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Skolem Mean Labeling of Four Star Graphs $K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ Where $\eta_1 + \eta_2 + 1 \leq \tau_1 + \tau_2 \leq \eta_1 + \eta_2 + 2$

J. Vinolin, D. S. T. Ramesh, S. Athisayanathan

Abstract: *Skolem mean labeling of the four star $G = K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ where $\eta_1 \leq \eta_2$ and $\tau_1 \leq \tau_2$ is a skolem mean graph if $1 \leq \left|\sum_{i=1}^{2} \tau_i - \sum_{i=1}^{2} \eta_i\right| \leq 2$ is the main purpose of this article . Here we partite the four star into two pairs and then found the labeling function which proves the four star to be skolem mean using mathematical calculations.*

Keywords: *Mean graph, Skolem mean graph, skolem mean labeling, star graphs.*

I. INRODUCTION

The idea of skolem mean labeling was first conceived by V.Balaji et. al.[3] in the year 2007. In that paper [3] he gave the definition of skolem mean labeling for the first time and also some basic properties for a graph to be a skolem mean graph. The most important properties are (i) If G is a graph with n vertices and m edges then G is said to be a skolem mean graph only if n ≥ m + 1, (ii) The graphs which satisfies the condition n ≥ m + 1 are paths and star graphs (iii) Every path is skolem mean (iv) G = $K_{1,n}$ where n ≥ 4 is not a skolem mean graph. Therefore, we used the n – star to get a detailed research about skolem mean labeling.

II. PRELIMINARIES

Definition 1: A graph label is the assigning of labels to edges and also vertices of a graph or simply either edges or vertices of a graph only by integers.

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* Correspondence Author
J.Vinolin*, Research Scholar, Department of Mathematics Reg .No. 12310, St. Xavier's College, Palayamkottai, Tirunelveli-627002, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, TamilNadu, India.judevino3010@gmail.com.
D.S.T.Ramesh, Department of Mathematics, Nazareth Margoschis College, Pillaiyanmanai,Thoothukudi-628617, Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli-627 012, TamilNadu, India. dstramesh@gmail.com
S.Athisayanathan, Department of Mathematics, Loyola College, Nungambakkam, Chennai -600034, Affiliated to University of Madras, Chennai – 600005, TamilNau, India athisxc@gmail.com

We define the skolem mean labeling of a graph G with vertex set V and edge set E of order n and size m as follows:

Definition 2: The vertex labeling f : V⟶{1, 2, …, n} and the induced edge labeling f^*: E⟶{2,3…, n} is a skolem mean labeling if both f and f^* are one – one functions such that f^*(e = uv) = [f(u)+f(v)] / 2 if the sum of the vertex label u and v is even and [f(u)+f(v)+1] / 2 if the sum of the vertex label u and v is odd.

III. MAIN RESULT

Theorem 1: Four star graph $G = K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ where $\eta_1 \leq \eta_2$ and $\tau_1 \leq \tau_2$ is skolem mean graph if $1 \leq |\sum_{i=1}^{2} \tau_i - \sum_{i=1}^{2} \eta_i| \leq 2$.

Proof: Let $N_k = \sum_{i=1}^{k} \eta_k ; 1 \leq k \leq 2; \ T_k = \sum_{i=1}^{k} \tau_k ; 1 \leq k \leq 2.$
That is, $N_1 = \eta_1; N_2 = \eta_1 + \eta_2$ and $T_1 = \tau_1; T_2 = \tau_1 + \tau_2$.
Consider the graph $G = K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ having $V = V_1 \cup V_2 \cup V_3 \cup V_4$ as its set of vertices of G where $V_k = \{v_{k,i}: 0 \leq i \leq \eta_k\}$ for $1 \leq k \leq 2$, $V_3 = \{v_{3,i} : 0 \leq i \leq \tau_1\}$, $V_4 = \{v_{4,i} : 0 \leq i \leq \tau_2\}$. Let $E = \bigcup_{i=1}^{2}\{v_{i,0}v_{i,j} : 1 \leq j \leq \eta_i\}$ $\bigcup_{i=1}^{2}\{v_{2+i,0}v_{2+i,j} : 1 \leq j \leq \tau_i\}$ be the set of edges of G.
The condition $\eta_1 + \eta_2 + 1 \leq \tau_1 + \tau_2 \leq \eta_1 + \eta_2 + 2 \Rightarrow N_2 + 1 \leq T_2 \leq N_2 + 2$.
That is, there are two cases viz. $T_2 = N_2 + 1$ and $T_2 = N_2 + 2$.

Case A: $T_2 = N_2 + 1$.

