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Power of 2 Decomposition of a Complete Tripartite Graph $K_{2,4,M}$ and a Special Butterfly Graph

V. G. Smilin Shali, S. Asha

Abstract: *Let G be a finite, connected simple graph with p vertices and q edges. If $G_1, G_2, \ldots, G_n$ are connected edge-disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $\{G_1, G_2, \ldots, G_n\}$ is said to be a decomposition of G. A graph G is said to have Power of 2 Decomposition if G can be decomposed into edge-disjoint subgraphs $\{G_2, G_4, \ldots, G_{2^n}\}$ such that each G_{2^i} is connected and $|E(G_i)| = 2^i$, for $1 \le i \le n$. Clearly, $q = 2[2^n - 1]$. In this paper, we investigate the necessary and sufficient condition for a complete tripartite graph $K_{2,4,m}$ and a Special Butterfly graph $BF_{\left[\frac{2^{2m+1}-5}{3}\right]}$ to accept Power of 2 Decomposition.*

Keywords : Decomposition of Graph, Power of 2 Decomposition, Complete tripartite graph, Special Butterfly graph.

I . INTRODUCTION

Let G be a simple, connected graph with p vertices and q edges. If $G_1, G_2, \ldots, G_n$ are connected edge-disjoint subgraphs of G with $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $\{G_1, G_2, \ldots, G_n\}$ is said to be a Decomposition of G. Different type of decomposition of G have been studied in the literature by imposing suitable conditions on the subgraphs G_i. In this paper, we investigate the necessary and sufficient condition for a Complete tripartite graph $K_{2,4,m}$ and a Special Butterfly graph to accept a new type of decomposition called Power of 2 Decomposition. Terms not defined here are used in the sense of Harary [2] .

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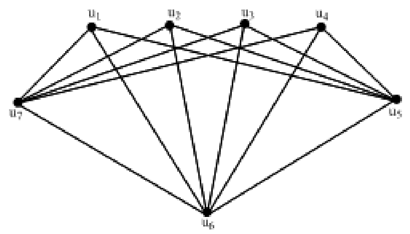
Definition 1.1. Let G be a simple graph of order p and size q. If $G_1, G_2, \ldots, G_n$ are edge-disjoint subgraphs of G such that $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $\{G_1, G_2, \ldots, G_n\}$ is said to be a Decomposition of G.

II. POWER OF 2 DECOMPOSITION OF GRAPHS

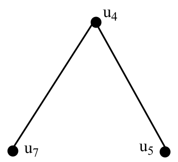
Definition 2.1. A graph G is said to have Power of 2 Decomposition [Po2D] , if G can be decomposed into n subgraphs $\{G_1, G_2, \ldots, G_n\}$ such that each G_i is connected and $|E(G_i)| = 2^i$, for $1 \le i \le n$. Clearly $q = 2[2^n-1]$ is the sum of $2, 2^2, 2^3, \ldots, 2^n$. Thus, we denote the Power of 2 Decomposition as $\{G_2, G_4, \ldots, G_{2^n}\}$.

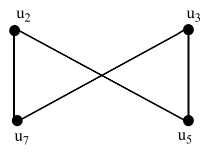
Example 2.2. Consider the graph.



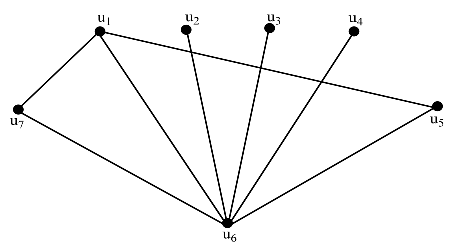
$K_{1,2,4}$ [(4,3) – Fan graph]

Complete tripartite graph $K_{1,2,4}$ admits Power of 2 Decomposition $\{G_2, G_4, G_8\}$ as follows.

