

# Enhanced CORDIC Based Rotator Design for Sinusoidal Transforms



Trivedi Pratik, Zaveri Tanish

**Abstract:** Transforms play an important role in conversion of information from one domain to the other. To be more specific transforms like Discrete Fourier transform (DFT) and Discrete Cosine transform (DCT) helps us to migrate from one time domain to frequency domain based on the basis function selected. The basis function of the every sinusoidal transform carries out a circular rotation to convert information from one domain to the other. There are applications related to communication which requires this rotation into the hyperbolic trajectory as well. Multiplierless algorithm like CORDIC improves the latency of the transforms by eliminating the number of multipliers in the basis function. In this paper we have designed and implemented enhanced version of CORDIC based Rotator design. The Enhanced version is simulated for order 1 to order 36 to emphasize on the results of the proposed algorithm. Results shows that the enhanced CORDIC rotator design surpasses the Mean square error after the order 18 compared to standard CORDIC. Unified CORDIC also can be implemented using the said algorithm to implement different three trajectories.

**Keywords:** CORDIC, DCT, DFT.

## I. INTRODUCTION

Multipliers play an important role in digital signal processing applications now a days as people are hungrier about data and hence the amount of information that needs to be processed is very large and the time required to process that amount is limited. In such kind of cases it becomes critical to use multiplier based algorithms and hence CORDIC[1] and related multiplier less algorithms are popular since their inception. However CORDIC based approaches have certain drawbacks like the number of iterations and scaling factor. Unified CORDIC[7] approach takes this algorithm further by adding just a parameter and increasing the trajectory from only circular to linear, circular and hyperbolic.

The unified approach is very crucial if large number of applications needs to be covered by a single tunable hardware for all three different trajectories. The moment Unified CORDIC[7] combines the three approached the problem of

ROC(region of convergence) comes into picture as all the three trajectories ROCs are not matched to a single architecture. It is extremely important to optimise the number of hardware components used by Unified CORDIC[7] and also their ROCs needs to be matched in all cases. Here we propose an architecture which combines the three trajectories and takes care of the ROCs as well.

Section II. gives an overview on the basics of CORDIC[1] algorithm, section III talks about Unified CORDIC[1], section IV discuss on our approach and V focuses on results and analysis followed by conclusion and references.

## II. CORDIC ALGORITHM[1]

CORDIC[1] stands for the Co-ordinate Rotation In Digital Computer. It tries to bring down the multipliers used in the rotators by shift and add approach. The equations (1) shows the normal rotation for points  $x_i$  and  $y_i$  by and angle  $\theta$  & equation (2) below show the normal operation of CORDIC[1] algorithm. As we can see that there are some assumptions taken into consideration in equation (1) to convert it into shift and add approach. It has an advantage of having only a single multiplication of scaling factor if the number of iterations are comparatively higher.

$$\begin{bmatrix} X_{i+1} \\ Y_{i+1} \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \begin{bmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} X_{i+1} \\ Y_{i+1} \end{bmatrix} = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \cos \theta_i \begin{bmatrix} 1 & -\tan \theta_i \\ \tan \theta_i & 1 \end{bmatrix} \quad (2)$$

Where  $\tan \theta_i = 2^{-i}$

## III. UNIFIED CORDIC[7]

CORDIC algorithms can then be computed with help of linear, circular, and hyperbolic trajectories. The equations (3) & (4) describes that by adding a parameter  $m$  to the basic CORDIC, it can be made to operate in three different trajectories viz. circular, linear and hyperbolic.

$$X_{i+1} = x_i - m d_i y_i 2^{-i} \quad (3)$$

$$Y_{i+1} = y_i + d_i x_i 2^{-i} \quad (4)$$

Where  $m=0, 1$  and  $-1$  for linear circular and hyperbolic trajectories. Circular and hyperbolic find various applications and used extensively. CORDIC in circular trajectory is mainly used for the computation of sinusoidal functions, it also finds it applications in transform calculations, whereas, hyperbolic version of CORDIC is mainly applicable for finding the exponents and neural networks.