G has $N_2 + T_2 + 4 = 2N_2 + 5$ vertices and $N_2 + T_2 = 2N_2 + 1$ edges. The vertex labeling $f : V \to \{1, 2, 3, \dots, N_2 + T_2 + 4 = 2N_2 + 5\}$ given as:

$f(v_{1,0}) = 1; \ f(v_{2,0}) = 2;$
$f(v_{3,0}) = N_2 + T_2 + 3 = 2N_2 + 4$
$f(v_{4,0}) = N_2 + T_2 + 4 = 2N_2 + 5$
$f(v_{1,i}) = 2i + 2 \qquad 1 \leq i \leq \eta_1$
$f(v_{2,i}) = 2N_1 + 2i + 2 \qquad 1 \leq i \leq \eta_2$
$f(v_{3,i}) = 2i + 1 \qquad 1 \leq i \leq \tau_1$
$f(v_{4,i}) = 2T_1 + 2i + 1 \qquad 1 \leq i \leq \tau_2$

Their induced labels for edges are as follows:

The induced label of $v_{1,0}v_{1,i}$ is 2 + i where $1 \leq i \leq \eta_1$ (edge labels are 3, 4,…., η_1 + 2 = N_1 + 2), $v_{2,0}v_{2,i}$ is N_1 + 2 + i for $1 \leq i \leq \eta_2$ (edge labels are N_1 + 3, N_1 + 4, …, $N_1 + \eta_2$ + 2 = N_2 + 2), $v_{3,0}v_{3,i}$ is N_2 + 3 + i for $1 \leq i \leq \tau_1$ (edge labels are N_2 + 4, N_2 + 5, …, $N_2 + \tau_1$ + 3), $v_{4,0}v_{4,i}$ is $N_2 + \tau_1$ + 3 + i for $1 \leq i \leq \tau_2$ (edge labels are $N_2 + \tau_1$ + 4, $N_2 + \tau_1$ + 5, …, $N_2 + \tau_1 + \tau_2$ + 3 = $N_2 + T_2$ + 3 = $2N_2$ + 4).The labels of edges induced by the labels of vertices of graph G are distinct. This shows that G is skolem mean .

Case B: $T_2 = N_2 + 2$ Let $T_2 = N_2 + 2$. G has $N_2 + T_2 + 4 = 2N_2 + 6$ vertices and $N_2 + T_2 = 2N_2 + 2$ edges. The vertex labeling $f: V \to \{1, 2, 3, \dots, N_2 + T_2 + 4 = 2N_2 + 6\}$ is as follows:

$f(v_{1,0}) = 1;\ f(v_{2,0}) = 2;$

$f(v_{3,0}) = N_2 + T_2 + 2 = 2N_2 + 4$

$f(v_{4,0}) = N_2 + T_2 + 4 = 2N_2 + 6$

$f(v_{1,i}) = 2i + 2 \qquad 1 \leq i \leq \eta_1$

$f(v_{2,i}) = 2N_1 + 2i + 2 \qquad 1 \leq i \leq \eta_2$

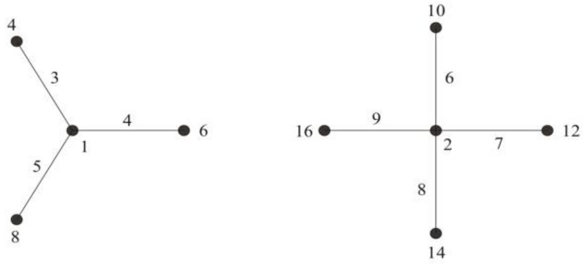
$f(v_{3,i}) = 2i + 1 \qquad 1 \leq i \leq \tau_1$

$f(v_{4,i}) = 2T_1 + 2i + 1 \qquad 1 \leq i \leq \tau_2$

Their induced labels for edges are as follows:

The induced label of $v_{1,0}v_{1,i}$ is 2 +i where $1 \leq i \leq \eta_1$ (edge labels are 3, 4,…., η_1+ 2 = N_1 + 2), $v_{2,0}v_{2,i}$ is N_1 + 2 + i for $1 \leq i \leq \eta_2$ (edge labels are N_1 + 3, N_1 + 4, …, $N_1 + \eta_2$ + 2 = N_2 + 2), $v_{3,0}v_{3,i}$ is N_2 + 3 + i for $1 \leq i \leq \tau_1$ (edge labels are N_2 + 4, N_2 + 5, …, $N_2 + \tau_1$ + 3), $v_{4,0}v_{4,i}$ is $N_2 + \tau_1$ + 4 + i for $1 \leq i \leq \tau_2$ (edge labels are $N_2 + \tau_1$ + 5, $N_2 + \tau_1$ + 6, …, $N_2 + \tau_1 + \tau_2$ + 4 = $N_2 + T_2$ + 4 = $2N_2$ + 6). The labels of edges induced by the labels of vertices of graph G are distinct. This shows that the four star graph $G = K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ is skolem mean . We illustrate the above two cases with the following four star graphs.

