International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-9 Issue-3, February 2020

Stochastic Modelling of an Aircraft Considering the Possibility of Precautionary Landing Due to Fuel - Filter Clogging

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***Abstract*: The present work deals with the stochastic and cost - benefit analysis of a helicopter taking into account the situation of precautionary landing caused by blockage in its fuel - filter. The system has been analysed by developing a model and finding the various indices of system effectiveness like mean time to filter clogging, expected up (flying) time, expected number of precautionary landings etc. The regenerative point technique has been used for the purpose. The obtained measures have been further used to analyse the profit generated by the system. Graphical study of the proposed model has also been done. The suggested methodology finds its application in commercial aviation sector.**

***Keywords*: Helicopter, Precautionary Landing, Fuel - Filter, Clogging, Regenerative Point Technique**

I. INTRODUCTION

Over the past few decades, the researchers in the field of reliability modelling have done a lot of research work in a variety of areas and the aviation industry is not an exception in this regard. Many researchers, including [1] – [11], have contributed to the field of aviation research by considering different aircrafts and the systems/devices necessary for adequate functioning of aircrafts. Bigel and Winsten [1] presented an historical overview of the reliability and maintainability growth of F-14A fighter aircraft. Gai et al. [2] analysed the reliability of a dual-redundant engine controller. In [3] and [5], authors modelled and analysed a two engine aeroplane and a GIV Gulf Stream aircraft respectively, using regenerative point technique. Jenab and Rashidi [4] assessed the operational reliability of an aircraft environment control system. Further, the aircraft fuel system and the situation of fuel contamination has also been studied by the researchers [6] – [8]. Besides the above, research work [9] – [12] has been carried out on helicopters considering various aspects and using different methodologies. However, the situation of precautionary landing of a helicopter due to fuel - filter clogging, which has not been analysed in earlier studies, also needs to be considered.

A single engine helicopter, with focus on its fuel- filter and bypass valve, has been taken into account in the present study. Engine is the most vital organ of any aircraft which provides the necessary power / thrust to execute a flight. For the engine to keep generating power during a flight, adequate fuel supply to it is quite essential. The fuel from the tank is passed through fuel - filter before entering the engine, in order to ensure that the fuel supplied to the engine is free from contaminants / foreign particles. Another component that forms an integral part of the fuel supply system is the bypass valve which allows the unfiltered fuel to bypass the filter, whenever filter gets clogged. Designing of the bypass valve is such that it opens as and when the pressure of accumulated oil (due to blockage in the filter) reaches a certain limit, thereby preventing fuel starvation. Impending bypass is indicated to the pilot, however the flight is continued as per schedule and helicopter is put under applicable maintenance after landing. In case, the bypass valve gets open during the flight, there is an indication of the same and in order to avoid further damage, precautionary landing is done as soon as practical. After such landing the helicopter is put under the corrective action (as per maintenance manual) for which there is provision of regular maintenance facility.

II. MATERIALS AND METHODS

Different performability measures of the studied system have been obtained using regenerative point technique, Laplace transform / convolution and Laplace - Stieltjes transform / convolution. Profit analysis of the system has also been done using the obtained measures. In addition, for calculation work and graphical analysis, software like MATLAB, Mathematica and Excel have been used.

III. ASSUMPTIONS

- Initially the helicopter is during flight.
- The situations of impending bypass as well as clogged filter are precisely indicated and bypass valve opens whenever the filter gets clogged.
- Repair / Maintenance work is not feasible during flight and is done only after landing.

Revised Manuscript Received on February 05, 2020.
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- Precautionary landings are caused by blockage of fuel -filter only (other faults /causes have not been considered in the proposed model). Such landings are followed by inspection (enquiry), under the supervision of concerned authority, to find out the exact cause of problem and actions to be taken accordingly.
- Inspection time includes the time taken by concerned personnel to reach the place of landing.
- Idle time includes the time to carry the helicopter (after maintenance), from place of precautionary landing to the hangar.
- Distribution of flying time till the fuel - filter gets blocked / bypassed, is exponential distribution and all the other distributions are general distributions.

