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A New Generalization of Quasi Gamma Distribution with Properties and Applications

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Abstract: ***we have introduced weighted technique for quasi gamma to convert known distribution into new model called weighted quasi gamma distribution. Finally, newly proposed distribution is examined with an application.***

Keywords: ***Quasi gamma, Order statistics, Statistical measures, Weighted model.***

I. INTRODUCTION

New Probability models used by researchers for the purpose of modelling or transforming known as weighted models proposed by Fisher in 1934, then after that in 1965 Rao makes significant changes in it and made it in other terms when he observe that the standard distributions are not suitable for modelling . For survival data analysis, Jing (2010) introduced the inverse version for both weighted Weibull and beta Weibull as a new lifetime models. Kilany (2016), have obtained the weighted version of lomax distribution. Ajami and Jahanshahi (2017) introduced weighted Rayleigh distribution as a new generalization of Rayleigh distribution and discussed its parameter estimation in broad. Para and Jan (2018) introduced the Weighted Pareto type II Distribution as a new model for handling medical science data and studied its statistical properties and applications. Khan et al. (2018) discussed the weighted modified weibull distribution. Reccently, Ganaie, Rajagopalan and Rather (2019), discussed A new extension of Ram Awadh distribution. The two parametric quasi gamma distribution was introduced by shanker (2018). Two parametric quasi gamma distribution also has better flexibility in handling real lifetime data over one parameter exponential, quasi exponential and two parameters gompertz, weibull and gamma distribution.

II. WEIGHTED QUASI GAMMA DISTRIBUTION (WQGD)

The distribution of quasi gamma having probability density function

$$f(x;\theta,\alpha)=\frac{2\theta^{\alpha}}{\Gamma(\alpha)}e^{-\theta x^2}x^{2\alpha-1};\quad x>0,\theta>0,\alpha>0 \qquad (1)$$

and distribution of quasi gamma having cumulative function given by

$$F(x;\theta,\alpha)=1-\frac{\Gamma(\alpha,\theta x^2)}{\Gamma(\alpha)};\quad \theta>0,\alpha>0,x>0 \qquad (2)$$

Let us suppose take X as a random variable which is non-negative having density function $f(x)$. Suppose w(x) be the function of weighted which is positive, then, the weighted random variable Xw has probability density function

$$f_w(x;\theta,c,\alpha)=\frac{w(x)f(x;\theta,\alpha)}{E(w(x))},\quad x>0$$

Where $w(x)$ be the positive weighted function

Now consider the function of weighted as w(x) = xc to obtain the model of weighted quasi gamma. having probability density function

$$f_w(x;\theta,c,\alpha)=\frac{x^c f(x;\theta,\alpha)}{E(x^c)},\ x>0 \qquad (3)$$

$$E(x^c)=\frac{\theta^{\alpha}\Gamma\frac{(2\alpha+c+1)}{2}}{\theta^{\frac{2\alpha+c+1}{2}}\Gamma(\alpha)} \qquad (4)$$

Probability density function is achieved for the distribution of weighted quasi gamma on Substituting equation (1) and (4) in equation (3)

$$f_w(x;\theta,c,\alpha)=\frac{2x^{2\alpha+c-1}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+c+1)}{2}} \qquad (5)$$

and cumulative function for weighted quasi gamma distribution is obtained as

$$F_w(x;\theta,c,\alpha)=\int_0^x f_w(x;\theta,c,\alpha)dx$$

$$F_w(x;\theta,c,\alpha)=\int_0^x\frac{2x^{2\alpha+c-1}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+c+1)}{2}}dx$$

$$F_w(x;\theta,c,\alpha)=\frac{2}{\Gamma\frac{(2\alpha+c+1)}{2}}\int_0^x x^{2\alpha+c-1}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}dx$$

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Put $\theta x^2 = t \;\Rightarrow\; 2\theta x dx = dt \;\Rightarrow\; dx = \dfrac{dt}{2\theta x}$, $dx = \dfrac{dt}{2\theta\left(\dfrac{t}{\theta}\right)^{\frac{1}{2}}}$

