International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-9 Issue-3, February 2020

Flexural Motions of Beams on Foundation Subjected to Moving Concentrated Force

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Abstract: ***In this present paper, the dynamic analysis of non-prismastic beams subjected to moving concentrated forces is investigated at constant speed. Two cases of load- beam interaction problems described by the Dirac delta function with constant and harmonic magnitude mobile forces are studied. The technique called Galerkin's method in conjunction with integral transform method was employed to solve the motion equation. From the numerical results, it is evidently seen that an increase in the foundation stiffness provides reduction on the beam deflection. And furthermore, the issue of resonance is closely monitored and observed to have reached earlier in constant magnitude than harmonic variable magnitude problem. Results presented in this work are useful in constructions engineering designs.***

Keywords: ***Vibrating, Non-Prismastic Beam, Concentrated forces, Harmonic load, foundation, Galerkin method.***

I. INTRODUCTION

In the past years, problem of the vibrating motions of elastic structures (such as beam, plates, or shells) subjected to mobile forces has been given considerable attention by several researchers in Physics, Applied Mathematics and related fields. In particular, problem of uniform beams under the passage of moving loads has been studied by numerous authors [1,2,3] Dynamic motions of beam with non-uniformity when subjected to moving load was first addressed by Kolousek et al [4]. Their problems were solved using normal mode analysis. Sadiku and Leipholz [5] examined the dynamics of a prismatic beam and the effect of inertia was considered. Knowles [6] also worked on the dynamic deflection of a beam to a randomly moving load. The analysis of the expected beam deflection and bending moment was carried out when the load was moving with a variable velocity. The inertia effect of the load on the vibration of the system was not considered. Gutierrez and Laura [7] analysed the displacement response of a beam with a non-uniform cross-section and the approximate determination of the dynamic vibration of a beam on a time varying concentrated load. Masoud [8] examined vibration interaction analysis of non-prismatic cross-sectional beam structure under moving vehicle. The problem of a non-uniform cross-section beam with different boundary condition subjected to moving loads, such as a moving concentrated mass and a simple quarter-car (SQC) planar model was studied.

Revised Manuscript Received on February 05, 2020.
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These investigations though impressive, have neglected the more practical situation where the elastic structure are given as variable cross-section. The dynamic response of non-prismatic beam-type resting on variable elastic subgrade and traversed by moving concentrated masses is examined by Ogunyebi et al [9]. In the paper, the flexural effect of rotatory inertia is neglected. This paper presents the problem of vibration analysis of non-prismastic beam with exponential decaying foundation to mobile concentrated masses. The Galerkin's technique is used in the first instance to lower the order and simplify the motion equation from the fourth order to second order differential equations called Galerkin equations. Analytical solution will be of interest as it sheds light on vital information about the vibrating system.

II. PROBLEM FORMULATION

This section seeks the closed form solutions to the dynamical problem. The equation that governs the system subjected to moving load is given as Frybal [9]

$$E^*\frac{\partial^4 y}{\partial x^4}+\mu(x)\frac{\partial^2 y}{\partial x^2}+\varepsilon_o\frac{\partial y}{\partial t}+K(x)y=P(x,t)$$

$$+r_o\mu(x)\frac{\partial^4 y}{\partial x^2\partial t^2}+N\frac{\partial^2 y}{\partial x^2}$$

$$P(x,t)+r_o\mu(x,t)\frac{\partial^4 u(x,t)}{\partial x^2\partial t^2}=EI(x)\frac{\partial^4 u(x,t)}{\partial x^4}+\mu(x)\frac{u(x,t)}{\partial x^2}+\varepsilon_o\frac{\partial u(x,t)}{\partial t}+K(x)u(x,t) \quad (2.0)$$

where, $y=u(x,t)$ $E^*=EI(x)$,x x represent spacial coordinate, t the time,$u(x,t)$ y the Displacement,E E the Young's modulus, $\mu(x)$ $\mu(x)$ the mass, $I(x)$ the inertia of the beam, $K(x)$ is the variable elastic foundation and r_o r_o is the measure of rotating inertia.

Defining moment of inertia (variable type) and the mass of the beam to be

For the variable moment of inertia I I and the mass per unit length μ μ of the beam adopt the example in [8] and take $I(x)$ $I(x)$ and $\mu(x)$ to be of the form,

$$I(x)=I_o(1+D_o)^3 \quad I(x)=I_o\left(1+\sin\frac{\pi x}{L}\right)^3 \quad (2.1)$$

and

$$\mu(x)=\mu_o(1+D_o)$$

$$\mu(x) = \mu_o(1 + \sin \pi x/L) \quad (2.2)$$

where

$$D_o = \sin\frac{\pi x}{L} \qquad (2.3)$$

The rigidity of exponential form is given by

$$K(x) = K_o e^{-\lambda x}$$

$$K(x) = K_0 e^{-\lambda x} \qquad (2.4)$$

where λ is a constant and K_o is the elastic foundation constant.