IV. NOTATIONS AND SYMBOLS

λ_1	rate at which impending bypass is indicated.
λ_2	rate at which filter gets bypassed (clogged) after the indication of impending bypass
i(t) / I(t)	p.d.f. / c.d.f. of inspection (enquiry) time
$g_1(t)$ / $G_1(t)$	p.d.f. / c.d.f. of time for which helicopter remains under maintenance following the indication of impending bypass.
$g_2(t)$ / $G_2(t)$	p.d.f. / c.d.f. of time for which helicopter remains under maintenance after filter gets clogged (bypassed).
$h_1(t)$ / $H_1(t)$	p.d.f. /c.d.f. of time required to complete the scheduled flight, after the indication of impending bypass
$h_2(t)$ / $H_2(t)$	p.d.f. /c.d.f. of that lapses before precautionary landing is done after fuel - filter gets bypassed
a(t) /A(t)	p.d.f. /c.d.f. of scheduled flight duration
b(t) / B(t)	p.d.f. /c.d.f. of the time for which the operable helicopter remains Idle (because of no demand, unfavorable flying conditions or being involved in scheduled maintenance /overhaul)
d(t)/D(t)	p.d.f. /c.d.f. of time taken for preflight (or turnaround) maintenance
$FC_i(t)$	c.d.f. of the first passage time from regenerative 'i' to the state of filter clogging
$FT_i(t)$ / $ID_i(t)$	probability that the helicopter is in upstate (on flight) / idle state at instant t, given that the system entered regenerative state 'i' at t = 0
$MI_i(t)$	probability that, at instant t, the system is under applicable maintenance following the indication of impending bypass, given that the system entered regenerative state 'i' at t = 0
$MC_i(t)$	probability that, at instant t, the system is under applicable maintenance following fuel - filter clogging, given that system entered regenerative state 'i' at t = 0
$PL_i(t)$	the expected number of precautionary landings in the time interval (0, t], given that the system entered regenerative state 'i 'at t = 0
$S_i(t)$	probability that system sojourns in state 'i' up to time t.
P_0	profit per hour yielded by the system
R_f	revenue per flight hour
C_f	direct operating cost per flight hour (expenses on fuel, lubricant, scheduled maintenance /overhaul etc.)
C_{ib} / C_{cl}	cost per hour of applicable maintenance following the indication of impending bypass / fuel – filter clogging
C_o	indirect operating cost per hour (expenses on staff salaries, insurance premium, regular maintenance facility, office/ hanger rent, and taxes etc.)
L	goodwill loss and expenses (other than expenses on maintenance) associated per precautionary landing
*/ **	symbol for Laplace/ Laplace - Stieltjes transform
© / ®	symbol for Laplace Convolution/ Laplace - Stieltjes Convolution
F_n / F_{ib} / F_c	fuel - filter operating normally/ about to be bypassed /after getting clogged, during flight
F_{ibm}	fuel-filter under applicable maintenance following the indication of impending bypass
F_{ci} / F_{cm}	fuel - filter under inspection /applicable maintenance after getting bypassed
F_{id} / F_{pf}	fuel - filter during idle / pre-flight state of helicopter
V_s / V_o	bypass valve in standby / operative (open) mode
V_i / V_m	bypass valve during inspection/ maintenance of system
V_{id} / V_{pf}	bypass valve during idle / pre-flight state of helicopter
H_f /H_{Id} / H_{pf}	helicopter during flight/ in idle state / in preflight state
H_i / H_m	helicopter under inspection / applicable maintenance

V. STATE TRANSITION DIAGRAM

Figure 1 is the state transition diagram depicting the different state transitions. The entry points into the states 'i' (i = 0 to 7) are regenerative, thus all the states are regenerative. The states S_0, S_1 and S_4 represent the helicopter during flight.