Also $\theta x^2 = t \Rightarrow x^2 = \dfrac{t}{\theta} \Rightarrow x = \left(\dfrac{t}{\theta}\right)^{\frac{1}{2}}$

As $x \to 0, t \to 0$ and $x \to x, t \to \theta x^2$

After simplification, we obtain the cumulative function for weighted quasi gamma distribution

$$F_w(x;\theta,c,\alpha) = \frac{1}{\Gamma\dfrac{(2\alpha+c+1)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+c+1)}{2},\theta x^2\right)\right) \qquad (6)$$

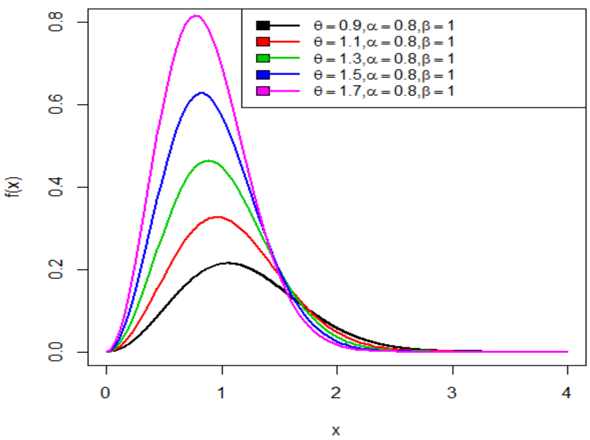

Fig.1:Pdf plot of WQGD

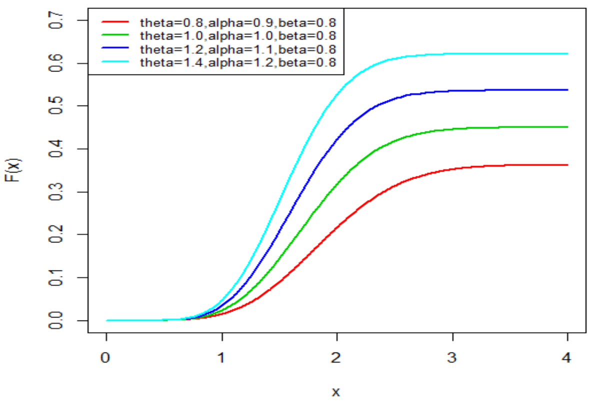

Fig.2 Cdf plot of WQGD

III. RELIABILITY ANALYSIS

A. Reliability function

The reliability function of newly obtained distribution is

$$R(x) = 1 - F_w(x;\theta,c,\alpha)$$

$$R(x) = 1 - \frac{1}{\Gamma\dfrac{(2\alpha+c+1)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+c+1)}{2},\theta x^2\right)\right)$$

B. Hazard function

$$h(x) = \frac{f_w(x;\theta,c,\alpha)}{R(x)}$$

$$h(x) = \frac{2x^{2\alpha+c-1}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}}{\Gamma\dfrac{(2\alpha+c+1)}{2} - \left(\theta^{\frac{1}{2}}\gamma\left(\dfrac{(2\alpha+c+1)}{2},\theta x^2\right)\right)}$$

C. Reverse hazard function

$$h^r(x) = \frac{f_w(x;\theta,c,\alpha)}{F_w(x;\theta,c,\alpha)}$$

$$h^r(x) = \frac{2x^{2\alpha+c-1}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}}{\left(\theta^{\frac{1}{2}}\gamma\left(\dfrac{(2\alpha+c+1)}{2},\theta x^2\right)\right)}$$