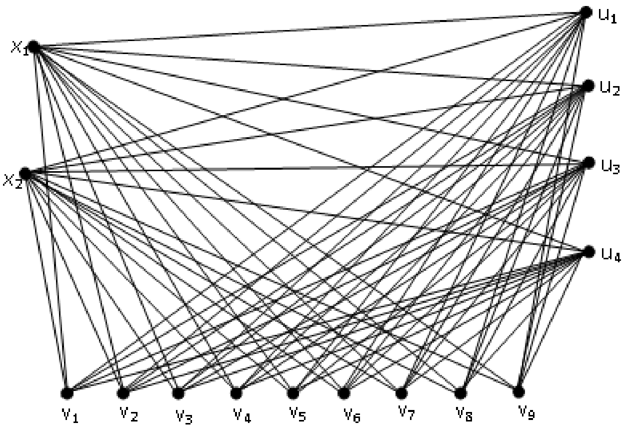




G_2 G_4



G_8



$K_{2,4,9}$

Theorem 2.3. A graph G admits Power of 2 Decomposition [Po2D] { $G_2, G_4, \ldots, G_{2^n}$} if and only if $q = 2[2^n-1]$ for each $n \in N$.

III. POWER OF 2 DECOMPOSITION OF $K_{2,4,m}$

Definition 3.1. A Complete tripartite graph is a tripartite graph whose vertices are decomposed into three disjoint sets such that no two vertices with in the same set are adjacent but every pair of vertices in the three sets are adjacent. A Complete tripartite graph is denoted as $K_{a,b,c}$ where a, b and c are three disjoint set of vertices of the graph.

Theorem 3.2. For an odd integer m, $K_{2,4,m}$ has a Power of 2 Decomposition $\{G_2, G_4, \ldots, G_{2^n}\}$ [n-decompositions] if and only if there exists an integer n satisfying the following properties.

1. $n = 2r + 1, r \geq 1$ and $r \in Z$.
2. $2^n = 5 + 3m$

Proof. Let $G = K_{2,4,m}$. Assume that G has Power of 2 Decomposition $\{G_2, G_4, G_8, \ldots, G_{2^n}\}$. By the definition, $q = 2[2^n - 1]$, where n denotes the total number of decompositions. By the definition of G, q = 8 + 6m. Hence $2[2^n - 1] = 8 + 6m$. This implies $2^n = 5 + 3m$. Since 2^n is even, 5 + 3m is even. Hence m is odd.

Case 1. n is even .

Then 5 + 3m should be a power value of 4. This is notpossible for any odd integer m. Hence n cannot be even.

Case 2. n is odd.

Clearly 5 +3m should be written as a power value of 2, as 5 + 3m is even. Hence n is odd.

Thus $n = 2r + 1, r \geq 1$ and $r \in Z$.

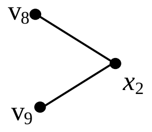
Conversely, assume that $n = 2r + 1, r \geq 1$ and $r \in Z$. Also, $2^n = 5 + 3m$. Hence m is odd. By the definition of G, q = 8 + 6m. This implies q =

$$8 + 6\left[\frac{2^n - 5}{3}\right] = 8 + 2[2^n - 5] = 2[2^n - 1].$$

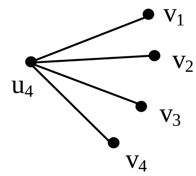
Since $q = 2[2^n - 1]$, G can be decomposed into $\{G_2, G_4, \ldots, G_{2^n}\}$. Hence G admit Power of 2 Decomposition.

Illustration 3.3. As an illustration, let us decompose the Complete tripartite graph $K_{2,4,9}$.

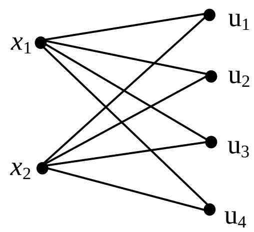
Let $G = K_{2,4,9}$. Here m = 9. Hence $2^n = 32$. Thus n = 5. Hence there will be 5 decompositions. The Power of 2 Decomposition of G is $\{G_2, G_4, G_8, G_{16}, G_{32}\}$ and is given as follows.