III. BEAM ON EXPONENTIAL DECAY FOUNDATION UNDER CONSTANT MAGNITUDE

In this section, we define

$$P(x,t) = P_o\delta(x - c_m t)$$

$$P(x,t) = P_o\delta(x - c_m t) \quad (3.0)$$

where c_m is the velocity of the m^{th} m^{th} particles of the system. Equation (3.0) describe the mobile concentrated load and I is taken to be of constant magnitude

Putting equations (2.2), (2.3), (2.4) and (3.0) are substituted into equation (2.0), one obtains

$$EI_o\frac{\partial^2}{\partial x^2}\left[\left(1+\sin\frac{\pi x}{L}\right)^3\frac{\partial^2 u(x,t)}{\partial x^2}\right] + \mu_o\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial^2 u(x,t)}{\partial t^2}$$

$$-r_o\mu_o\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial^4 u(x,t)}{\partial x^2 \partial t^2}$$

$$+\varepsilon_o\frac{\partial u(x,t)}{\partial t} - N\frac{\partial^2 u(x,t)}{\partial x^2} + K_0 e^{-\lambda x}u(x,t) = P_o\delta(x - c_m t) \qquad (3.1)$$

IV. METHOD OF SOLUTION

The versatile solution technique called Galerkin's method to solve the beam problem above is employed. To this end, it is given in the form

$$\theta(u) - P_o = 0 \quad \theta u(x,t) - P_0(x,t) = 0 \qquad (3.2)$$

Where θ represent differential operator, u the structural deflection and P_o the load on the beam. The solution of the beam problem is expressed as

$$y = \sum_{i=1}^{n} P_i Q_i \qquad u(x,t) = \Sigma Y_i(t) H_i(x) \qquad (3.3)$$

where P_i and Q_i are functions of t and x respectively. P_i is given as modal coordinates and Q_i is

$$Q_i(x) = \sin\alpha_a + A_i\cos\alpha_a + B_i\sinh\alpha_a + C_i\cosh\alpha_a$$

$$Q_i(x) = sin\alpha_i x + A_i cos\alpha_i x + B_i sinh\alpha_i x + C_i cosh\alpha_i x (3.4)$$

where the constant $A_i, B_i,$ and C_i are amplitude of vibration and $\alpha_a = \alpha_i x$.

Noting that the problem at hand is simply supported, equation (3.3) upon consideration of equation (3.4) can be re-written in the form

$$y = \sum_{i=1}^{\infty} P_i \sin\frac{i\pi x}{L}$$

u_a (x,t) = ∑_(i = 1)^∞ P_i (t)sin iπx/L(3.5)

Putting equation (3.5) above into equation (3.1) gives

$$\frac{EI_o}{4}\left[\left(10 - 6\cos\frac{\pi x}{L} + 15\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)\frac{\partial^4}{\partial x^4}\sum_{i=1}^{\infty}P_i(t)\sin\frac{i\pi x}{L}\right]$$

$$+\frac{3\pi}{L}\left(\sin\frac{3\pi x}{L} + 5\cos\frac{\pi x}{L} - \cos\frac{3\pi x}{L}\right)$$

$$\cdot\frac{\partial^3}{\partial x^3}\sum_{i=1}^{\infty}P_i(t)\sin\frac{i\pi x}{L} 3\left(\frac{\pi}{L}\right)^2$$

$$\left(8\cos\frac{2\pi x}{L} - 5\sin\frac{\pi x}{L} + 3\sin\frac{3\pi x}{L}\right)\frac{\partial^2}{\partial x^2}\sum_{i=1}^{\infty}P_i(t)\sin\frac{i\pi x}{L}$$

$$+\mu_o\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial^2}{\partial x^2}\sum_{i=1}^{n}P_i(t)\sin\frac{i\pi x}{L}$$

$$-r_o\mu_o\left(1+\sin\frac{\pi x}{L}\right)\frac{\partial^4}{\partial x^2\partial t^2}\sum_{i=1}^{n}P_i(t)\sin\frac{i\pi x}{L}$$

$$+\varepsilon_o\frac{\partial}{\partial t}\sum_{i=1}^{n}P_i(t)\sin\frac{i\pi x}{L} - N\frac{\partial^2}{\partial x^2}\sum_{i=1}^{n}P_i(t)\sin\frac{i\pi x}{L}$$

$$+K_0 e^{-\lambda x}\sum_{i=1}^{n}P_i(t)\sin\frac{i\pi x}{L} = P_o\delta(x - c_i t)$$

$$\frac{EI_o}{4}[(10 - 6\cos\cos\frac{2\pi x}{L} + 15\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L})\frac{\partial^4}{\partial x^4}\sum_{i=1}^{n} P_i(t)\sin\frac{i\pi x}{L} + +\frac{3\pi}{L}\left(4\sin\frac{2\pi x}{L} + 5\cos\frac{\pi x}{L} - \cos\frac{3\pi x}{L}\right)\frac{\partial^3}{\partial x^3}\sum_{i=1}^{n} P_i(t)\sin\frac{i\pi x}{L} + 3\left(\frac{\pi}{L}\right)^2\left(8\cos\frac{2\pi x}{L} - 5\sin\frac{\pi x}{L} + 3\sin\frac{3\pi x}{L}\right)\frac{\partial^2}{\partial x^2}\sum_{i=1}^{n} P_i(t)\sin\frac{i\pi x}{L}] + \mu_0\left(1 + \sin\frac{\pi x}{L}\right)\frac{\partial^2}{\partial t^2}\sum_{i=1}^{n} P_i(t)\sin\frac{i\pi x}{L} - -r_0\mu_0\left(1 + \sin\frac{\pi x}{L}\right)\frac{\partial^4}{\partial x^2\partial t^2}\sum_{i=1}^{n} P_i(t)\sin\frac{i\pi x}{L} + \varepsilon_0\frac{\partial}{\partial t}\sum_{i=1}^{n} P_i(t)\sin\frac{i\pi x}{L} + K_0 e^{-\lambda x}\sum_{i=1}^{n} P_i(t)\sin\frac{i\pi x}{L} = P_0\delta(x - c_i t)$$