Retrieval Number: C6396029320 /2020©BEIESP
DOI: 10.35940/ijeat.C6396.029320
Journal Website: www.ijeat.org

Published By:
Blue Eyes Intelligence Engineering
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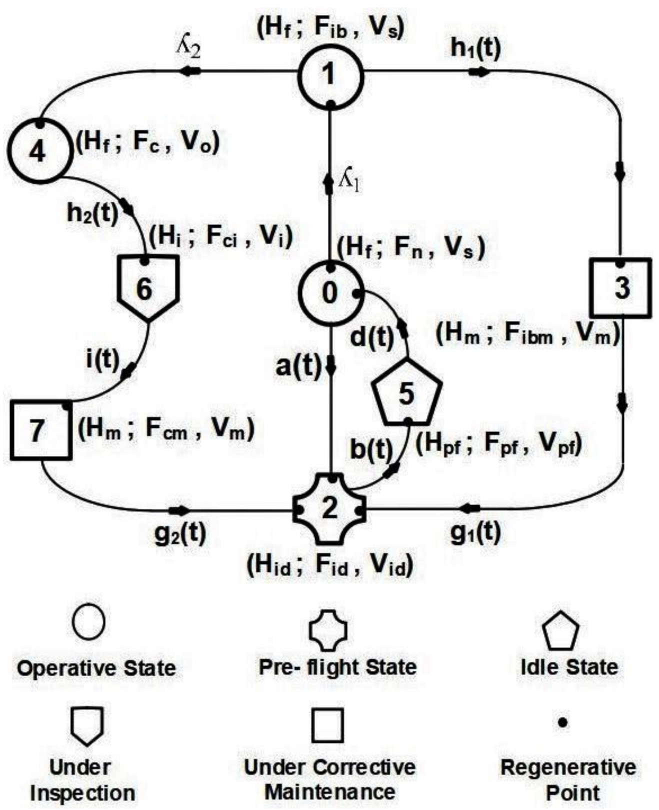

Fig. 1. State transition diagram

VI. TRANSITION TIMES AND PROBABILITIES

Applying the mathematical arguments, p.d.f. of transition times i.e. $q_{ij}(t)$ are given by:

$q_{01}(t) = \lambda_1 e^{-\lambda_1 t}\ \overline{A(t)},$
$q_{02}(t) = e^{-\lambda_1 t}\ a(t),$
$q_{13}(t) = e^{-\lambda_2 t}\ h_1(t),$
$q_{14}(t) = \lambda_2 e^{-\lambda_2 t}\ \overline{H_1(t)},$
$q_{25}(t) = b(t),$
$q_{32}(t) = g_1(t),$
$q_{46}(t) = h_2(t),$
$q_{50}(t) = d(t),$
$q_{67}(t) = i(t),$
$q_{72}(t) = g_2(t).$

(1) - (10)

Using, $p_{ij} = q_{ij}^{*}(0)$, transition probabilities (p_{ij}) are given by:

$p_{01} = 1 - a^{*}(\lambda_1),$
$p_{02} = a^{*}(\lambda_1),$
$p_{13} = h_1^{*}(\lambda_2),$
$p_{14} = 1 - h_1^{*}(\lambda_2),$
$p_{25} = p_{32} = p_{46} = p_{50} = p_{67} = p_{72} = 1.$

(11) - (15)

It follows from the equations (11) to (14) that:

$$p_{01} + p_{02} = p_{13} + p_{14} = 1$$

(16)

VII. SOJOURN TIMES

As defined in [5], following expressions for mean sojourn times (μ_i) and contribution to mean sojourn times (m_{ij}) have been obtained:

$\mu_0 = \{1 - a^{*}(\lambda_1)\}/\lambda_1,$
$\mu_1 = \{1 - h_1^{*}(\lambda_2)\}/\lambda_2,$
$\mu_2 = -b^{*\prime}(0),$
$\mu_3 = -g_1^{*\prime}(0),$
$\mu_4 = -h_2^{*\prime}(0),$
$\mu_5 = -d^{*\prime}(0),$
$\mu_6 = -i^{*\prime}(0),$
$\mu_7 = -g_2^{*\prime}(0),$
$m_{01} = a^{*\prime}(\lambda_1) + \{1 - a^{*}(\lambda_1)\}/\lambda_1,$
$m_{02} = -a^{*\prime}(\lambda_1),$
$m_{13} = -h_1^{*\prime}(\lambda_2),$
$m_{14} = h_1^{*\prime}(\lambda_2) + \{1 - h_1^{*}(\lambda_2)\}/\lambda_2,$
$m_{25} = -b^{*\prime}(0),$
$m_{32} = -g_1^{*\prime}(0),$
$m_{46} = -h_2^{*\prime}(0),$
$m_{50} = -d^{*\prime}(0),$
$m_{67} = -i^{*\prime}(0),$
$m_{72} = -g_2^{*\prime}(0).$

