

# Fractional Order Control Methodology for the Self-Balance Robot.



Sandip A Mehta, Dipak M Adhyaru

**Abstract :** *In this research paper the control algorithms like LQR and PID has been proposed for the integer and fractional order system. In this research paper the modeling of the self-balance robot system has been carried out in integer domain and fractional domain. This research paper presents the simulation analysis of control algorithms for two wheel self-balancing robot using Linear Quadratic Regulator, Proportional-Integral-Derivative and Fractional order Proportional-Integral-Derivative control algorithm. These all control algorithm are applied on the integer order system and the fractional order system and comparative analysis has been done. The comparison between integer order PID against the fractional order PID is also been made for the self-balance robot. It has been demonstrated through simulation that fractional order controller gives better response as compared to integer order controller. Further it has been found out that fractional order controller gives better results when applied to fractional order system compared to its integer order counterpart.*

**Index Terms-** PID, FOPID, Linear quadratic regulator, two wheeled robot, self-balance system.

## I. INTRODUCTION

In modern calculus the differentiation term  $D = (d/dx)$  is a well-known tool for modeling and analysis of many physical phenomenon. In case of integer differentiation the derivative order is in terms of integer i.e. one or two. For second order systems like dc motor, spring mass damper system, the resulting ODE has the derivative of 2. However in fractional order processes, this order is non-integer. i.e. 0.5 or 0.9. Also in classical control theory, the PID is most popular control algorithm in industries. In the integer order PID again derivative and integer order is 1, where as in fractional order it is non integer. Before two decades, the fractional order controller was not popular, mainly because of the implementation issues. However in last two decades the fractional order calculus gain the momentum and proves many advantages in scientific fields. It starts with system modelling, identification and automatic control. The rise of interest to the topic of fractional differentiation is also related to accessibility of more efficient and powerful computational tools. The introduction of computer algebra systems in professional computing tools like MATLAB and Maple led to new possibilities for evaluating the practical aspects of fractional calculus in specific application. For the study purpose, interested reader can refer the tutorial given by [1]

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The classical PID control algorithm have wide acceptance and it is used as a controller in many process industries and dynamical systems. The basic limitations of integer order PID is easily overcome by fractional order PID. Some of the glimpses of fractional PID are found in [2] magnetic bearing system, in [3] for the dc motor control, for structures in [4], and in [5] for air craft pitch control and in [6] for turbine regulation system. The more detailing about the fractional PID methods and tuning part can be found in [7]. For the process control point of view, in [8] the fractional PID is designed for the distillation column. From all these literature review one can say that the fractional order PID control is now applied in almost every engineering field. Fractional PID gives faster response compared to the integer order PID [9–11], however for nonlinear and unstable systems, fractional PID controller tuning is difficult, and hence control algorithms like LQR (Linear Quadratic Regulator)/LQG (Linear Quadratic Gaussian) is used. The fractional LQR is first proposed in [12], where the analytical solution method is proposed to design the fractional order LQR. After that the LQR theory is explored for the civil structure [13]. Here the integer order LQR along with fractional order filters are used for the control purposes. For the discrete time one can study the reference [14]. In [15], the LQR and HJB formulation is done. In this case the author has used simple first order fractional dynamical system. Recently a practical method is proposed for the design of fractional LQR [16] for fractional order system. In this work author has proposed the state feedback control which is equivalent to the integer order LQR formulation and Riccati equation formulation. Based upon the above mentioned literature it is interesting to carry out research related to the effect of LQR algorithm for the fractional order system. This paper is organized as follows: First the system modeling in integer order domain and then in fractional order domain has been presented. After that the LQR (Linear Quadratic Regulator) algorithm is applied to integer order and fractional order system. The LQR algorithm is designed using state feedback control law and Riccati equation. In the last section PID and FOPID algorithm is applied to integer order model and fractional model. For the simulation purpose some of codes available in [17] and in [18].

## II. SYSTEM MODELLING

The self-balancing robot is balancing on the two wheeled and this wheeled are connected with the two DC motor, here the modelling is divided into three parts, first is modelling of DC motor, modelling of two wheeled and modelling of robot pendulum, after the mathematical derivation, apply different law on the system one can get state space model of the self-balancing system, which shown below [19][20].

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Here  $\Phi$  represents the pendulum angle,  $x$  represents the distance or displacement. Further the assumption is made that system is at rest and initial conditions are zero.i.e. $x(0)=0$  and  $\Phi(0) = 0$ .