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\* Correspondence Author

**Pratik Trivedi\***, Electronics and Communication engineering department, Institute of Technology, Nirma University, Ahmedabad, India Email: erpratik84@gmail.com

**Zaveri Tanish**, Electronics and Communication engineering department, Institute of Technology, Nirma University, Ahmedabad, India Email: er\_pratik84@yahoo.co.in

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IV. ENHANCED SCALING FREE CORDIC[3]

The sine and cosine components present in the normal CORDIC equations can also be represented in the form on Taylor series expansion and can be written as

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad (5)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad (6)$$

This brings down the enhanced CORDIC equations as

$$X_{i+1} = x_i \left( 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \right) - y_i \left( x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \right) \quad (7)$$

$$Y_{i+1} = y_i \left( 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \right) + x_i \left( x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \right) \quad (8)$$

V. RESULTS AND DISCUSSIONS

In order to evaluate the performance of standard and enhanced CORDIC for better accuracy and feasibility of transforms, the coefficients from both the approaches were calculated and their mean square error were found with respect to the expected value of coefficients. The equations depict the formula used for the MSE.

$$MSE_{std\_cordic} = \sqrt{(X_{expected} - X_{std\_cordic})^2 + (Y_{expected} - Y_{std\_cordic})^2} \quad (9)$$

$$MSE_{enhanced\_cordic} = \sqrt{(X_{expected} - X_{enhanced\_cordic})^2 + (Y_{expected} - Y_{enhanced\_cordic})^2} \quad (10)$$

The MSE for 0-4096 points were calculated. Order 0,2,4,6,8,10,12,14,16 and 18 were considered for calculating enhanced CORDIC.

A. Results for Basic CORDIC[1] Algorithm

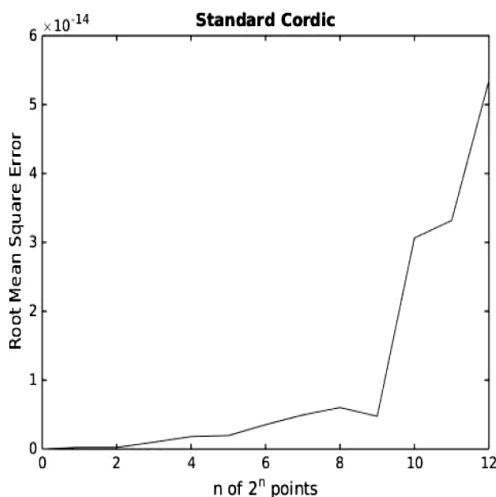


Fig.1.Mean Square error for N-point FFT for CORDIC Algorithm

From the above results, we could see that the MSE for standard CORDIC is in the order of 10<sup>-14</sup> but the error goes on increasing as we increase the number of points for FFT.

This is only due to the error accumulation as we increase the number as the iterated number will be used to calculate the next one.

B. Results for Enhanced CORDIC

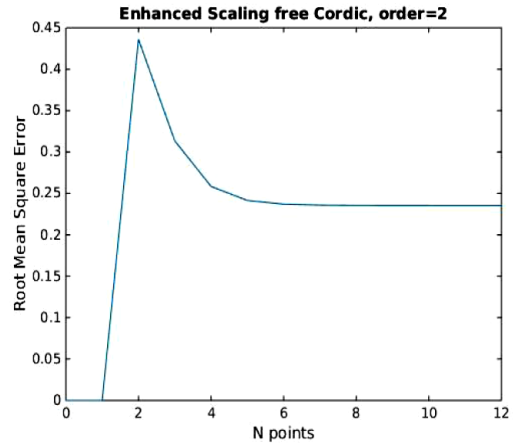


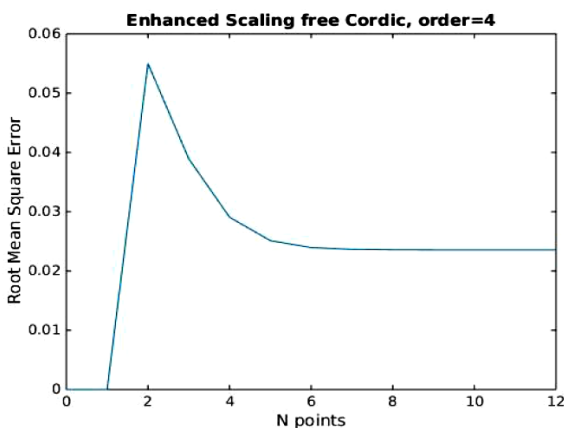
Fig. 2. Mean Square error for N-point FFT for Enhanced CORDIC Algorithm (2<sup>nd</sup> Order accuracy)

