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Further Results on Dual Domination in Graphs

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Abstract: Let G = (V, E) be a simple graph. A set $S \subseteq V(G)$ is a dual dominating set of G (or bi-dominating set of G) if S is a dominating set of G and every vertex in S dominates exactly two vertices in V-S. The dual-domination number $\gamma_{du}(G)$ (or bi-domination number $\gamma_{bi}(G)$) of a graph G is the minimum cardinality of the minimal dual dominating set (or dual dominating set). In this paper dual domination number and relation with other graph parameters are determined.

Keywords: Domination, dual-domination, chromatic number and connectivity.

I. INTRODUCTION

Let G(V,E) be a simple, connected graph where V(G) is its vertex set and E(G) is its edge set. The degree of any vertex v in G is the number of edges incident with v and is denoted by deg v. The minimum degree of a graph is denoted by δ(G) and the maximum degree of a graph G is denoted by $\Delta(G)$. A vertex of degree 1 is called a pendent vertex. In this paper, dual domination number with other parameters are determined. For graph theoretic notations, Harary [1]and Gray chartand [2] are referred to.

II. PRELIMINARIES

Definition 2.1:[1] The chromatic number χ(G) is defined as the minimum n for which G has an n-coloring. A graph G is n-colorable if $\chi(G) \leq n$ and is n-chromatic if $\chi(G) = n$.

Definition 2.2:[1] The connectivity $\kappa = \kappa(G)$ of a graph G is the minimum number of points whose removal results in a disconnected or trivial graph.

Definition2. 3:[5] A set $S \subseteq V(G)$ is a dual dominating set of G if S is a dominating set of G and every vertex in S dominates exactly two vertices in V-S.

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Remark 2.4: The dual domination number γ_{du}(G) of a graph G is the minimum cardinality of all minimal dual dominating sets. The maximum cardinality of a dual dominating set of G is called the upper dual domination number of G and it is denoted by Γ_{du}(G).

Theorem 2.5[5]: Let G be a connected graph, If $G = K_n$ then $\gamma_{du}(G)$ = n - 2.

III. MAIN RESULT

Theorem 3.1: For any connected graph G with $n \geq 5$ vertices, $\gamma_{du}(G) + \chi(G) \leq 2n - 2$ and the bound is sharp if and only if
$G \cong K_n$.

Proof: Let G be a connected graph with $n \geq 5$ vertices. We know that $\chi(G) \leq n$ and by theorem[1.5], $\gamma_{du}(G) \leq n - 2$. Hence $\gamma_{du}(G) + \chi(G) \leq 2n - 2$. Suppose G is isomorphic to K_n. Then clearly $\gamma_{du}(G) + \chi(G) = 2n - 2$. Conversely, let $\gamma_{du}(G)+\chi(G) = 2n- 2$.

Case(i): Suppose $\chi(G) = n - r$, $r \geq 1$. Since $\gamma_{du}(G) + \chi(G) = 2n - 2$, $\gamma_{du}(G) = n + r - 2$, a contradiction.

Case(ii): Suppose $\gamma_{du}(G) = n - r$, $r \geq 3$. Since $\gamma_{du}(G) + \chi(G) = 2n - 2$, $\chi(G) = n + r - 2$, $r \geq 3$, a contradiction. From both cases it is observed that $\gamma_{du}(G) + \chi(G) = 2n - 2$ is possible only if $\gamma_{du}(G) = n - 2$ and $\chi(G) = n$. Hence G is isomorphic to K_n .

Theorem 3.2: For any connected graph G with $n \geq 3$ $\gamma_{du}(G) + \Delta(G) \leq 2n - 3$

Proof: Let G be a connected graph with $n \geq 5$ vertices. We know that for any connected graph G, $\Delta(G) \leq n - 1$. Since $\gamma_{bi}(G) \leq n - 2$, $\gamma_{du}(G) + \Delta(G) \leq 2n - 3$.

Example 3.3: Consider the following graph G is given in the following figure 1

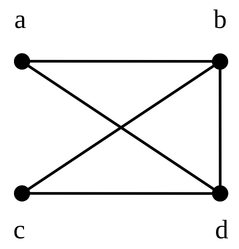

Figure1

Let $S_1 = \{a, c\}$ and $S_2 = \{b, d\}$, every vertex of the set S_i , $1 \leq i \leq 2$ dominates exactly two vertices in V - S_i. Hence S_i , $1 \leq i \leq 2$ are the dual dominating set of G, $\gamma_{du}(G) \leq 2$. Since G is not isomorphic to either C_3 or P_3, $\gamma_{du}(G) \geq 2$. Hence $\gamma_{du}(G) = 2$ and $\Delta(G) = 3$, $\gamma_{du}(G) + \Delta(G) = 5 = 2n - 3$.

Theorem 3.4: Let G be a graph of order $n \geq 5$. Then $\gamma_{du}(G) + \gamma_{du}(\bar{G}) \leq 2n - 6$ and the bound is sharp.

Proof: Case(1): Suppose $\gamma_{du}(G) = n - 2$. Let S be a γ_{du} – set. Let V - S = {u, v}, the two vertices u and v may or may not be adjacent with both u and v in G. Let H = $< S >$. Hence $\bar{G} = \bar{H} \cup K_2$ or $\bar{H} \cup 2K_1$. Since K_2 and K_1 do not have dual dominating set, $\bar{G}$ has no dual dominating set.

Subcase(1a): Suppose $\gamma_{du}(G) = n - 3$. Since $\gamma_{du}(\bar{G}) \neq n - 1$, $\gamma_{du}(G) + \gamma_{du}(\bar{G}) \neq 2n - 4$.