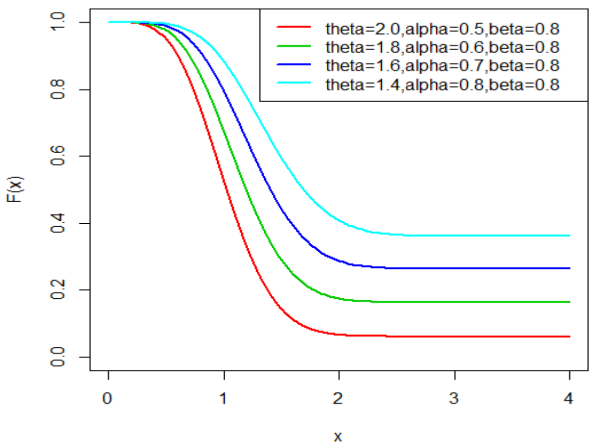

Fig.3 Reliability plot of WQGD

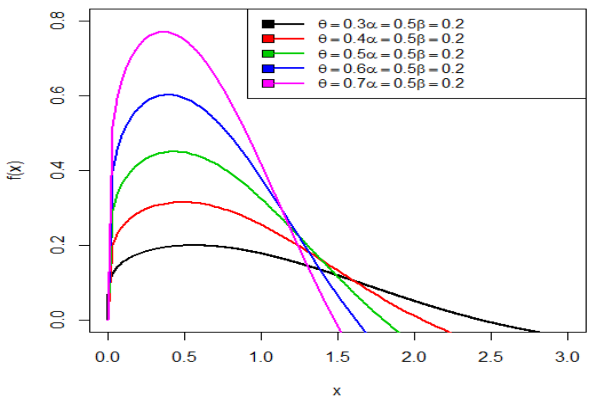

Fig.4:Showing Hazard function of WQGD

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IV. STATISTICAL MEASURES

In this Part, different statistical properties of weighted quasi gamma distribution including its moments, Harmonic mean, moment generating function and characteristics function are explained

A. Moments

Let X denotes the random variable of Weighted quasi gamma distribution, the rth order of weighted quasi gamma is

$$E(X^r)=\int_0^\infty x^r f_w(x;\theta,c,\alpha)dx$$

$$=\int_0^\infty x^r \frac{2x^{2\alpha+c-1}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+c+1)}{2}}dx$$

$$=\frac{2\theta^{\frac{2\alpha+c+1}{2}}}{\Gamma\frac{(2\alpha+c+1)}{2}}\int_0^\infty x^{2\alpha+c+r-1}e^{-\theta x^2}dx \qquad (7)$$

Put $x^2=t\Rightarrow x=t^{\frac{1}{2}}$

Also $2xdx=dt\Rightarrow dx=\frac{dt}{2x}=\frac{dt}{2t^{\frac{1}{2}}}$

After simplification, we obtain from equation (7)

$$=\frac{\theta^{\frac{2\alpha+c+1}{2}}}{\Gamma\frac{(2\alpha+c+1)}{2}}\left(\int_0^\infty t^{\frac{(2\alpha+c+r+1)-3}{2}}e^{-\theta t}dt\right)$$

$$E(X^r)=\frac{\Gamma\frac{(2\alpha+c+r+1)}{2}}{\theta^{\frac{r}{2}}\Gamma\frac{(2\alpha+c+1)}{2}} \qquad (8)$$

Moments of weighted quasi gamma distribution are obtained by substituting values 1,2,3 & 4 in (8)

$$E(X)=\mu_1'=\frac{\Gamma\frac{(2\alpha+c+2)}{2}}{\theta^{\frac{1}{2}}\Gamma\frac{(2\alpha+c+1)}{2}}$$

$$E(X^2)=\mu_2'=\frac{\Gamma\frac{(2\alpha+c+3)}{2}}{\theta\Gamma\frac{(2\alpha+c+1)}{2}}$$

$$E(X^3)=\mu_3'=\frac{\Gamma\frac{(2\alpha+c+4)}{2}}{\theta^{\frac{3}{2}}\Gamma\frac{(2\alpha+c+1)}{2}}$$

$$E(X^4)=\mu_4'=\frac{\Gamma\frac{(2\alpha+c+5)}{2}}{\theta^2\Gamma\frac{(2\alpha+c+1)}{2}}$$