In Fig.2 the error reduces with the number of points except that there is a spike at 2 point where we have already assumed the values to be 0 and 1 as the angle would be 0 and pi. But the error is very high in comparison with standard CORDIC[1] as it is in the order 10<sup>-2</sup>.

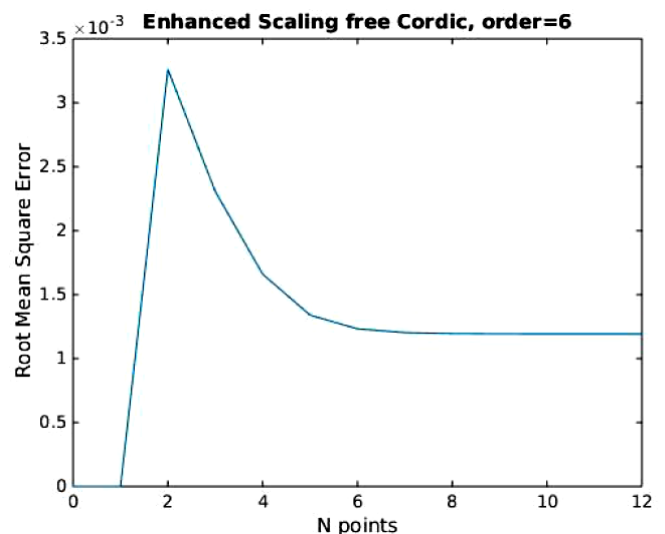
**Table-I Error Analysis for Standard CORDIC[1] and Enhanced CORDIC**

Without Shifter	WoSh	Error analysis for standard and Enhanced CORDIC with and without shifters							
With Shifters	WSh								
	Enhanced CORDIC								Standard CORDIC[1]
N Points	Order 2		Order 4		Order 6		Order 18		
	WoSh	WSh	WoSh	WSh	WoSh	WSh	WoSh	WSh	
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	2.5 e-16
4	0.4361	0.4361	0.055	0.0683	0.0033	0.125	3.10E-14	0.1211	2.30E-16
8	0.3135	0.3135	0.0389	0.05	0.0023	0.0895	2.19E-14	0.0868	9.90E-16
16	0.2584	0.2584	0.0291	0.044	0.0017	0.0726	1.55E-14	0.0707	1.81E-16
32	0.2416	0.2416	0.0251	0.0425	0.0013	0.0671	1.10E-14	0.0656	1.95E-15
64	0.2371	0.2371	0.0239	0.0422	0.0012	0.0656	8.42E-15	0.0642	3.54E-15
128	0.2359	0.2359	0.0236	0.0421	0.0012	0.0652	7.40E-15	0.0639	4.94E-15
256	0.2356	0.2356	0.0236	0.0421	0.0012	0.0652	7.11E-15	0.0638	6.02E-15
512	0.2356	0.2356	0.0235	0.0421	0.0012	0.0651	7.03E-15	0.0638	4.76E-15
1024	0.2356	0.2356	0.0235	0.0421	0.0012	0.0651	7.01E-15	0.0638	3.06E-14
2048	0.2355	0.2355	0.0235	0.0421	0.0012	0.0651	7.00E-15	0.0638	3.32E-14
4096	0.2355	0.2355	0.0235	0.0421	0.0012	0.0651	7.00E-15	0.0638	5.35E-14