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\Phi} \\ \ddot{\Phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{2k_m.k_e(M_p l r - M_p l^2)}{R r^2 \alpha} & \frac{M_p^2 g l^2}{\alpha} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2k_m.k_e.(r\beta - M_p l)}{R r^2 \alpha} & \frac{M_p g l \beta}{\alpha} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \Phi \\ \dot{\Phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2k_m(I_p + M_p l^2 - M_p l r)}{R r \alpha} \\ 0 \\ \frac{2k_m(M_p l - r\beta)}{R r \alpha} \end{bmatrix} V_a \dots \quad (1)$$

Substituting the values of  $g = 9.81$ ,  $r = 0.052$ ,  $M_w = 0.04$ ,  $M_p = 1.14$ ,  $I_w = 0.000049$ ,  $I_p = 0.0042$ ,  $l = 0.08$ ,  $K_m = 0.006131$ ,  $K_e = 0.006091$ ,  $R = 4$ ,

One can obtain the following model.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0076 & 13.3230 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.0292 & 183.5182 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0.0650 \\ 0 \\ -0.2491 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

### III. CONVERTING INTEGER ORDER TO FRACTIONAL ORDER SYSTEM

Representation of state space model for integer order system is

$$\dot{x} = Ax + Bu$$

Where  $x(t)$  is state vector,  $x(t) \in \mathbb{R}^n$  is the state vector,  $u(t) \in \mathbb{R}^p$  is the control vector and  $y(t) \in \mathbb{R}^q$  is the output vector.

The dimension of  $A$  is  $n \times n$ , and  $B$  is  $n \times p$ .

Representation of state space model for integer order system is

$$D_x^\alpha = Ax + Bu$$

Fractional order Laplace transform have one properties is

$$L[aD_t^\alpha . F(t)] = s^\alpha L[F(t)] \dots (2)$$

Rearranging above Laplace transform properties

$$[aD_t^\alpha . F(t)] = L^{-1}[s^\alpha L[F(t)]]$$

$$[aD_t^\alpha . F(t)] = L^{-1}[s^\alpha] L^{-1}[L[F(t)]]$$

$$[aD_t^\alpha . F(t)] = L^{-1}[s^\alpha] . [F(t)] \dots (3)$$

As per the Laplace transform properties

$$L^{-1}[s^\alpha] = \frac{\Gamma(\alpha)}{t^{\alpha-1}}$$

Put this above Laplace transform properties into the equation (3)

$$[aD_t^\alpha . F(t)] = \frac{\Gamma(\alpha)}{t^{\alpha-1}} . [F(t)] \dots (4)$$

As per the integer order state space model one can write these two state equation

$$\ddot{x} = -0.0076\dot{x} + 13.3230\Phi + 0.0650V_a \dots (5)$$

$$\ddot{\Phi} = -0.0292\dot{x} + 183.5182\Phi + 0.2491V_a \dots (6)$$

For the fractional state space model next one has to find out  $[aD_t^\alpha . \dot{x}]$  and  $[aD_t^\alpha . \ddot{\Phi}]$  using equation (4), and following fractional state equations are obtained

$$[aD_t^\alpha . \dot{x}] = -0.0076 \frac{\Gamma(\alpha)}{t^{\alpha-1}} \dot{x} + 13.3230 \frac{\Gamma(\alpha)}{t^{\alpha-1}} \Phi + 0.0650 \frac{\Gamma(\alpha)}{t^{\alpha-1}} V_a \dots (7)$$

$$[aD_t^\alpha . \ddot{\Phi}] = -0.0292 \frac{\Gamma(\alpha)}{t^{\alpha-1}} \dot{x} + 183.5182 \frac{\Gamma(\alpha)}{t^{\alpha-1}} \Phi + 0.2491 \frac{\Gamma(\alpha)}{t^{\alpha-1}} V_a \dots (8)$$

From the above two equation (7) and (8) the fractional state space model is obtained as shown below

$$\begin{bmatrix} aD_t^\alpha . x \\ aD_t^\alpha . \dot{x} \\ aD_t^\alpha . \Phi \\ aD_t^\alpha . \ddot{\Phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0076 \frac{\Gamma(\alpha)}{t^{\alpha-1}} & 13.3230 \frac{\Gamma(\alpha)}{t^{\alpha-1}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.0292 \frac{\Gamma(\alpha)}{t^{\alpha-1}} & 183.5182 \frac{\Gamma(\alpha)}{t^{\alpha-1}} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \Phi \\ \dot{\Phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 0.0650 \frac{\Gamma(\alpha)}{t^{\alpha-1}} \\ 0 \\ 0.2491 \frac{\Gamma(\alpha)}{t^{\alpha-1}} \end{bmatrix} v_a \dots (9)$$

Here it is assumed that fractional order parameter may be related to motor dynamics like motor damping coefficient or ageing effect of the motor components. Fractional order model can also be attributed to battery voltage ripples or the high frequency noise impulses which is not previously considered in the integer order state space representation.