$$\text{Variance }(\mu_2)=\frac{\Gamma\frac{(2\alpha+c+3)}{2}}{\theta\Gamma\frac{(2\alpha+c+1)}{2}}-\left(\frac{\Gamma\frac{(2\alpha+c+2)}{2}}{\theta^{\frac{1}{2}}\Gamma\frac{(2\alpha+c+1)}{2}}\right)^2$$

$$S.D(\sigma)=\sqrt{\left(\frac{\Gamma\frac{(2\alpha+c+3)}{2}}{\theta\Gamma\frac{(2\alpha+c+1)}{2}}-\frac{\left(\Gamma\frac{(2\alpha+c+2)}{2}\right)^2}{\left(\theta^{\frac{1}{2}}\Gamma\frac{(2\alpha+c+1)}{2}\right)^2}\right)}$$

B. Harmonic mean

The Harmonic mean is the reciprocal of the arithmetic mean of the reciprocals. The harmonic mean for the proposed weighted quasi gamma distribution can be obtained as

$$H.M=E\left(\frac{1}{x}\right)=\int_0^\infty\frac{1}{x}f_w(x;\theta,c,\alpha)dx$$

$$=\int_0^\infty\frac{2x^{2\alpha+c-2}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+c+1)}{2}}dx$$

$$=\frac{2}{\Gamma\frac{(2\alpha+c+1)}{2}}\left(\int_0^\infty x^{2\alpha+c-2}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}dx\right) \qquad (9)$$

Put $\theta x^2=t\Rightarrow x^2=\frac{t}{\theta}\Rightarrow x=\left(\frac{t}{\theta}\right)^{\frac{1}{2}}$

Also $2\theta xdx=dt\Rightarrow dx=\frac{dt}{2\theta x}\Rightarrow dx=\frac{dt}{2\theta\left(\frac{t}{\theta}\right)^{\frac{1}{2}}}$

After simplification, we obtain from equation (9)

$$=\frac{1}{\Gamma\frac{(2\alpha+c+1)}{2}}\left(\theta\int_0^\infty t^{\frac{(2\alpha+c+1)-4}{2}}e^{-t}dt\right)$$

$$H.M=\frac{1}{\Gamma\frac{(2\alpha+c+1)}{2}}\left(\theta\gamma\left(\frac{(2\alpha+c+1)}{2},\theta x^2\right)\right)$$

C. Moment Generating Function

The moment generating function is the expectation of a function of the random variable. Generating function of moment is given by

$$M_X(t) = E\left(e^{tx}\right) = \int_0^\infty e^{tx} f_w(x;\theta,c,\alpha)dx$$

$$= \int_0^\infty \left[1 + tx + \frac{(tx)^2}{2!} + \ldots\right] f_w(x;\theta,c,\alpha)dx$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!}\mu_j'$$

$$= \sum_{j=0}^\infty \frac{t^j}{j!}\left[\frac{\Gamma\frac{(2\alpha+c+j+1)}{2}}{\theta^{\frac{j}{2}}\Gamma\frac{(2\alpha+c+1)}{2}}\right]$$

$$M_X(t) = \frac{1}{\Gamma\frac{(2\alpha+c+1)}{2}} \sum_{j=0}^\infty \frac{t^j}{j!\theta^{\frac{j}{2}}}\left(\Gamma\frac{(2\alpha+c+j+1)}{2}\right)$$

D. Characteristics function

In probability theory and statistics characteristic function is defined as the function of any real-valued random variable completely defines the probability of a random variable.

$$\varphi_X(t) = M_X(it)$$

$$M_X(it) = \frac{1}{\Gamma\frac{(2\alpha+c+1)}{2}} \sum_{j=0}^\infty \frac{(it)^j}{j!\theta^{\frac{j}{2}}}\left(\Gamma\frac{(2\alpha+c+j+1)}{2}\right)$$