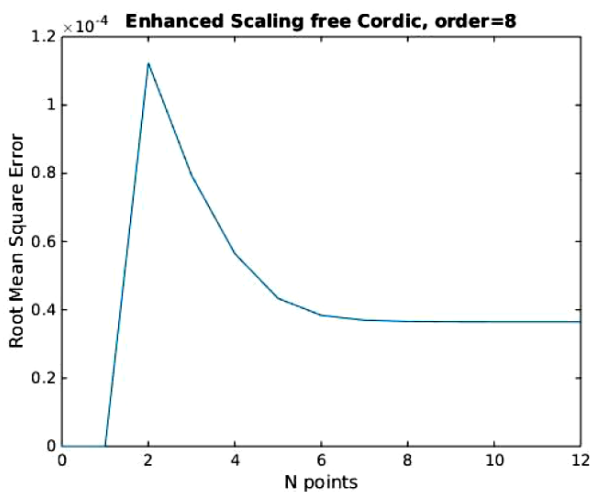
Table-I indicates the analysis of the Means square error calculated for standard CORDIC[1] and the proposed enhanced CORDIC starting from N=1 till 4096 points and we can see till the 18<sup>th</sup> order the error seems to be higher for enhanced CORDIC but it surpasses the Standard CORDIC after that as in case of standard CORDIC the error accumulates at each iteration.



**Fig. 3. Mean Square error for N-point FFT for Enhanced CORDIC Algorithm (4th Order accuracy)**

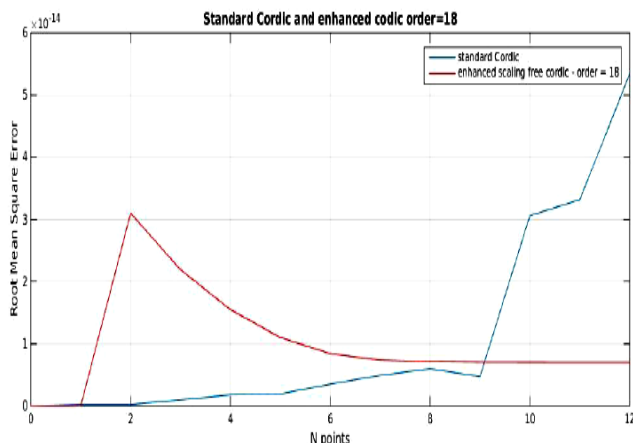


**Fig. 4. Mean Square error for N-point FFT for Enhanced CORDIC Algorithm (6th Order accuracy).**



**Fig. 5. Mean Square error for N-point FFT for Enhanced CORDIC Algorithm (8th Order accuracy)**

Fig.3, 4 and 5 depicts the MSE for 4<sup>th</sup>, 6<sup>th</sup> and 8<sup>th</sup> order enhanced CORDIC which indicates that the MSE reduces considerably to the order of  $10^{-4}$  for the 8<sup>th</sup> order. The figures shows us that using enhanced CORDIC the error goes on reducing as the number if points and the order goes on increasing.



**Fig. 5. Comparison for MSE for standard CORDIC and enhanced scaling free CORDIC**

Fig.5 clearly indicates that after the order 18 the enhanced scaling free CORDIC outperforms the standard CORDIC.

## VI. CONCLUSIONS

The results and discussions above shows that by increasing the order there is a considerable increase in accuracy compared to the basic CORDIC[1] algorithm. The Enhanced Scaling free CORDIC surpasses the basic CORDIC[1] at 18<sup>th</sup> order of accuracy and also it is scaling free and hence one need not to multiply the constant at the end as well. This improves the latency of the transforms used in signal processing algorithms.

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## AUTHORS PROFILE

**Pratik Trivedi**, is a Doctoral student at Institute of Technology, Nirma



University and Senior lecturer at School of engineering and applied sciences, Ahmedabad University. He has published 08 papers in various international conferences and journals. His areas of interest are architectures for signal processing Algorithms, Digital design etc. He is member of IEEE and lifetime member

of ISTE.



**Dr. Tanish Zaveri**, is a Professor at Institute of Technology, Nirma University. He completed his PhD from Sardar Vallabhbhai National Institute of Technology, Surat. He has published 25 papers in international conference and journals. His areas of interest are architectures for signal processing algorithms, Image processing, Machine learning etc. He is a member of IEEE.