(17) - (34)

From the above expression, it follows that:

$m_{01} + m_{02} = \mu_0,$
$m_{13} + m_{14} = \mu_1,$
$m_{25} = \mu_2,$
$m_{32} = \mu_3,$
$m_{46} = \mu_4,$
$m_{50} = \mu_5.$
$m_{67} = \mu_6,$
$m_{72} = \mu_7.$

(35) - (42)

VIII. MEASURES OF SYSTEM EFFECTIVENESS

A. Mean Time to Filter Clogging (MTFC)

Reckoning the state of filter clogging, i.e. State'4', as the absorbing state, we get following recursive relations for $FC_i(t)$:

$FC_0(t) = Q_{01}(t) Ⓢ FC_1(t) + Q_{02}(t) Ⓢ FC_2(t)$
$FC_1(t) = Q_{13}(t) Ⓢ FC_3(t) + Q_{14}(t)$
$FC_2(t) = Q_{25}(t) Ⓢ FC_5(t)$
$FC_3(t) = Q_{32}(t) Ⓢ FC_2(t)$
$FC_5(t) = Q_{50}(t) Ⓢ FC_0(t)$

(43) - (47)

Taking Laplace - Stieltjes transform of equations (43) – (47) and solving for $FC_0^{**}(s)$, we have

$$FC_0^{**}(s) = N_1(s)/D_1(s)$$

where, $N_1(s) = q_{01}^{*}(s)\, q_{14}^{*}(s)$

$D_1(s) = 1 - \{q_{01}^{*}(s)\, q_{13}^{*}(s) q_{32}^{*}(s) + q_{02}^{*}(s)\}\, q_{25}^{*}(s)\, q_{50}^{*}(s)$

(48) - (50)

The mean time to filter clogging (MTFC), when the system starts from state '0', is given by

$$MTFC = \lim_{s \to 0}[\{1 - FC_0^{**}(s)\}/s] = N_1/D_1$$

where,

$N_1 = \mu_0 + p_{01}\mu_1 + (p_{01}\, p_{13} + p_{02})(\mu_2 + \mu_5) + p_{01}\, p_{13}\, \mu_3$
$D_1 = p_{01}\, p_{14}$

(51) - (53)

B. Expected Uptime / Flying Time

Using the arguments of probability theory and Laplace convolution, we get:

$$FT_0(t) = S_0(t) + q_{01}(t) \copyright FT_1(t) + q_{02}(t) \copyright FT_2(t)$$
$$FT_1(t) = S_1(t) + q_{13}(t) \copyright FT_3(t) + q_{14}(t) \copyright FT_4(t)$$
$$FT_2(t) = q_{25}(t) \copyright FT_5(t)$$
$$FT_3(t) = q_{32}(t) \copyright FT_2(t)$$
$$FT_4(t) = S_4(t) + q_{46}(t) \copyright FT_6(t)$$
$$FT_5(t) = q_{50}(t) \copyright FT_0(t)$$
$$FT_6(t) = q_{67}(t) \copyright FT_7(t)$$
$$FT_7(t) = q_{72}(t) \copyright FT_2(t)$$

(54) - (61)

where,

$$S_0(t) = e^{-\lambda_1 t}\ \overline{A(t)},\ S_1(t) = e^{-\lambda_2 t}\ \overline{H_1(t)},\ S_4(t) = \overline{H_2(t)}$$

(62) - (64)

Following equation is obtained by taking Laplace transform of equations (54) – (61) and solving for $FT_0^*(s)$:

$$FT_0^*(s) = \mathcal{N}_2(s)/\mathcal{D}_2(s)$$

where,

$$\mathcal{N}_2(s) = S_0^*(s) + q_{01}^*(s)\, S_1^*(s) + q_{01}^*(s) q_{14}^*(s) S_4^*(s)$$
$$\mathcal{D}_2(s) = 1 - \{q_{01}^*(s) q_{13}^*(s) q_{32}^*(s) + q_{01}^*(s) q_{14}^*(s)$$
$$q_{46}^*(s)\, q_{67}^*(s) q_{72}^*(s) + q_{02}^*(s)\}\, q_{25}^*(s)\, q_{50}^*(s)$$