Fig.1. is an illustartion of Case A where $T_2 = N_2 + 1$



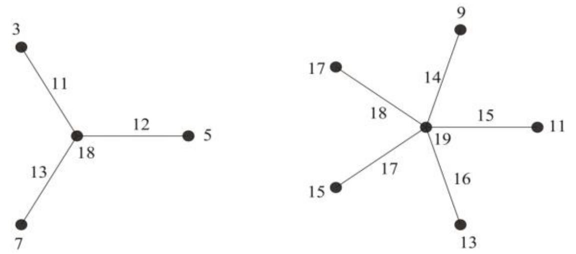
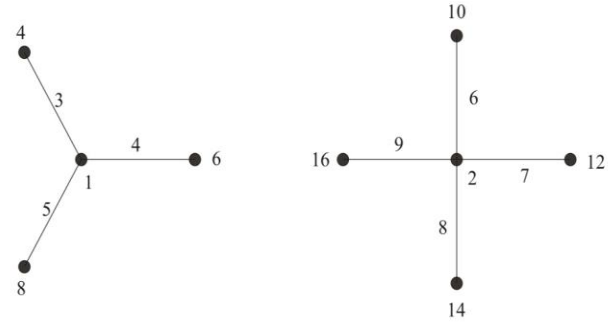


Fig. 1. G = $K_{1,3} \cup K_{1,4} \cup K_{1,3} \cup K_{1,5}$

In this graph , $\eta_1 + \eta_2$ = 3 + 4 = 7 and $\tau_1 + \tau_2$ = 3 + 5 = 8 = N_2 + 1.

Fig. 2. is an illustration of Case B where $T_2 = N_2 + 1$



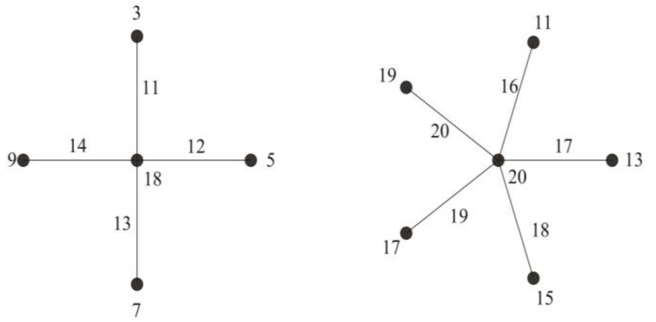


Fig. 2. G = $K_{1,3} \cup K_{1,4} \cup K_{1,4} \cup K_{1,5}$

In this graph , $\eta_1 + \eta_2$ = 3 + 4 = 7 and $\tau_1 + \tau_2$ = 4 + 5 = 9 = N_2 + 2.

IV. CONCLUSION

Skolem mean labeling of a four star graph $G = K_{1,\eta_1} \cup K_{1,\eta_2} \cup K_{1,\tau_1} \cup K_{1,\tau_2}$ with a partition of four into 2, 2 is discussed in this paper . We mainly discussed the two cases $T_2 = N_2 + 1$ and $T_2 = N_2 + 2$ and gave the labeling which stasifies the condition of skolem mean labeling exisits for graph G. We are in further research to find upto how many cases the graph G will allow skolem mean labeling.

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AUTHORS PROFILE

J.Vinolin, M.Sc., M.Phil., Research Scholar, St. Xavier’s College, Palayamkottai, Tamil Nadu. Published two papers in National and International Journals. Presented 2 papers in two National Conferences.

Dr. D.S.T.Ramesh, M.Sc., M.Phil., Ph.D., H**e** is an Associate Professor of Mathematics in Nazerth Margochis College, Pillayanmanai, Tuticorin, Tamil Nadu. He has 31 years of teaching experience. He guided 8 Ph.D."s and 5 M. Phil.. His area of specialization is Graph Theory. He published 51 research papers in National and International Journals. He was invited as a resource person for National conferences organized by Sacred Heart College, Tirupattur and Pope's College, Sawyerpuram.

Dr. S. Athisayanathan, M.Sc., M.Phil., PGDCA., Ph.D., Head of the Department of Mathematics (S.S), Loyola College, Nungambakkam, Chennai, TamilNadu. He has 34 years of teaching experience. He guided 4 Ph.D."s and more than 40 M. Phil.. His area of specialization is Graph Theory. He published 55 research papers in National and International Journals. He is one of the reviewers of Journal of King Saud University – Science. ICACM-2018, Erode Arts and Science College, Erode, TamilNadu.