(3.6)

Subjecting equation (3.6) to further simplification, one obtains

$$\frac{EI_o}{4}\left[R_1(x)(A_1)^4\sin\phi x - R_2(x)(A_1)^3\cos\phi x - R_3(x)(A_1)^2\sin\phi x\right]P_i(t)$$

$$+\mu_o R_4(x)\ddot{P}_i(t)\sin\phi x + (\phi)^2 r_o\mu_o R_4(x)\ddot{P}_i(t)\sin\phi x + \varepsilon_o\dot{P}_i(t)\sin\phi x$$

$$+N(\phi)^2 P_i(t)\sin\phi x + K_0 e^{-\lambda x}P_i(t)\sin\phi x = P_o\delta(x - c_i t) \qquad (3.7)$$

where

$$A_1 = \frac{i\pi x}{L},\ A_2 = \frac{\sin\pi x}{L} \qquad (3.8)$$

$$\phi = \frac{i\pi}{L} \qquad (3.9)$$

and

$$R_1(x) = \left(10 - 6\cos\frac{\pi x}{L} + 15\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)$$

$R_1(x) = 10 - 6cos\frac{2\pi x}{L} + 15sin\frac{\pi x}{L} - sin\frac{3\pi x}{L}$ (3.9a)

$$R_2(x) = \frac{3\pi}{L}\left(\sin\frac{3\pi x}{L} + 5\cos\frac{\pi x}{L} - \cos\frac{3\pi x}{L}\right)$$

$R_2(x) = \frac{3\pi}{2}\left(4sin\frac{2\pi x}{L} + 5cos\frac{\pi x}{L} - cos\frac{3\pi x}{L}\right)$ (3.9b)

$$R_3(x) = 3\left(\frac{\pi}{L}\right)^2\left(8\cos\frac{2\pi x}{L} - 5\sin\frac{\pi x}{L} + 3\sin\frac{3\pi x}{L}\right) \quad (3.9c)$$

$$R_4(x) = \left(1 + \sin\frac{\pi x}{L}\right) \quad (3.9d)$$

By the imposition of orthogonality conditions, equation (3.7) becomes

$$\int_0^L \sum_{i=1}^n \left\{ \frac{EI_o}{4}\left[R_1(x)\left(\frac{i\pi x}{L}\right)^4 \sin\frac{i\pi x}{L} - R_2(x)\left(\frac{i\pi x}{L}\right)^3 .\cos\frac{i\pi x}{L} - R_3(x)\left(\frac{i\pi x}{L}\right)^2 \sin\frac{i\pi x}{L}\right] P_i(t)$$

$$+\mu_o R_4(x)\ddot{P}_i(t)\sin\frac{i\pi x}{L}$$

$$+\left(\frac{i\pi}{L}\right)^2 r_o\mu_o R_4(x)\ddot{P}_i(t)\sin\frac{i\pi x}{L}$$

$$+N\left(\frac{i\pi}{L}\right)^2 P_i(t)\sin\frac{i\pi x}{L} + \varepsilon_o\dot{P}_i(t)\sin\frac{i\pi x}{L}$$

$$+K_0 e^{-\lambda x}P_i(t)\sin\frac{i\pi x}{L}\right\}\sin\frac{k\pi x}{L} = \int_0^l P_o\delta(x - c_i t)\sin\frac{k\pi x}{L} \quad (3.10)$$

Equation (3.10) after some rearrangements and simplifications yields

$$D_1(i,k)\ddot{P}_i(t) + D_2(i,k)\dot{P}_i(t) + D_3(i,k)P_i(t) = P_o\sin\frac{k\pi ct}{L} \quad (3.11)$$

where

$$D_1(i,k) = \mu_o[I_1 + I_3]\left[1 + \left(\frac{i\pi}{L}\right)^2 r_o\right] \quad (3.12a)$$

$D_2(i,k) = \varepsilon_o I_1$ $D_2 = \varepsilon_0 I_1$ (3.12b)

$D_3(i,k) = \frac{EI_o}{4}[E_1 - E_2 - E_3] + E_4$ $D_3 = \frac{EI_0}{4}[E_1 - E_2 - E_3] + E_4$ (3.12c)

$E_1 = \left(\frac{i\pi}{L}\right)^4[10I_1 - 6I_2 + 15I_3 - I_4]$ $E_1 = \left(\frac{i\pi}{4}\right)^4[10I_1 - 6I_2 + 15I_3 - I_4]$ (3.13)

$E_2 = 3i^3\left(\frac{\pi}{L}\right)^4[4I_5 + 5I_6 - I_7]$ $E_2 = 3i^3\left(\frac{\pi}{L}\right)^4[4I_5 + 5I_6 - I_7]$ (3.14)