(65) - (67)

In steady–state, the fraction of time for which the helicopter is on flight, is given by

$$FT_0 = \lim_{t\to\infty} FT_0(t) = \lim_{s\to 0} s\ FT_0^*(s) = \mathcal{N}_2/\mathcal{D}_2$$

where,

$$\mathcal{N}_2 = \mu_0 + p_{01}\,\mu_1 + p_{01}\, p_{14}\,\mu_4$$
$$\mathcal{D}_2 = \mu_0 + p_{01}\,\mu_1 + \mu_2 + p_{01}\, p_{13}\,\mu_3 + \mu_5$$
$$+ p_{01}\, p_{14}\,(\mu_4 + \mu_6 + \mu_7)$$

(68) - (70)

C. Expected Time for Which the Operable Helicopter remains Idle (non-operational)

Proceeding as above, we have:

$$ID_0(t) = q_{01}(t) \copyright ID_1(t) + q_{02}(t) \copyright ID_2(t)$$
$$ID_1(t) = q_{13}(t) \copyright ID_3(t) + q_{14}(t) \copyright ID_4(t)$$
$$ID_2(t) = S_2(t) + q_{25}(t) \copyright ID_5(t)$$
$$ID_3(t) = q_{32}(t) \copyright ID_2(t)$$
$$ID_4(t) = q_{46}(t) \copyright ID_6(t)$$
$$ID_5(t) = q_{50}(t) \copyright ID_0(t)$$
$$ID_6(t) = q_{67}(t) \copyright ID_7(t)$$
$$ID_7(t) = q_{72}(t) \copyright ID_2(t)$$

where, $S_2(t) = \overline{B(t)}$

(71) - (79)

It follows from the above equations that

$$ID_0^*(s) = \mathcal{N}_3(s)/\mathcal{D}_2(s)$$

Where,

$$\mathcal{N}_3(s) = \{q_{01}^*(s)\, q_{13}^*(s)\, q_{32}^*(s) + q_{01}^*(s) q_{14}^*(s)$$
$$q_{46}^*(s)\, q_{67}^*(s)\, q_{72}^*(s) + q_{02}^*(s)\}\ S_2^*(s)$$

(80) - (81)

Thus, the fraction of time for which the helicopter remains idle, is given by

$$ID_0 = \lim_{t\to\infty} ID_0(t) = \lim_{s\to 0} s\ ID_0^*(s) = \mathcal{N}_3/\mathcal{D}_2$$

where, $\mathcal{N}_3 = \mu_2$

(82) - (83)

D. Expected Time for which the System is under Applicable Maintenance Following the Indication of Impending Bypass

To find the required time, we have obtained the following recursive relations for $MI_i(t)$:

$$MI_0(t) = q_{01}(t) \copyright MI_1(t) + q_{02}(t) \copyright MI_2(t)$$
$$MI_1(t) = q_{13}(t) \copyright MI_3(t) + q_{14}(t) \copyright MI_4(t)$$
$$MI_2(t) = q_{25}(t) \copyright MI_5(t)$$
$$MI_3(t) = S_3(t) + q_{32}(t) \copyright MI_2(t)$$
$$MI_4(t) = q_{46}(t) \copyright MI_6(t)$$
$$MI_5(t) = q_{50}(t) \copyright MI_0(t)$$
$$MI_6(t) = q_{67}(t) \copyright MI_7(t)$$
$$MI_7(t) = q_{72}(t) \copyright MI_2(t)$$

Where, $S_3(t) = \overline{G_1(t)}$

(84) - (92)

Proceeding in the similar manner, as before, we have

$$MI_0^*(s) = \mathcal{N}_4(s)/\mathcal{D}_2(s)$$

where,

$$\mathcal{N}_4(s) = q_{01}^*(s)\ q_{13}^*(s)\ S_3^*(s)$$

(93) - (94)