### IV. RESULTS FOR LQR, PID AND FOPID CONTROL ALGORITHM

This section is divided into two parts where in the first part LQR algorithm is applied to integer order system and then it is applied to fractional order system.

**A. Applying LQR control on integer order system**

The LQR control is a well-known state space technique for designing robust and optimal regulators. LQR control refers only linear system and quadratic performance index as per the,[8]

$$\dot{x}(t) = A \cdot x(t) + B \cdot u(t) \dots (10)$$

$$X(0) = X0 \dots (11)$$

Here below equation (12) is representing integral performance index

$$J = \frac{1}{2} \int_0^{\infty} [x'(t)Qx(t) + U'(t)RU(t)]dt \dots (12)$$

Linear quadratic regulator control law is specified as

$$u = -R^{-1}B'P \bar{x} \dots (13)$$

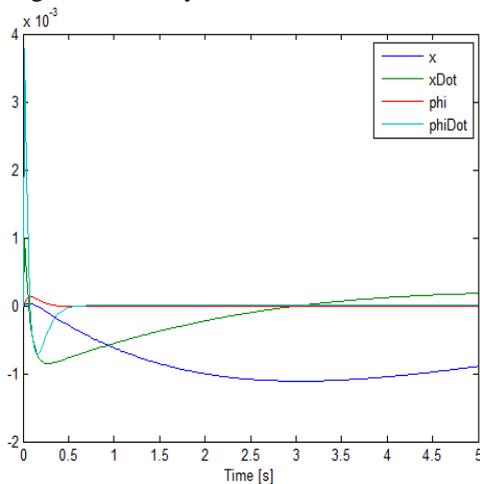
Where  $P=P' \geq 0$  solve the following algebraic Ricatti equation

$$0 = PA + A'P - PBR^{-1}B'P + Q \dots (14)$$

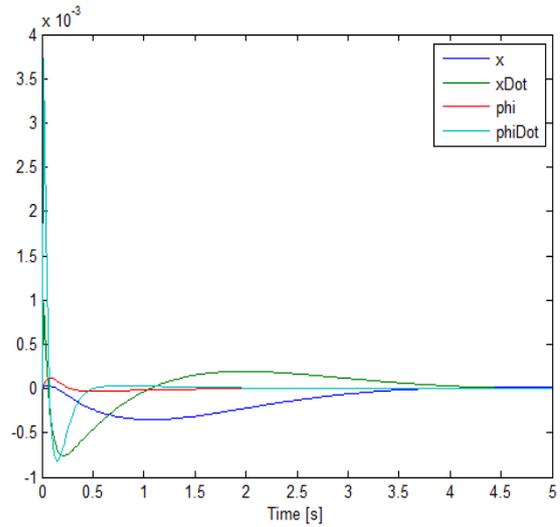
Here in LQR control have one gain vector is K, and its determine some amount of feedback into the system and  $K = R^{-1}B'P$ . Here for LQR have two another tuning parameters are Q and R, and its value are always positive, Q matrix value is depend on the size of the system state matrix, and R matrix size depend on the system's control input.

**A. Results for LQR control on fractional order system**

For position weighting 1000, pendulum's angle weighting 5000 and R=1 the obtained response is shown in Fig. 1. This value Q entries and R entries are obtained using hit and try method. Reducing the weighing either in position or pendulum angle weighing results in unstable operation and also it has been observed that by keeping the pendulum angle fixed, and decreasing the position weighing allowing the distance (x) value to be settled out less quickly than for the current value of the system. Here it should be note dthat Q and R matrix parameter is tuned using hit and try method but it can be also tuned using GA or PSO algorithm. Fig. 2. Shows the response of the system with high weighting of control input R and value of R is 100. One can see here the less motor control operation[5]. Hence it has been concluded that increasing the R value will make system to took more time to stabilize than the R=1. Further it can be deduced from figure 2 that as R value increased, the settling time of the system is also increased.