V. ORDER STATISTICS

Order statistics are sample values placed in order of increasing just as like x1, x2,…, xn then rth order statistics has probability density function

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left(F_X(x)\right)^{r-1}\left(1-F_X(x)\right)^{n-r} \tag{10}$$

Using equation (5) and (6) in equation (10), we obtain rth weighted model of quasi gamma distribution which is given by

$$f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\left(\frac{2x^{2\alpha+c-1}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+c+1)}{2}}\right)$$

$$\times\left(\frac{1}{\Gamma\frac{(2\alpha+c+1)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+c+1)}{2},\theta x^2\right)\right)\right)^{r-1}$$

$$\times\left(1-\frac{1}{\Gamma\frac{(2\alpha+c+1)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+c+1)}{2},\theta x^2\right)\right)\right)^{n-r}$$

Therefore, Ist order statistics X(1) of weighted quasi gamma distribution is given by

$$f_{x(1)}(x) = \frac{2nx^{2\alpha+c-1}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+c+1)}{2}}$$

$$\times\left(1-\frac{1}{\Gamma\frac{(2\alpha+c+1)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+c+1)}{2},\theta x^2\right)\right)\right)^{n-1}$$

and the higher order statistics X(n) of weighted quasi gamma is

$$f_{x(n)}(x) = \frac{2nx^{2\alpha+c-1}\theta^{\frac{2\alpha+c+1}{2}}e^{-\theta x^2}}{\Gamma\frac{(2\alpha+c+1)}{2}}$$

$$\times\left(\frac{1}{\Gamma\frac{(2\alpha+c+1)}{2}}\left(\theta^{\frac{1}{2}}\gamma\left(\frac{(2\alpha+c+1)}{2},\theta x^2\right)\right)\right)^{n-1}$$

VI. LIKELIHOOD RATIO TEST

Let the sample of size n randomly drawn from the distribution of quasi gamma or distribution of weighted quasi gamma. For testing purpose setting up hypothesis

$$H_o : f(x) = f(x;\theta,\alpha) \text{ VS } H_1 : f(x) = f_w(x;\theta,c,\alpha)$$

For testing, likelihood ratio function is applied whether the randomly selected sample comes for quasi gamma or weighted quasi gamma

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^n \frac{f_w(x;\theta,c,\alpha)}{f(x;\theta,\alpha)}$$

$$\Delta = \frac{L_1}{L_o} = \prod_{i=1}^{n} \left[\frac{x_i^{\,c}\, \theta^{\frac{c+1}{2}}\, \Gamma\alpha}{\Gamma \frac{(2\alpha + c + 1)}{2}} \right]$$

$$\Delta = \frac{L_1}{L_o} = \left(\frac{\theta^{\frac{c+1}{2}}\, \Gamma\alpha}{\Gamma \frac{(2\alpha + c + 1)}{2}} \right)^n \prod_{i=1}^{n} x_i^{\,c}$$

Null hypothesis is not accepted if

$$\Delta = \left(\frac{\theta^{\frac{c+1}{2}}\, \Gamma\alpha}{\Gamma \frac{(2\alpha + c + 1)}{2}} \right)^n \prod_{i=1}^{n} x_i^{\,c} > k$$

Equivalently, null hypothesis is also rejected when

$$\Delta^* = \prod_{i=1}^{n} x_i^{\,c} > k \left(\frac{\Gamma \frac{(2\alpha + c + 1)}{2}}{\theta^{\frac{c+1}{2}}\, \Gamma\alpha} \right)^n$$

$$\Delta^* = \prod_{i=1}^{n} x_i^{\,c} > k^*, \text{Where } k^* = k \left(\frac{\Gamma \frac{(2\alpha + c + 1)}{2}}{\theta^{\frac{c+1}{2}}\, \Gamma\alpha} \right)^n$$

Further, null hypothesis should not be accepted, when the value of probability is

$p\left(\Delta^* > \gamma^*\right)$ and $\gamma^* = \prod_{i=1}^{n} x_i^{\,c}$ is minimum than a particular level of significance and $\prod_{i=1}^{n} x_i^{\,c}$ is the experimental value of the Statistic Δ^*.