Therefore, we get

$$MI_0 = \lim_{t\to\infty} MI_0(t) = \lim_{s\to 0} s\ MI_0^*(s) = \mathcal{N}_4/\mathcal{D}_2$$

where, $\mathcal{N}_4 = p_{01}\ p_{13}\ \mu_3$

(95) - (96)

E. Expected Time for which the System is under Applicable Maintenance after the Fuel - filter gets Bypassed (Clogged)

Again proceeding in the similar fashion, we get:

$$MC_0(t) = q_{01}(t) \copyright MC_1(t) + q_{02}(t) \copyright MC_2(t)$$
$$MC_1(t) = q_{13}(t) \copyright MC_3(t) + q_{14}(t) \copyright MC_4(t)$$
$$MC_2(t) = q_{25}(t) \copyright MC_5(t)$$
$$MC_3(t) = q_{32}(t) \copyright MC_2(t)$$
$$MC_4(t) = q_{46}(t) \copyright MC_6(t)$$
$$MC_5(t) = q_{50}(t) \copyright MC_0(t)$$
$$MC_6(t) = q_{67}(t) \copyright MC_7(t)$$
$$MC_7(t) = S_7(t) + q_{72}(t) \copyright MC_2(t)$$

Where, $S_7(t) = \overline{G_2(t)}$

(97) - (105)

Following expression for $MC_0^*(s)$ has been obtained from the equations (97) - (104):

$$MC_0^*(s) = \mathcal{N}_5(s)/\mathcal{D}_2(s)$$

where, $\mathcal{N}_5(s) = q_{01}^*(s)\ q_{14}^*(s)\ q_{46}^*(s)\ q_{67}^*(s)\ S_7^*(s)$

(106) - (107)

Thus, we have

$$MC_0 = \lim_{t\to\infty} MC_0(t) = \lim_{s\to 0} s\ MC_0^*(s) = \mathcal{N}_5/\mathcal{D}_2$$

Where, $\mathcal{N}_5 = p_{01}\ p_{14}\ \mu_7$

(108) - (109)

Retrieval Number: C6396029320 /2020©BEIESP
DOI: 10.35940/ijeat.C6396.029320
Journal Website: www.ijeat.org

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F. Expected Number of Precautionary Landings

Applying arguments as above, we have the following recursive relations for $PL_i(t)$:

$$PL_0(t) = Q_{01}(t) \circledR PL_1(t) + Q_{02}(t) \circledR PL_2(t)$$
$$PL_1(t) = Q_{13}(t) \circledR PL_3(t) + Q_{14}(t) \circledR PL_4(t)$$
$$PL_2(t) = Q_{25}(t) \circledR PL_5(t)$$
$$PL_3(t) = Q_{36}(t) \circledR PL_6(t)$$
$$PL_4(t) = Q_{46}(t) \circledR \{1 + PL_6(t)\}$$
$$PL_5(t) = Q_{50}(t) \circledR PL_0(t)$$
$$PL_6(t) = Q_{67}(t) \circledR PL_7(t)$$
$$PL_7(t) = Q_{72}(t) \circledR PL_2(t)$$

(110) - (117)

Advancing as earlier, we get

$$PL_0^{**}(s) = N_6(s)/D_2(s)$$

where,

$$N_6(s) = q_{01}^{*}(s)\, q_{14}^{*}(s)\, q_{46}^{*}(s)$$

(118) - (119)

Thus, expected number of precautionary landings per unit time is given by

$$PL_0 = \lim_{t\to\infty}[PL_0(t)/t] = \lim_{s\to 0} s\, PL_0^{**}(s) = N_6/D_2$$

where, $N_6 = p_{01}\, p_{14}$

(120) - (121)

IX. PROFIT ANALYSIS

Profit yielded by the system can be expressed as:

$$P_0 = (R_f - C_f)FT_0 - \{C_{ib}\, MI_0 + C_{cl}\, MC_0 + L\; PL_0 + C_o\}$$

(122)

where, R_f, C_f, C_{ib}, C_{cl}, C_o, L are as defined in Section IV and expressions for FT_0, MI_0, MC_0, PL_0 have been obtained in previous section.