Subcase(1b): Suppose $\gamma_{du}(G) = n - 4$. Since $\gamma_{du}(\bar{G}) \neq n$, $\gamma_{du}(G) + \gamma_{du}(\bar{G}) \neq 2n - 4$.

Subcase(1c): Suppose $\gamma_{du}(G) = n - r$, $r \geq 5$. Since $\gamma_{du}(\bar{G}) \neq n + s$, $s \geq 1$, $\gamma_{du}(G) + \gamma_{du}(\bar{G}) \neq 2n - 4$.

From the cases (1), (1a) and (1b), $\gamma_{du}(G) + \gamma_{du}(\bar{G}) \neq 2n - 4$.

Case(2): Suppose either $\gamma_{du}(G)$ or $\gamma_{du}(\bar{G})$ is equal to $n - 2$. As in case(1) dual dominating set doesnot exist for G or $\bar{G}$.

Subcase(2a): Suppose $\gamma_{du}(G) = n - 4$. Since $\gamma_{du}(\bar{G}) \neq n - 1$. Hence $\gamma_{du}(G) + \gamma_{du}(\bar{G}) \neq 2n - 5$.

Subcase(2b): Suppose $\gamma_{du}(G) = n - r$, $r \geq 5$. Since $\gamma_{du}(\bar{G}) \neq n + s$, $s \geq 0$. Hence $\gamma_{du}(G) + \gamma_{du}(\bar{G}) \neq 2n - 5$.

case(3): Suppose $\gamma_{du}(G) = n - 5$ and $\gamma_{du}(\bar{G}) \neq n - 1$. Hence $\gamma_{du}(G) + \gamma_{du}(\bar{G}) \neq 2n - 6$.

Subcase(3a): Suppose $\gamma_{du}(G) = n - r$, $r \geq 6$ and $\gamma_{du}(\bar{G}) \neq n + s$, $s \geq 0$. Hence $\gamma_{du}(G) + \gamma_{du}(\bar{G}) \neq 2n - 6$.

Case(4): Let G = C_5 and $\bar{G}$ is also C_5 and $\gamma_{du}(C_5) = 2$. Hence $\gamma_{du}(G) + \gamma_{du}(\bar{G}) = 4 = 2n - 6$.

From all the cases $\gamma_{du}(G) + \gamma_{du}(\bar{G}) \leq 2n - 6$.

Remark 3.5: Let |V(G)| = 4. G has a dual dominating set if and if G is isomorphic to C_4, K_4 and K_4 – e. Hence $\bar{G} = 2K_2$, $4K_1$ and $K_2 \cup 2K_1$ respectively. Hence $\bar{G}$ has no dual dominating set.

Remark 3.6: Let |V(G)| = 3. G has a dual dominating set if and if G is isomorphic to P_3 and C_3. Hence $\bar{G} = 3K_1$ and $K_2 \cup K_1$ respectively. Hence $\bar{G}$ has no dual dominating set.

Theorem 3.7: Let G be a connected graph with $n \geq 3$ vertices, $\gamma_{du}(G) + \kappa(G) \leq 2n - 3$ and the bound is sharp if and only if G is isomorphic to K_n.

Proof: Let G be a connected graph with $n \geq 3$. We know that $\kappa(G) \leq n - 1$ and $\gamma_{du}(G) \leq n - 2$. Hence $\gamma_{du}(G) + \kappa(G) \leq 2n - 3$. Suppose G is isomorphic to K_n. Then clearly $\gamma_{du}(G) + \kappa(G) = 2n - 3$. Conversly, Let $\gamma_{du}(G) + \kappa(G) = 2n - 3$. This is possible only if $\gamma_{du}(G) = n - 2$ and $\kappa(G) = n - 1$. Hence G is isomorphic to K_n.

Theorem 3.8: Let G be a connected graph with $n \geq 4$ vertices. Let S be a minimum dual dominating set of G . If $\kappa(G) = n - 2$ or $n - 1$, $\Delta(G) = n-1$, $\chi(G) = n-1$ or n, and diam(G) = 2 or 1 iff |S| = n – 2 and $< S >$ is complete graph.

Proof: Let G be a connected graph with $n \geq 4$ vertices. S = { $v_1, v_2, \ldots, v_{n-2}$} is the dual dominating set of G and $< S >$ is complete graph.

Case(i): Suppose the vertices v_{n-1} and v_n belong to V – S is adjacent with each other. Then clearly $\kappa(G) = n - 1$, $\Delta(G) = n-1$, $\chi(G) = n$, and diam(G) = 1.

Case(ii): Suppose the vertices v_{n-1} and v_n belong to V – S not adjacent with each other. Then clearly $\kappa(G) = n - 2$, $\Delta(G) = n-1$, $\chi(G) = n - 1$, and diam(G) = 2.

Conversely,

Case(i): Suppose $\kappa(G) = n - 1$ then G is isomorphic to K_n. Clearly $\Delta(G) = n-1$, $\chi(G) = n$, and diam(G) = 1.

Let V(G) = {$v_1, v_2, \ldots, v_n$}. S = {$v_1, v_2, \ldots, v_{n-2}$ } is the minimum dual dominating set of G. |S| = n – 2 and $< S >$ is complete graph.

Case(ii): Suppose $\chi(G) = n - 1$ then G is isomorphic to K_n - e. Clearly $\Delta(G) = n-1$, $\kappa(G) = n - 2$, and diam(G) = 2.

Let V(G) = {$v_1, v_2, \ldots, v_n$}. S = {$v_1, v_2, \ldots, v_{n-2}$ }is the minimum dual dominating set of G. The vertices v_{n-1} and v_n not adjacent with each other. |S| = n – 2 and $< S >$ is complete graph.

IV. CONCLUSION

In this paper, dual domination number with chromatic number, connectivity and Nordhaus-Gaddum type result are discussed .

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