mean graph.

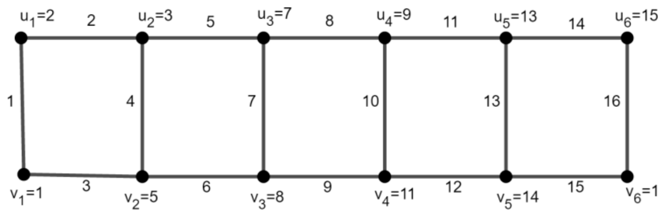

Figure 6: $P_6 \times P_2$

IV. CONCLUSION

In this paper, we had introduced the notion of Power-3 Heronian Mean Labeling and studied for some standard graphs.

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