Every commercial operator desires to earn certain minimum profit. If P_{min} denotes the desired minimum profit, then it follows from equation (122) that:

- $P_0 \geq P_{min}$ if and only if $R_f \geq X_1 / FT_0$

where,

$$X_1 = P_{min} + \{C_f\, FT_0 + C_{ib}\, MI_0 + C_{cl}\, MC_0 + L\; PL_0 + C_o\}$$

(123)

- $P_0 \geq P_{min}$ if and only if $C_o \leq X_2$

where,

$$X_2 = (R_f - C_f)\, FT_0 - \{C_{ib}\, MI_0 + C_{cl}\, MC_0 + L\; PL_0 + P_{min}\}$$

(124)

X. GRAPHICAL ANALYSIS

Following particular case, wherein all the time distributions are assumed to be exponential, is considered for the purpose of graphical analysis of model under consideration:

$$a(t) = \alpha\, e^{-\alpha t} \qquad b(t) = \beta\, e^{-\beta t} \qquad d(t) = \sigma\, e^{-\sigma t}$$
$$h_1(t) = \gamma_1\, e^{-\gamma_1 t} \qquad h_2(t) = \gamma_2\, e^{-\gamma_2 t} \qquad i(t) = \delta\, e^{-\delta t}$$
$$g_1(t) = \delta_1\, e^{-\delta_1 t} \qquad g_2(t) = \delta_2\, e^{-\delta_2 t}$$

Assumed values have been assigned to various parameters as per **Table I**.

Table- I: Assumed numerical values

Rates (per hour)				Revenue/ Costs / Loss (INR)	
α	1	ʎ1	0..00067	R_f	90000
β	0.0635	ʎ2	0.25	P_{min}	1000
γ_1	1.2	δ	0.0028	C_f	22000
γ_2	3	δ_1	1	C_{ib}	7000
σ	1.5	δ2	0.0083	C_{cl}	2100
				C_o	2870
				L	50000

Different graphs demonstrating the behavior of MTFC, expected uptime and profit, have been plotted, as shown below:

a) Varying the values of ʎ1 and ʎ2 while keeping the other parameters constant as per Table 1, the following graph of MTFC versus ʎ1, for different values of ʎ2, has been plotted:

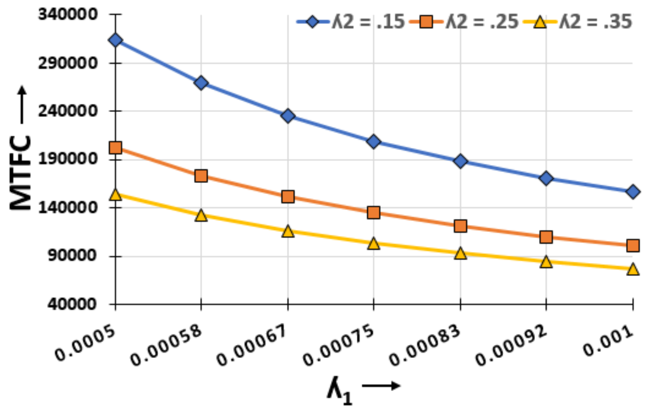

Fig. 2. Graph of MTFC vs. ʎ1

b) Fig. 3, given below, shows change in expected uptime (flying time) with variation in value of β, for different values of ʎ1:

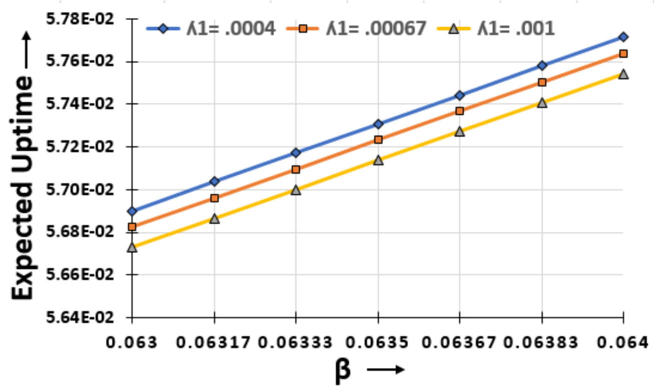

Fig. 3. Graph of FT_0 vs. β

c) The following graph depicts the behavior of profit per hour with variation in value of revenue per flight hour, for different values of α:

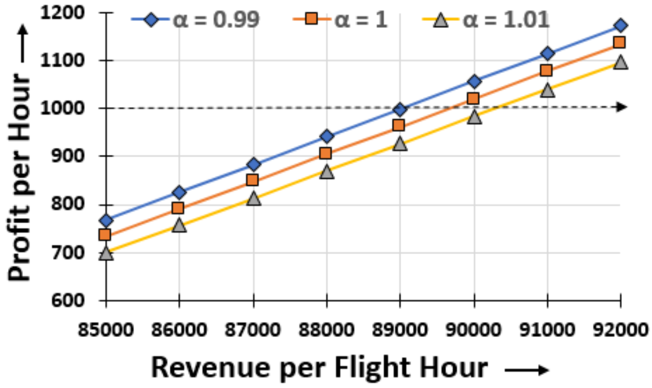

Fig. 4. Graph of P_0 vs. R_f

d) Again, varying the values of C_o and β and taking the numerical values of other parameters as per Table 1, the following graph has been plotted:

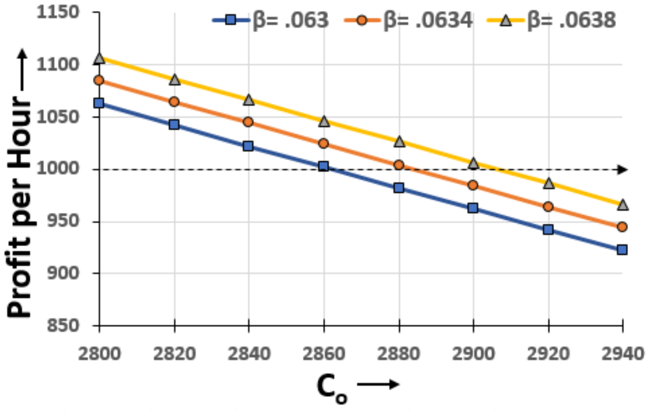

Fig. 5. Graph of P_0 vs. C_o

XI. RESULT AND DISCUSSION

From the graphical analysis as done in the above Section, following may be observed:

1) MTFC, uptime /flying time and profit vary in expected manner with respect to change in different parameters as shown in Figures 2 to 5.
2) Figures 4 and 5 reveal the numerical outcomes as tabulated below:

Table - II: Obtained cut - off values with interpretations

S. N.	Fixed parameters (with values as per Table I)	Varied parameters	Cut – off value For	Cut – off value Value obtained (Approx.)	Interpretation
1	$\beta, \gamma_1, \gamma_2, \sigma, \delta, \delta_1, \delta_2, \lambda_1, \lambda_2, C_f, C_{ib}, C_{cl}, C_o$, L	α and R_f	α = 0.99	R_f = 89025.1342	In order to get the desired minimum profit, for different values of α, R_f must be greater than or equal to the corresponding value obtained
			α = 1	R_f = 89660.0652	
			α = 1.01	R_f = 90294.9962	
2	$\alpha, \gamma_1, \gamma_2, \sigma, \delta, \delta_1, \delta_2, \lambda_1, \lambda_2, R_f, C_f, C_{ib}, C_{cl}$, L	β and C_o	β = .0630	C_o = 2861.8266	In order to get the desired minimum profit, for different values of β, C_o must be less than or equal to the corresponding value obtained
			β = .0634	C_o = 2883.9325	
			β = .0638	C_o = 2906.0109	

XII. CONCLUSION

The studied system has been modelled and analysed taking into consideration the situation of precautionary landing due to fuel - filter clogging. Various measures of the system performance and profit equation have been obtained in **Sections VIII** and **IX**. For numerical illustration and graphical analysis of the proposed model, particular case has been considered. The effect of variation in different parameters on MTFC, expected uptime and profit has been graphically observed and discussed in **Sections X** and **XI**. In addition to above, for getting the desired minimum profit, the cut-off values for some parameters have also been obtained. The suggested methodology can be adopted by the commercial aircraft operators to accomplish the same or to find the cut –off values related to other parameters of interest.

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