VII. BONFERRONI AND LORENZ CURVES

The proposed measure of weighted quasi gamma is

$$B(p) = \frac{1}{p\mu_1'} \int_0^q x f_w(x;\theta,c,\alpha)dx$$

and $L(p) = pB(p) = \frac{1}{\mu_1'} \int_0^q x f_w(x;\theta,c,\alpha)dx$

Where $\mu_1' = \frac{\Gamma \frac{(2\alpha + c + 2)}{2}}{\theta^{\frac{1}{2}}\Gamma \frac{(2\alpha + c + 1)}{2}}$ and $q = F^{-1}(p)$

$$B(p) = \frac{\theta^{\frac{1}{2}}\Gamma \frac{(2\alpha + c + 1)}{2}}{p\left(\Gamma \frac{(2\alpha + c + 2)}{2}\right)} \int_0^q \frac{2x^{2\alpha+c}\theta^{\frac{2\alpha+c+1}{2}} e^{-\theta x^2}}{\Gamma \frac{(2\alpha + c + 1)}{2}} dx$$

$$B(p) = \frac{2\theta^{\frac{1}{2}}}{p\left(\Gamma \frac{(2\alpha + c + 2)}{2}\right)} \left(\int_0^q x^{2\alpha+c}\theta^{\frac{2\alpha+c+1}{2}} e^{-\theta x^2} dx \right) \qquad (11)$$

Put $\theta x^2 = t \Rightarrow x^2 = \frac{t}{\theta} \Rightarrow x = \left(\frac{t}{\theta}\right)^{\frac{1}{2}}$

Also $2\theta x dx = dt \Rightarrow dx = \frac{dt}{2\theta x} = \frac{dt}{2\theta\left(\frac{t}{\theta}\right)^{\frac{1}{2}}}$

After simplification, equation (11) becomes

$$B(p) = \frac{\theta^{\frac{1}{2}}}{p\left(\Gamma \frac{(2\alpha + c + 2)}{2}\right)} \left(\int_0^q t^{\frac{(2\alpha+c+1)-2}{2}} e^{-t} dt \right)$$

$$B(p) = \frac{\theta^{\frac{1}{2}}}{p\left(\Gamma \frac{(2\alpha + c + 2)}{2}\right)} \left(\gamma\left(\frac{(2\alpha + c + 1)}{2}, \theta q \right) \right)$$

$$L(p) = \frac{\theta^{\frac{1}{2}}}{\left(\Gamma \frac{(2\alpha + c + 2)}{2}\right)} \left(\gamma\left(\frac{(2\alpha + c + 1)}{2}, \theta q \right) \right)$$

VIII. MAXIMUM LIKELIHOOD ESTIMATION AND FISHER'S INFORMATION MATRIX

Parameters of the distribution of weighted quasi gamma are calculated by using estimation of maximum likelihood, and then corresponding function of likelihood is

$$L(x;\theta,c,\alpha) = \prod_{i=1}^{n} f_w(x;\theta,c,\alpha)$$

$$L(x;\theta,c,\alpha) = \prod_{i=1}^{n} \left[\frac{2x_i^{\,2\alpha+c-1}\theta^{\frac{2\alpha+c+1}{2}} e^{-\theta x_i^2}}{\Gamma \frac{(2\alpha + c + 1)}{2}} \right]$$

$$L(x;\theta,c,\alpha) = \frac{2^n \theta^{n\left(\frac{2\alpha+c+1}{2}\right)}}{\left(\Gamma \frac{(2\alpha + c + 1)}{2}\right)^n} \prod_{i=1}^{n} \left[x_i^{\,2\alpha+c-1} e^{-\theta x_i^2} \right]$$