$E_3 = 3i^2\left(\frac{\pi}{L}\right)^4[8I_2 - 5I_3 + 3I_4]$ $E_3 = 3i^2\left(\frac{\pi}{L}\right)^4[8I_2 - 5I_3 + 3I_4]$ (3.15)

The integrals I_i are as follow

$I_1 = \int_0^L \sin\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx$, $I_1 = \int_0^L Sin\frac{i\pi x}{L}Sin\frac{k\pi x}{L}dx$

$I_2 = \int_0^L \cos\frac{2\pi x}{L}\sin A_1 \sin A_3 dx$ $I_2 = \int_0^L Cos\frac{2\pi x}{L}Sin\frac{i\pi x}{L}sin\frac{k\pi x}{L}dx$

$I_3 = \int_0^L \sin\frac{\pi x}{L}\sin A_1 \sin A_3 dx$ $I_3 = \int_0^L Sin\frac{\pi x}{L}Sin\frac{i\pi x}{L}Sin\frac{k\pi x}{L}dx$

$I_4 = \int_0^L \sin\frac{3\pi x}{L}\sin A_1 \sin A_3 dx$ $I_4 = \int_0^L Sin\frac{3\pi}{L}Sin\frac{i\pi x}{L}Sin\frac{k\pi x}{L}dx$

$$I_5 = \int_0^L \sin\frac{2\pi x}{L}\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx$$

$I_6 = \int_0^L \cos\frac{\pi x}{L}\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx$ $I_5 = \int_0^L Sin\frac{2\pi x}{L}Cos\frac{i\pi x}{L}Sin\frac{k\pi x}{L}dx$

$$I_7 = \int_0^L \cos\frac{3\pi x}{L}\cos\frac{i\pi x}{L}\sin\frac{k\pi x}{L}dx \quad (3.16a)$$

$$A_3 = \frac{k\pi x}{L} \quad (3.16b)$$

Evaluating the integrals in (3.16a), equations (3.12a) to (3.15) yield the following respectively

$$E_1 = \left(\frac{i\pi}{L}\right)^4\left[\frac{5L}{2} + \frac{3iL}{2} - \frac{60ikL}{\pi\left[1-(i-k)^2\right]\left[1-(i+k)^2\right]} + \frac{12ikL}{\pi\left[9-(i-k)^2\right]\left[9-(i+k)^2\right]}\right] \quad (3.17)$$

$$E_2 = 3i^3\left(\frac{\pi}{L}\right)^4\left[kL - \frac{60ikL\left[1+i^2+k^2\right]}{\pi\left[1-(i-k)^2\right]\left[1-(i+k)^2\right]} + \frac{2kL\left[9+i^2-k^2\right]}{\pi\left[9-(i-k)^2\right]\left[9-(i+k)^2\right]}\right] \quad (3.18)$$

$$E_3 = 3i^2\left(\frac{\pi}{L}\right)^4\left[-2iL - \frac{20ikL}{\pi\left[1-(i-k)^2\right]\left[1-(i+k)^2\right]} + \frac{36ikL}{\pi\left[9-(i-k)^2\right]\left[9-(i+k)^2\right]}\right] \quad (3.19)$$

And

$$E_4 = K_o\left(\frac{\left(\pi(i-k)/L\right)}{2\left[(i-k)^2\left(\frac{\pi}{L}\right)^2+\lambda^2\right]}\right)\left[e^{-\lambda L}\sin\pi(i-k)-\frac{e^{-\lambda L}\cos\pi(i-k)}{\left(\pi(i-k)/L\right)}+\frac{\lambda}{\left(\pi(i-k)/L\right)}\right]$$

-

$$K_o\left(\frac{\left(\pi(i+k)/L\right)}{2\left[(i+k)^2\left(\frac{\pi}{L}\right)^2+\lambda^2\right]}\right)$$

$$.\left[e^{-\lambda L}\sin\pi(i+k)-\frac{e^{-\lambda L}\cos\pi(i+k)}{\left(\pi(i+k)/L\right)}+\frac{\lambda}{\left(\pi(i+k)/L\right)}\right]$$

$$E_4 = K_0\left\{\frac{(i-k)\frac{\pi}{L}}{2\left[(i-k)^2\left(\frac{\pi}{L}\right)^2+\lambda^2\right]}\left[e^{-\lambda L}\ \ Sin(i-k)\pi - \frac{\lambda e^{-\lambda L}Cos(i-k)\pi}{(i-k)\frac{\pi}{2}}+\frac{\lambda}{(i-k)\frac{\pi}{L}}\right] - \frac{(i+k)\frac{\pi}{L}}{2\left[(i+k)^2\left(\frac{\pi}{L}\right)^2+\lambda^2\right]}\times\left[e^{-\lambda L}Sin(i+k)\pi-\frac{\lambda e^{-\lambda L}Cos(i+k)\pi}{(i-k)\frac{\pi}{L}}+\frac{\lambda}{(i+k)\frac{\pi}{L}}\right]\right\} \quad (3.20)$$