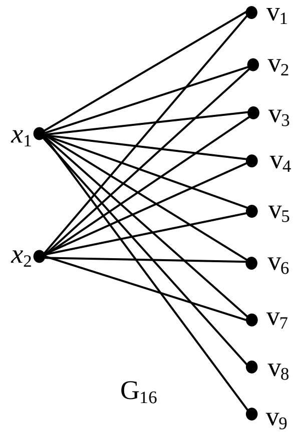
G_2



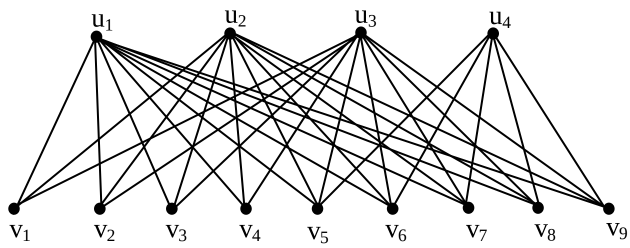
G_4



G_8



G_{16}



G_{32}

Table 3.4. List of first 10 $K_{2,4,m}$'s which accept Power of 2 Decomposition and their decompositions are given in the following table.

m	Power of 2 Decompositions
1	$G_2, \ldots, G_8$
9	$G_2, \ldots, G_{32}$
41	$G_2, \ldots, G_{128}$
169	$G_2, \ldots, G_{512}$
681	$G_2, \ldots, G_{2048}$
2729	$G_2, \ldots, G_{8192}$
10,921	$G_2, \ldots, G_{32768}$
43,689	$G_2, \ldots, G_{131072}$
174,761	$G_2, \ldots, G_{524288}$
699,049	$G_2, \ldots, G_{2097152}$

IV. POWER OF 2 DECOMPOSITION OF SPECIAL BUTTERFLY GRAPH

Definition [5] 4.1. Consider the cycle C_{2t+2}, $t \geq 4$. Let a_0, a_1a_2, . . . ,a_{2t+1} be the vertices of the cycle C_{2t+2}. Join a_0 and a_{2i-1}, $2 \leq i \leq t$. Attach two pendant edges at a_t and a_{t+2}. Let the vertices attached with those edges be a_{2t+2}, a_{2t+3} respectively. This is called the Special Butterfly Graph and it is denoted by BF_t. The graph BF_t has 2t + 4 vertices and 3t + 3 edges.

Theorem 4.2. A Special Butterfly Graph $BF_{\left[\frac{2^{2m+1}-5}{3}\right]}$ admit Power of 2 Decomposition $\{G_2, G_4, \ldots, G_{2^n}\}$ if and only if n = 2m , m $\in$ N.

Proof. Assume that G = $BF_{\left[\frac{2^{2m+1}-5}{3}\right]}$ admit Power of 2 Decomposition $\{G_2, G_4, \ldots, G_{2^n}\}$. By the definition of G, $q = 3\left[\frac{2^{2m+1}-5}{3}\right] + 3 = 2^{2m+1} - 2$. Since G admit Power of 2 Decomposition, q = $2[2^n - 1]$. Hence $2^{2m+1}-2 = 2[2^n-1]$. This implies n = 2m.

Conversely, assume that n = 2m, m $\in$ N. Let G = $BF_{\left[\frac{2^{2m+1}-5}{3}\right]}$. Then q(G) = 2^{2m+1} - 2. This implies q = $2[2^n - 1]$. Decompose the edges of G as follows:

$G_2 = a_{t+2}\, a_{t+3} \cup a_{t+3}\, a_{t+4}$,

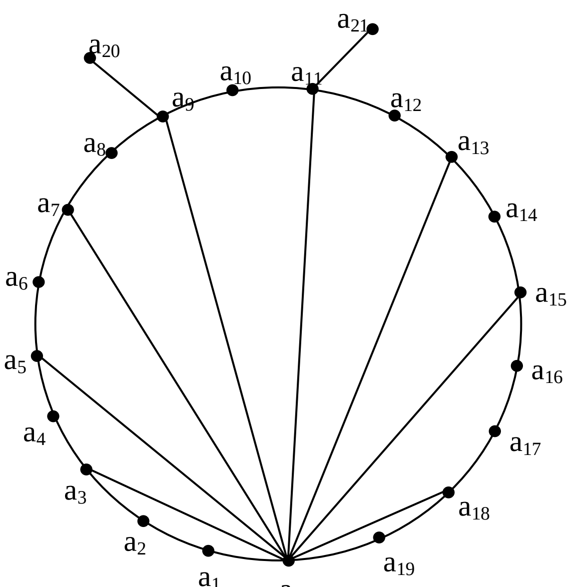
$G_4 = a_{2t+2}\, a_t \cup a_t\, a_{t+1} \cup a_{t+1}\, a_{t+2} \cup a_{t+2}\, a_{2t+3}$,