The function of log likelihood is

$$\log L = n\log 2 + n\left(\frac{2\alpha+c+1}{2}\right)\log\theta - n\log\left(\Gamma\frac{(2\alpha+c+1)}{2}\right) + (2\alpha+c-1)\sum_{i=1}^{n}\log x_i - \theta\sum_{i=1}^{n}x_i^2 \quad (12)$$

Now equation (12) is differentiated with parameters .The normal equations are

$$\frac{\partial\log L}{\partial\theta} = \frac{n(2\alpha+c+1)}{2\theta} - \sum_{i=1}^{n}x_i^2 = 0$$

$$\frac{\partial\log L}{\partial\alpha} = n\log\theta - n\psi\left(\frac{2\alpha+c+1}{2}\right) + 2\sum_{i=1}^{n}\log x_i = 0$$

$$\frac{\partial\log L}{\partial c} = \frac{n}{2}\log\theta - n\psi\left(\frac{2\alpha+c+1}{2}\right) + \sum_{i=1}^{n}\log x_i = 0$$

ψ (.) is the function of digamma

Because of complicated form of likelihood equation, algebraically it is very difficult to solve the system of non-linear equations. Therefore for estimating the required parameters by using R and wolfram mathematics.

To obtain confidence interval, we use the asymptotic normality results. We have that, if $\hat{\gamma} = (\hat{\theta}, \hat{\alpha}, \hat{c})$ denotes the MLE of $\gamma = (\theta, \alpha, c)$. We can state the result as

$$\sqrt{n}(\hat{\gamma}-\gamma) \to N_3(0, I^{-1}(\gamma))$$

Where $I(\gamma)$ is the Fisher's Information matrix.

Fisher's 3×3 Information matrix is given below as

$$I(\gamma) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2\log L}{\partial\theta^2}\right) & E\left(\frac{\partial^2\log L}{\partial\theta\partial\alpha}\right) & E\left(\frac{\partial^2\log L}{\partial\theta\partial c}\right) \\ E\left(\frac{\partial^2\log L}{\partial\alpha\partial\theta}\right) & E\left(\frac{\partial^2\log L}{\partial\alpha^2}\right) & E\left(\frac{\partial^2\log L}{\partial\alpha\partial c}\right) \\ E\left(\frac{\partial^2\log L}{\partial c\partial\theta}\right) & E\left(\frac{\partial^2\log L}{\partial c\partial\alpha}\right) & E\left(\frac{\partial^2\log L}{\partial c^2}\right) \end{pmatrix}$$

Where

$$E\left(\frac{\partial^2\log L}{\partial\theta^2}\right) = -\frac{n(2\alpha+c+1)}{2\theta^2}$$

$$E\left(\frac{\partial^2\log L}{\partial\alpha^2}\right) = -n\psi'\left(\frac{(2\alpha+c+1)}{2}\right)$$

$$E\left(\frac{\partial^2\log L}{\partial c^2}\right) = -n\psi'\left(\frac{(2\alpha+c+1)}{2}\right)$$

$$E\left(\frac{\partial^2\log L}{\partial\theta\partial\alpha}\right) = \frac{n}{\theta}$$

$$E\left(\frac{\partial^2\log L}{\partial\theta\partial c}\right) = \frac{n}{2\theta}$$

$$E\left(\frac{\partial^2\log L}{\partial\alpha\partial c}\right) = -n\psi''\left(\frac{(2\alpha+c+1)}{2}\right)$$

Where $\psi(.)'$ and $\psi(.)''$ is the first and second order derivative of digamma function.S ince γ being unknown we estimate $I^{-1}(\hat{\gamma})$ by $I^{-1}(\gamma)$ and this can be used to obtain asymptotic confidence intervals for θ, α and c.