At this stage, equation (3.11) is subjected to a Laplace transform and after further simplification give

$$\left(D_1(i,k)S^2+D_2(i,k)S+D_3(i,k)\right)P_i(s)$$

$$=P_0\frac{S}{S^2+\theta^2} \quad (3.21b)$$

where

$$\theta=\left(\frac{k\pi c_i}{L}\right) \qquad \theta=\frac{k\pi c_i}{L} \quad (3.22)$$

Subjecting equation (3.21) to some simplifications and rearrangements gives

$$P_i(s)=P_0\frac{\theta}{S^2+\theta^2}\frac{1}{\left(D_1(i,k)S^2+D_2(i,k)S+D_3(i,k)\right)} \quad (3.23)$$

which reduces to

$$P_i(S)=\frac{P_0}{(\beta_1-\beta_2)}\left(\frac{S}{S^2+\theta^2}\cdot\frac{1}{S-\beta_1}-\frac{S}{S^2+\theta^2}\cdot\frac{1}{S-\beta_2}\right)$$

$$P_{i(s)}=\frac{P_0}{(\beta_1-\beta_2)}\cdot\frac{\theta}{S^2+\theta^2}\left(\frac{1}{S-\beta_1}-\frac{1}{S-\beta_2}\right) \quad (3.24)$$

where

$$\beta_1=\frac{-D_2+\sqrt{D_2^2-4D_1D_3}}{2D_1}$$

$$\beta_1=\frac{-D_2+\sqrt{{D_2}^2-4D_1D_3}}{2D_1}\ and\ \beta_2=\frac{-D_2-\sqrt{{D_2}^2-4D_1D_3}}{2D_1}$$

and

$$\beta_2=\frac{-D_2-\sqrt{D_2^2-4D_1D_3}}{2D_1} \quad (3.25)$$

Adopting the representation below, the Laplace inversion of equation (3.24) is given as

$$g(s)=\frac{\theta}{S^2+\theta^2},\ f_1(s)=\frac{1}{S-\beta_1} \quad \text{and}$$

$$f_2(s)=\frac{1}{S-\beta_2} \quad (3.26)$$

Further simplification and arrangements of (3.26) gives

$$P_i(t)=P_p\left[\frac{e^{\beta_1 t}}{\beta_1}I_1-\frac{e^{\beta_2 t}}{\beta_2}I_2\right] \quad (3.27)$$

where

$$P_p=\frac{P_0}{(\beta_1+\beta_2)}\quad P_p=\frac{P_0}{\beta_1+\beta_2} \quad (3.28)$$

$$I_1=\int_0^t e^{-\beta_1 u}\sin\theta u du \quad \text{and} \quad I_2=\int_0^t e^{-\beta_2 u}\sin\theta u du \quad (3.29)$$

Evaluating the integrals (3.29) above, we obtain

$$I_1=\frac{1}{(\theta^2+\beta_1^2)}\left(-\theta e^{-\beta_1 t}\cos\theta t+\theta-\beta_1 e^{-\beta_1 t}\sin\theta t\right) \quad (3.30)$$

$$I_2=\frac{1}{(\theta^2+{\beta_2}^2)}\{-\theta e^{-\beta_2 t}\cos\cos\theta t+\theta-\beta_2 e^{-\beta_2 t}\sin\sin\theta t\ \}$$

$$I_2=\frac{1}{(\theta^2+\beta_2^2)}\left(-\theta e^{-\beta_2 t}\cos\theta t+\theta-\beta_2 e^{-\beta_2 t}\sin\theta t\right) \quad (3.31)$$

Subjecting equation (3.27) to some simplification and rearrangement yields,

$$P_i(t)=\frac{P_p}{\beta_1(\theta^2+\beta_1^2)}\left(\theta\left(e^{\beta_1 t}-\cos\theta t\right)-\beta_1\sin\theta t\right) - \frac{P_p}{\beta_2(\theta^2+\beta_2^2)}\left(\theta\left(e^{\beta_2 t}-\cos\theta t\right)-\beta_2\sin\theta t\right) \quad (3.32)$$

which on invasion yields

$$u(x,t)=\sum_{i=1}^{n}\left\{\frac{P_p}{\beta_1(\theta^2+\beta_1^2)}\left(\theta\left(e^{\beta_1 t}-\cos\theta t\right)-\beta_1\sin\theta t\right) - \frac{P_p}{\beta_2(\theta^2+\beta_2^2)}\left(\theta\left(e^{\beta_2 t}-\cos\theta t\right)-\beta_2\sin\theta t\right)\right\}\sin\frac{i\pi x}{L}$$

$$u(x.t)=\sum_{i=1}^{n}\ \left\{\frac{p_p\beta_1}{\theta^2+{\beta_1}^2}\left[\theta\left(e^{\beta_1 t}-\cos\cos\theta t\ \right)-\sin\beta_1\sin\theta t\ \right]-\frac{p_p\beta_2}{(\theta^2+{\beta_2}^2)}\left[\theta\left(e^{\beta_2 t}\ \ -\beta_2\cos\cos\theta t\ \right)-\sin\beta_2\sin\theta t\ \right]\right\}\sin\sin\frac{i\pi x}{L} \quad (3.33)$$

which is the solution of the governing differential equation of the non-prismatic element at constant mobile distributed masses on exponential decaying subgrade.