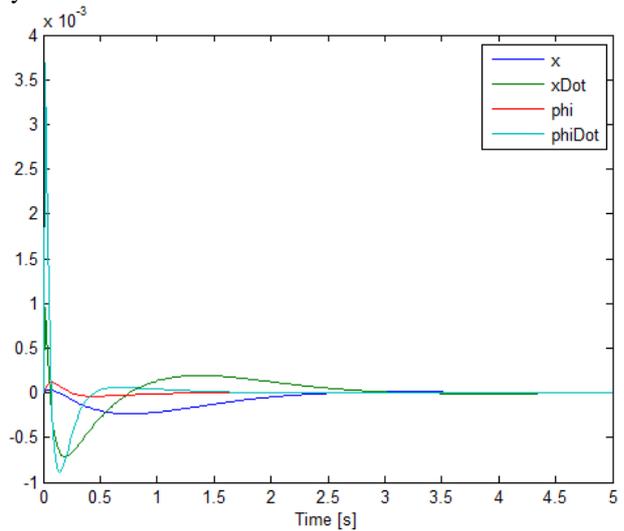


**Fig. 1. Response of the state with appropriate weighting**



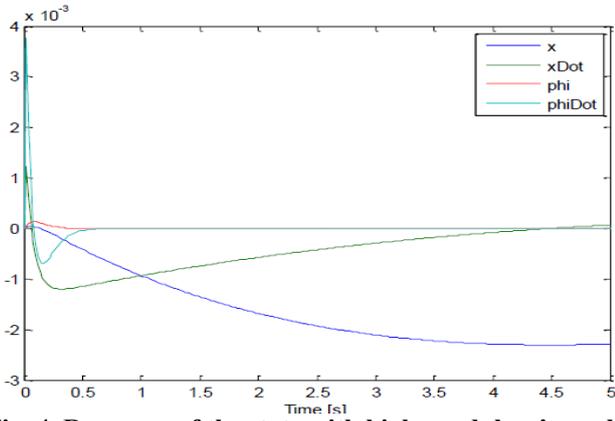
**Fig. 2. Response of the state with high R weighting**

Fig. 3. Shows the response with high value of position, one can note that the settling time is very less to every state response, but motor will not get appropriate response because motor required maximum torque to balance the system.

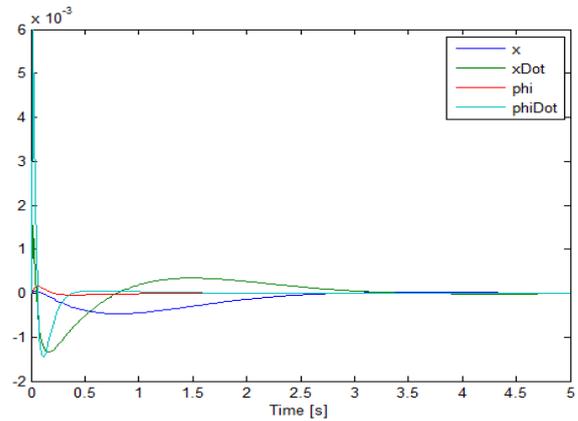


**Fig. 3. Response of the state with high position weighting**

Fig. 4. Shows the response with high value of pendulum's angle, here when while increasing weighting of pendulum's angle then one has to compromise with response of the state and settling time. In practical scenario this leads to continuous forward and backward movement of robot to maintain the balance of the system which may leads to instability.



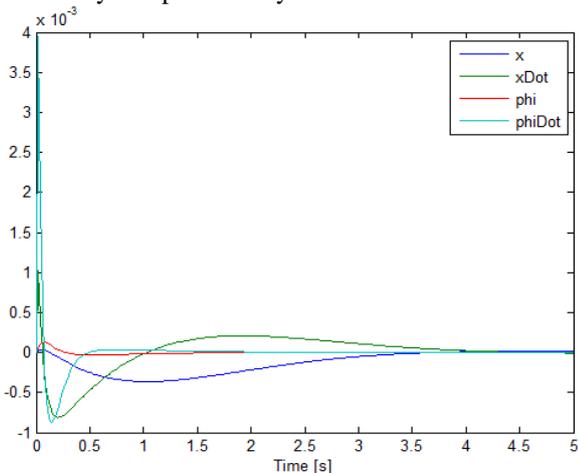
**Fig. 4. Response of the state with high pendulum's angle weighting**