IX. DATA EVALUATION

In the application portion analysis, we have used data sets for indicating that weighted quasi gamma distribution fits better over quasi gamma, exponential and one parameter lindley distribution. The following two data sets are given below as

Table-1: Strength of data by Naylor and Smith (1987) regarding 1.5cm glass fibers

0.55	1.70	1.77	2.24	1.11	1.28	1.42	1.50	1.60
1.62	1.66	2.00	0.93	1.78	1.84	0.81	1.13	1.48
1.50	1.55	1.61	1.62	1.82	0.74	1.25	1.89	1.24
1.30	1.48	1.51	1.55	1.61	1.63	1.76	2.01	1.36
1.49	1.52	1.58	1.61	1.64	1.68	1.73	1.81	1.84
0.77	1.27	1.39	1.49	1.53	1.59	1.61	1.66	1.76
1.54	1.29	0.84	1.04	1.70	1.68			

Table-2: Data regarding relief times (in minutes) of patients by Gross and Clark (1975)

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7
4.1	1.8	1.5	1.2	1.4	3.0	1.7	2.3	1.6	2.0

In order to compare the weighted quasi gamma distribution with quasi gamma, exponential and one parameter lindley distribution, we are using the criterion values like AIC, BIC and AICC. The distribution is good if it has minimum AIC, BIC, AICC and -2logL values than the others which we have mentioned for comparing. The formulas for calculation of values are

$$\text{AIC} = 2k - 2\log L, \quad \text{AICC} = AIC + \frac{2k(k+1)}{n-k-1}$$

and $\text{BIC} = k\log n - 2\log L$

Actually, the number of parameters is *k*, the size of sample is *n* and maximised function of log-likelihood is *-2logL*. The parameters of the distribution are obtained by the estimation of maximum likelihood. It is found from results in table 3 given below that weighted quasi gamma have lower criterion values than quasi gamma, exponential and one parameter lindley distribution which clearly indicates that the weighted quasi gamma distribution fits better than the quasi gamma, exponential and one parameter lindley distribution for the two data sets given. Finally, our conclusion reached that the distribution of weighted quasi gamma fits good than quasi gamma, exponential and one parameter lindley distribution.

Table-3: Comparison of the weighted quasi gamma distribution Vs quasi gamma, Exponential and one Parameter lindley distribution.

Data sets	Distribution	MLE	S.E	- 2logL	AIC	BIC	AICC
1	Weighted quasi gamma	$\hat{\alpha}=0.001$ $\hat{\theta}=0.842$ $\hat{c}=2.582$	$\hat{\alpha}=0.27$ $\hat{\theta}=0.07$ $\hat{c}=0.40$	9.3	15.3	21.7	15.7
	Quasi gamma	$\hat{\alpha}=5.042$ $\hat{\theta}=2.124$	$\hat{\alpha}=0.87$ $\hat{\theta}=0.38$	41.6	45.6	49.9	45.8
	Exponential	$\hat{\theta}=0.663$	$\hat{\theta}=0.08$	177.6	179.6	181.8	179.7
	Lindley	$\hat{\theta}=0.99$	$\hat{\theta}=0.09$	162.5	164.5	166.7	164.6
2	Weighted quasi gamma	$\hat{\alpha}=0.001$ $\hat{\theta}=0.490$ $\hat{c}=2.582$	$\hat{\alpha}=0.47$ $\hat{\theta}=0.07$ $\hat{c}=0.72$	29.66	35.66	38.64	37.16
	Quasi gamma	$\hat{\alpha}=2.347$ $\hat{\theta}=0.575$	$\hat{\alpha}=0.69$ $\hat{\theta}=0.19$	38.3	42.3	44.3	43.0
	Exponential	$\hat{\theta}=0.526$	$\hat{\theta}=0.11$	65.6	67.6	68.6	67.8
	Lindley	$\hat{\theta}=0.81$	$\hat{\theta}=0.13$	60.4	62.4	63.4	62.7

X. CONCLUSION

In the present manuscript, weighted technique is applied and taking the two parameter quasi gamma distribution as the base distribution called as weighted quasi gamma and also determines its parameters. The distribution which is introduced newly demonstrated with application. Then after that the results are compared over quasi gamma, exponential and one parameter lindley distribution and finally weighted quasi gamma proved and gives best results over quasi gamma, exponential and one parameter lindley distribution.

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