V. BEAM ON EXPONENTIAL DECAY FOUNDATION UNDER HARMONIC MAGNITUDE

This section seeks the solution of the non-prismatic structure with exponentially decaying foundation under harmonic magnitude at constant speed. The load on the structure is given as

$$P(x,t) = P_o \cos\omega t\delta(x - c_m t) \quad P(x,t) = Pcos\omega t\delta(x - c_m t) \qquad (4.1)$$

Using equation (2.0) in (4.1), the differential equation gives

$$EI(x)\frac{\partial^4 u_a(x,t)}{\partial x^4} + \mu(x)\frac{\partial^2 u_a(x,t)}{\partial x^2} - r_o\mu(x)\frac{\partial^4 u_a(x,t)}{\partial x^2\partial t^2} + \varepsilon_o \frac{\partial u_a(}{\partial}$$

$$+K(x)u_a(x,t) = P_o \cos\omega t\delta(x - c_m t) \qquad (4.2)$$

$$u_a(x,t) = \sum_{m=1}^{\infty} P_m(t)Q_m(x) \quad u_b(x,t) = \sum_{m=1}^{\infty} P_m(t)H_m(x) \qquad (4.3)$$

And for the simply supported study, equation (4.3) is re-written in the form

$$u_a(x,t) = \sum_{m=1}^{\infty} P_m(t)\sin\frac{m\pi x}{L} \quad u_b(x,t) = \sum_{m=1}^{\infty} P_m(t)sin\frac{m\pi x}{L} \qquad (4.\ 4)$$

Putting equation (4. 4) into equation (4. 2) and using the same procedures as in section (3.0) one obtains

$$\sum_{m=1}^{\infty}\left\{D_1\frac{d^2P_m}{dt^2} + D_2\frac{dP_m}{dt} + D_3P_m\right\} = P_o\cos\omega t\sin\frac{k\pi c_m t}{L} \qquad (4.5)$$

where c_m c_m is the speed and $P_m(t)$.

Considering the mth particle of the system under consideration, equation (4.5) becomes

$$D_1\frac{d^2P_m}{dt^2} + D_2\frac{dP_m}{dt} + D_3P_m = P_o\cos\omega t\sin\theta \qquad (4.6)$$

Subjecting equation (4.6) as defined previously yields

$$y_m(S) = \frac{P_o}{2(d_1 - d_2)}\left\{\frac{1}{\alpha_1}\left(\frac{\Omega_1}{S^2+\Omega_1^2}\cdot\frac{\alpha_2}{S-\alpha_1} - \frac{\Omega_2}{S^2+\Omega_2^2}\cdot\frac{\alpha_1}{S-\alpha_1}\right)\right.$$

$$\left.-\frac{1}{\alpha_2}\left(\frac{\Omega_1}{S^2+\Omega_1^2}\cdot\frac{\alpha_2}{S-\alpha_2} - \frac{\Omega_2}{S^2+\Omega_2^2}\cdot\frac{\alpha_2}{S-\alpha_2}\right)\right\} \qquad (4.7)$$

where

$$\Omega_1 = \omega + \frac{k\pi c_m}{L} \quad \text{and} \quad \Omega_2 = \omega - \frac{k\pi c_m}{L} \qquad (4.8)$$

$$\alpha_1 = \frac{-D_2 + \sqrt{D_2^2 - 4D_1D_3}}{2D_1} \quad \beta_1 = \frac{-D_2+\sqrt{D_2{}^2-4D_1D_3}}{2D_1} \ and\ \beta_2 = \frac{-D_2-\sqrt{D_2{}^2-4D_1D_3}}{2D_1}$$

(4.9a)

and

$$\alpha_2 = \frac{-D_2 - \sqrt{D_2^2 - 4D_1D_3}}{2D_1} \qquad (4.9b)$$

Following the argument in section (3.0), equation (4.6) gives

$$P_m(t) = \frac{P_p}{\beta_1(\Omega_1^2+\beta_1^2)}\left(\Omega_1\left(e^{\beta_1 t} - \cos\Omega_1 t\right) - \beta_1\sin\Omega_1 t\right)$$
$$-\frac{P_p}{\beta_1(\Omega_2^2+\beta_1^2)}\left(\Omega_2\left(e^{\beta_1 t} - \cos\Omega_2 t\right) - \beta_1\sin\Omega_2 t\right)$$
$$-\frac{P_H\alpha_2}{(\Omega_1^2+\alpha_2^2)}\{\Omega_1(e^{\alpha_2 t} - cos\Omega_1 t) - \alpha_2 sin\Omega_1 t\} + \frac{P_H\alpha_2}{(\Omega_2^2+\alpha_2^2)}\{\Omega_2(e^{\alpha_2 t} - cos\Omega_2 t) - \alpha_2 sin\Omega_2 t\}$$
$$-\frac{P_p}{\alpha_2(\alpha_1^2+\alpha_2^2)}\left(\Omega_1\left(e^{\alpha_2 t} - \cos\Omega_1 t\right) - \alpha_2\sin\Omega_1 t\right)$$
$$+\frac{P_p}{\alpha_2(\Omega_2^2+\alpha_2^2)}\left(\Omega_2\left(e^{\alpha_2 t} - \cos\Omega_2 t\right) - \alpha_2\sin\Omega_2 t\right) \qquad (4.10)$$