**Fig. 6. Response of the state for  $\alpha=0.5$**

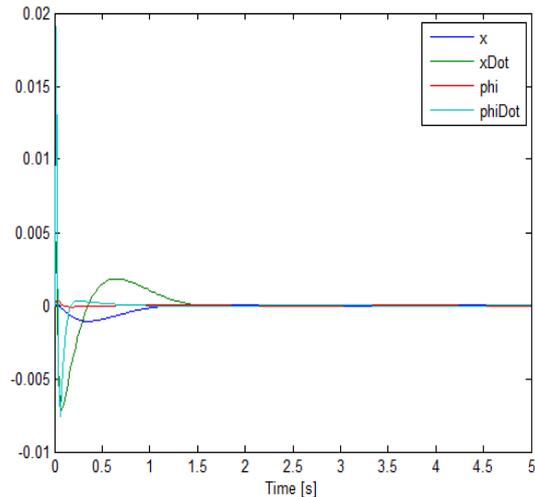
## B. Results for LQR control on fractional order system

Using the equation (45) one can apply the LQR control algorithm for the fractional controller as well. Here one has to the three different parameters for tuning, like Q and R are same as integer order system and one new parameter is  $\alpha$ , in commensurate order fractional model,  $\alpha$  varies between 0 to 1. Fig. 5. Shows the response of the state for fractional model where  $\alpha=0.9$ , here response is good but maximum peak overshoot and settling time is more. For the practical setup point of view, this behavior is desirable as most of the auto balance system have very narrow control span i.e.  $0 < \Phi < 5^\circ$ . So initial overshoot positive or negative direction can be easily compensated by the robot movement.



**Fig. 5. Response of the state for  $\alpha=0.9$**

Fig. 6. Shows the response of the state for fractional model where  $\alpha=0.5$ , here settling time is less compare to above response but peak overshoot is more. Fig. 7. Shows the response of the state for fractional model where  $\alpha=0.1$ , here system is more faster and less peak overshoot compared to above two response, it means when  $\alpha$  is nearest to 0 then good response resulted, but here when system become faster that time high torque is required for self -balancing system. This can also be visualized as because of lower fractional order the system has more disturbance, oscillations and ripple voltage variations and to overcome this effect more robust output is required.



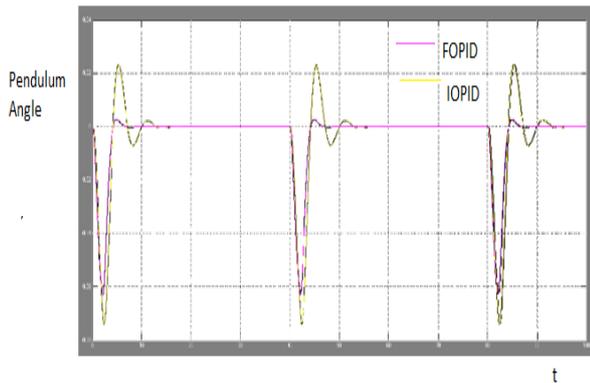
**Fig. 7. Response of the state for  $\alpha=0.1$**

## C. Applying PID and FOPID control algorithm on integer and fractional order system.

For applying PID and FOPID control algorithm on the system one needs to carried out the modelling into the MATLAB Simulink environment[6]. As per the state equation of the integer model and fractional model, the Simulink based simulation has been carried out.

Here the PID controller has been applied on integer order system and FOPID control on fractional order system, closed loop model of the system and afterwards the comparative analysis has been carried out.

Response of the system which shown in Fig. 8. Where yellow line is for the PID control response and pink line is for the FOPID control response, from this response one can observe that FOPID have less oscillation, faster stability and less peak overshoot compare to PID control algorithm.



**Fig. 9 Response of PID and FOPID for pendulum angle for self balance robot**

### V. CONCLUSION

In this research paper the PID and FOPID algorithm for the self-balance robot system is successfully applied. For the novelty purpose, modeling of the self-balance robot is been carried out in fractional domain using the assumption of commensurate order fractional order system. Also along with the analytical part the simulation for the LQR control algorithm is been proposed and successfully carried out. After applying PID and FOPID control algorithm on the self-balancing system it can be concluded that the FOPID controller response is better than PID control algorithm, also FOPID have more stability, less oscillation and gives faster response compare to PID control algorithm. While applying the LQR control algorithm for fractional order system, it has been noted that response of the system is governed by the alpha value. When alpha value is increased, the system become slower and when alpha decreases, the system response become faster. Further it has been concluded that LQR design provides faster response for the fractional order representation than the integer order representation.

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