$G_8, G_{32}, \ldots, G_{2^{n-1}} = \cup\, a_0\, a_{2i-1}$, $2 \leq i \leq t$ and

$G_{16}, G_{64}, \ldots, G_{2^n} = a_0\, a_{2t+1} \cup a_{i-1}\, a_i$, $1 \leq i \leq t$ and $t + 5 \leq i \leq 2t + 1$

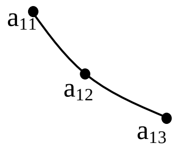
Thus G can be decomposed using Power of 2 Decomposition.

Illustration 4.3. As an illustration, let us decompose the Special Butterfly Graph BF_9.

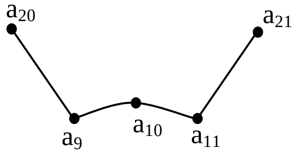


Let G = BF_9. Here t = 9 = $\frac{2^{4+1}-5}{3}$. Hence 2m = n = 4.

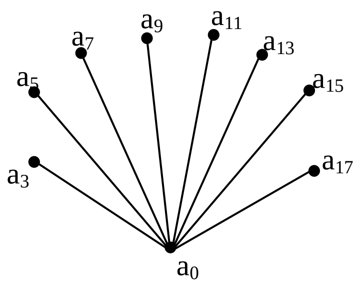
There will be 4 decompositions $\{G_2, G_4, G_8, G_{16}\}$. The Power of 2 Decomposition of BF_9 is given below.



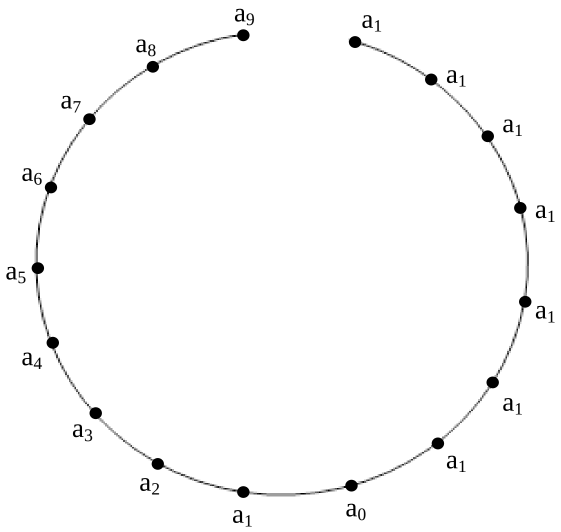
G_2



G_4



G_8



G_{16}

Table 4.4. List of first 10 Special Butterfly Graphs $BF_{\left[\frac{2^{2m+1}-5}{3}\right]}$ which accept Power of 2 Decomposition and its decompositions are given in the following table.

m	Power of 2 Decompositions
1	G_2, G_4
2	G_2, G_4, G_8, G_{16}
3	$G_2, \ldots, G_{64}$
4	$G_2, \ldots, G_{256}$

5	$G_2, \ldots, G_{1024}$
6	$G_2, \ldots, G_{4096}$
7	$G_2, \ldots, G_{16384}$
8	$G_2, \ldots, G_{65536}$
9	$G_2, \ldots, G_{262144}$
10	$G_2, \ldots, G_{1048576}$

V . CONCLUSION

Thus we had investigated the properties of two special graphs to accept a new type of graph decomposition called Power of 2 Decomposition.Also we can extend the study to various types of special graphs.

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