which on inversion yields

$$u_b(x,t) = \sum_{m=1}^{n}\left[\frac{P_p}{\alpha_1(\Omega_1^2+\alpha_1^2)}\left(\Omega_1\left(e^{\alpha_1 t} - \cos\Omega_1 t\right) - \alpha_1\sin\Omega_1 t\right)\right.$$
$$-\frac{P_p}{\alpha_1(\Omega_2^2+\alpha_1^2)}\left(\Omega_2\left(e^{\alpha_1 t} - \cos\Omega_2 t\right) - \alpha_1\sin\Omega_2 t\right)$$
$$-\frac{P_p}{\alpha_2(\alpha_1^2+\alpha_2^2)}\left(\Omega_1\left(e^{\alpha_2 t} - \cos\Omega_1 t\right) - \alpha_2\sin\Omega_1 t\right)$$
$$\left.+\frac{P_p}{\alpha_2(\Omega_2^2+\alpha_2^2)}\left(\Omega_2\left(e^{\alpha_2 t} - \cos\Omega_2 t\right) - \alpha_2\sin\Omega_2 t\right)\right]\sin\frac{m\pi x}{L} \qquad (4.11)$$

which is the deflection response of the structure subjected to actions of harmonic magnitude load at constant velocity.

VI. DISCUSSION

Effects of Resonance

The dynamic effect of resonance conditions is paramount in the study of dynamic system such as beam problem. These are the conditions under which the non-prismatic beam response grow without bound.

In studying a dynamic problem such as this, it is desirable to examine the phenomenon of resonance condition. These are the conditions under which the non- prismatic beam responses grow without bound.

Evidently, equation (3.34) shows that the non- prismastic beam response under constant magnitude mobile concentrated forces will grow without bound whenever

$$\beta_1 = \beta_2, \qquad \theta^2 = -\beta_1^2, or \quad \theta^2 = -\beta_2^2 \qquad (5.1)$$

And the velocity at which this occurs is termed the critical condition, while from equation (4.8) the same non-prismastic beam traversed by harmonic variable magnitude mobile forces will experience resonance effects whenever

$$\alpha_1 = \alpha_2, \ \Omega_1^2 = -\alpha_1^2, \text{or } \Omega_2^2 = -\alpha_2^2 \qquad (5.2)$$

It can be deduced by equations (5.1) and (5.2) that the critical velocity of the structure when on exponentially decaying subgrade is smaller than when the system involving non-pragmatic beam subjected to harmonic variable magnitude mobile force. Therefore, resonance is reached earlier in constant magnetic than harmonic variable magnitude.

VII. NUMERICAL CALCULATIONS AND DISCUSSION OF RESULTS

As an illustration, the non-prismatic beam is taken to be 12.192m long and the mass is taken to travel at the constant velocity 8.12m/s. Furthermore, we assumed that force travels at the EI is 6068242m^2/s^2, μ =2758.291kg/m. The deflection of the beam with exponential decaying foundation for the values of K and R_0 are plotted against t. The analysis was carried out separately for both cases of constant magnitude and harmonic magnitude of mobile forces problems.

Figure 1 displays the effect of foundation modulus K_o K_0on the transverse deflection of non-prismastic elastic beam when traversing concentrated forces are of constant magnitudes. The figure depict that as K_o increases the displacement response of beam decreases.

Figure 2 depicts the deflection of FM for constant force for fixed value of Rotatory inertia R_0 and axial force N. It is seen that an increase in FM reduces the deflection profile of the beam.

Figure 3 Figure 3 shows deflection of axial force on the transverse deflection of non-prismastic elastic beam when traversing concentrated forces is of constant magnitudes. An increase in N decreases the deflection of the elastic beam.

Figure 4 displays the response amplitude of the harmonic mobile forces. It is clearly seen the response amplitude decreases as rotatory inertia increases.

Figure 5 displays the deflection of the harmonic mobile forces for the beam. The response amplitude decreases as the FM of the beam increases.

Figure 6 displays the response amplitude of harmonic mobile forces for the beam. The response amplitude decreases as axial force N increases.

Figure 7 compares the deflection profiles of the non-prismatic structure for constant magnitude and harmonic magnitude at constant speed for fixed values of FM=4000000, N=,500000 K_o0.5. Evidently, the deflection amplitude of constant magnitude mobile load is higher than that of the harmonic variable mobile forces.

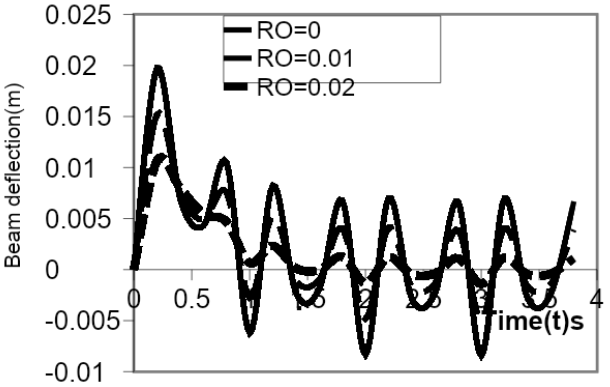

Fig. 1. Deflection of beam (constant magnitude) for fixed FM=40000000, N=500000 and various values of Ro.

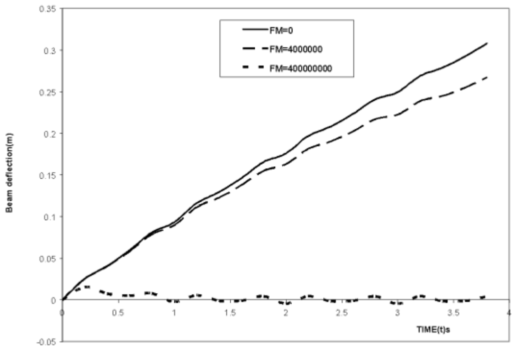

Fig. 2. Deflection of beam (constant magnitude) for fixed Ro=0.5, N=500000 and various values of FM.

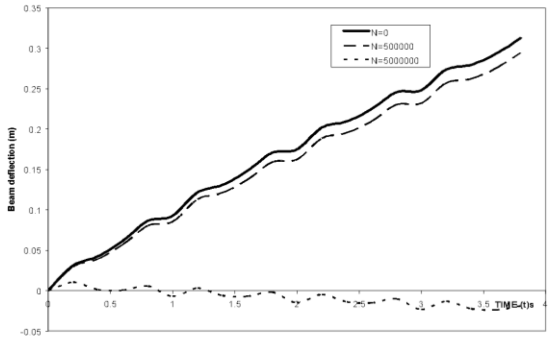

Fig. 3. Deflection of beam (constant magnitude) for fixed Ro=0.5, FM=4000000 and various values of N

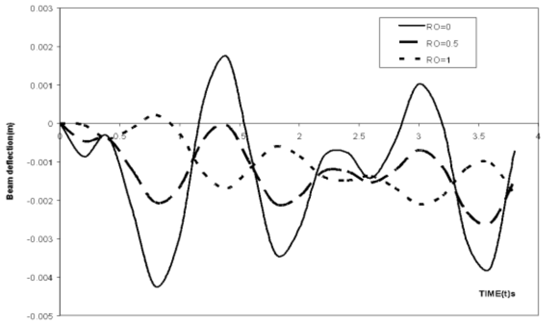

Figure 4. Displacement response of beam (harmonic magnitude) for fixed FM=4000000, N=500000 and various values of Ro.

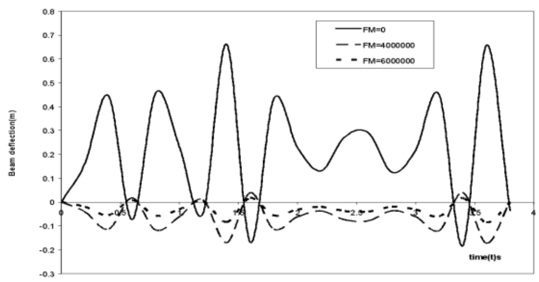

Figure 5. Displacement response of beam (harmonic magnitude) for fixed Ro=0.5, N=500000 and various values of FM.

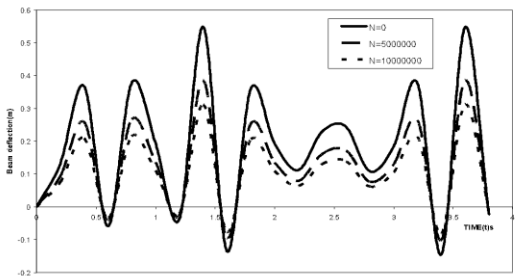

Figure 6. Displacement response of beam (harmonic magnitude) for fixed FM=4000000, Ro =0.5 and various values of N.

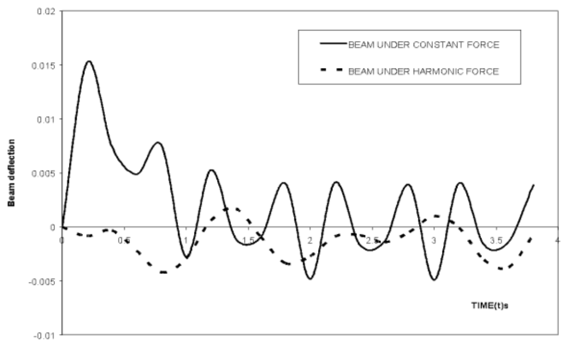

Figure 7. Comparison deflection of the beam on exponentially decaying foundation for constant and harmonic magnitude.

VIII. CONCLUSIONS

The problem of the vibration procedure of prestressed non-uniform beam having exponential decaying subgrade under mobile concentrated forces for both constant and harmonic variable forces is considered is examined. In this present paper. Solutions have been provided for the motion equation by the help of Galerkin's method which reduced it to second order ordinary differential equation. Numerical analysis for both constant and harmonic variable mobile forces is presented and various curves displayed.

It is observed that:

(i) as the Ro increases, the deflection of beam under the action of mobile concentration forces with constant velocity decrease for both constant and harmonic magnitude loads.

(ii) when the rotatory inertia r_0 is fixed, the amplitude of an exponential decaying foundation beam and traversed by concentrated forces travelling with constant speed decreases as the foundation modulus increases for both constant and harmonic magnitude loads.

(iii) increase in values of N reduce the deflection profile of non-prismastic beams for mobile concentration forces with constant velocity

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