

Flow of Micropolar Fluid Between Two Parallel Plates with Different Periodic Suction and Injection

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Abstract: The unsteady stokes flow of incompressible micropolar fluid between two porous plates is considered. The lower plate is subjected to periodic suction and different periodic injection is applied at the upper plate. Stream function for the flow is obtained and the variation of velocity function f' & g with η is shown graphically. The effects of the dimensionless parameters p_∞ , frequency parameter p_i , micropolarity parameter p_l and the microrotation parameter p_j on the velocity functions f' and microrotation velocity function g are discussed and shown through the graphs.

Keywords: Unsteady stokes flow, micropolar fluid, porous plates, periodic suction and injection.

I. INTRODUCTION

Eringen [1] has presented the theory of micropolar fluids. The theory of micropolar fluids become the very active field of research for the last few decades as this class of fluids represents, mathematically, many industrially important fluids such as paints, body fluids, polymers, colloidal fluids, suspension fluids etc. These fluids display the effects of local rotatory inertia and couple stress and may form suitable non-Newtonian fluid models, which can be used to analyze the behaviour of the exotic lubricants, animal bloods, etc. The study of micropolar fluid mechanics has received the attention of many researchers. A list of published papers for this fluid can be found in Eringen [2] and Ishak et.al [3]. The mathematical theory of equations of micropolar fluids and application of these fluids in the theory of lubrication and in the theory of porous media is presented by G. Lukaszewicz [4] in this book.

MICROPOLAR FLUIDS – THEORY AND APPLICATIONS

The problem of steady flow of an incompressible viscous fluid through a porous channel was considered by Berman [5]. He obtained a perturbation solution assuming normal wall velocities to be equal. Sellers [6], Terrill [7] and Yuan [8] have extended the analysis of Berman for various values of suction and injection Reynolds numbers. Terrill and Shretha [9] have examined the same problem, assuming different normal velocities at the walls. Flow of a second-order fluid between torsionally oscillating discs of different permeability was studied by Singh and Agarwal [10].

Asharaf and Bashir [11] have obtained the numerical solution of MHD stagnation point flow and heat transfer of a micropolar fluid towards a heated shrinking sheet. Srinivasacharya, D. et.al [12] have solved the problem of stokes flow of micropolar fluid between two parallel porous plates. They have studied the effect of p_j , p_l & p_i on the skin friction. The purpose of the present paper is to study the flow of micropolar fluid between two parallel plates with different periodic suction. The effects of dimensionless parameter p_j , p_l & p_i on the velocity functions f' & g have shown graphically and the conclusions have been drawn from the behaviour of the velocity functions with respect to different dimensionless parameters.

II. FORMULATION OF THE PROBLEM

Assuming the flow to be stokesian, neglecting the inertial and gyroinertial terms and ignoring the body force and body couple, the field equations of the micropolar fluid are :

$$\text{div } \vec{V} = 0 \tag{1}$$

$$\rho \frac{\partial \vec{V}}{\partial t} = -\text{grad } P + k \nabla \times \vec{\Omega} - (\mu + k) \nabla \times (\nabla \times \vec{V}) \tag{2}$$

$$p_j \frac{\partial \vec{\Omega}}{\partial t} = -2k \vec{\Omega} + k \nabla \times \vec{V} - \gamma \nabla \times (\nabla \times \vec{\Omega}) + (\alpha + \beta + \gamma) \nabla (\nabla \cdot \vec{\Omega}) \tag{3}$$

where \vec{V} is the velocity vector, $\vec{\Omega}$ is the micro-rotation vector and P is fluid pressure, ρ and j are the fluid density and micro-gyration parameter, $\{\mu, k\}$ and $\{\alpha, \beta, \gamma\}$ are viscosity and gyroviscosity coefficients.

The stress tensor τ_{ij} and the couple stress tensor m_{ij} are given by

$$\tau_{ij} = -P \delta_{ij} + (2\mu + k) e_{ij} + k \epsilon_{ijm} (w_m - Q_m), \tag{4}$$

$$m_{ij} = \alpha \Omega_{k,k} \delta_{ij} + \beta \Omega_{i,j} + \gamma \Omega_{j,i}, \tag{5}$$

where \vec{w} is the vorticity vector δ_{ij} is kronecker delta ϵ_{ijm} is the alternating symbol e_{ij} is the strain rate tensor.

Consider the two dimensional flow of micropolar fluid through two porous parallel plates $y=0$ and $y=h$ along the direction of x axis. Since the flow is along the x -direction, all the variables are independent of z . Let us take a periodic suction of velocity $v_0 e^{i\omega t}$ at the lower plate and periodic injection of velocity $nv_0 e^{i\omega t}$ at the upper plate.

Hence we choose the velocity vector \vec{V} , micro-rotation vector $\vec{\Omega}$ and the pressure P in the form

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$$\vec{V} = [u(x, y)\vec{i} + v(x, y)\vec{j}]e^{i\omega t},$$

$$\vec{\Omega} = N(x, y)\vec{k}e^{i\omega t} = (0, 0, N(x, y))e^{i\omega t}$$

and $P(x, y) = p(x, y)e^{i\omega t}$ (6)

We introduce the stream function $\psi(x, y)$ satisfying the continuity equation (1) through

$$u(x, y) = \frac{\partial \psi}{\partial y}, \quad v(x, y) = -\frac{\partial \psi}{\partial x} \quad (7)$$

Using expression (6), (7) in equation (2) and (3) comparing the i, j th component of equation (2) and k th components (which is only existing non zero component) of equation (3) respectively, we get;

$$i\rho\omega \frac{\partial \psi}{\partial y} = -\frac{\partial p}{\partial x} + k \frac{\partial N}{\partial y} + (\mu + k) \frac{\partial}{\partial y} \nabla^2 \psi \quad (8)$$

$$i\rho\omega \frac{\partial \psi}{\partial x} = \frac{\partial p}{\partial y} + k \frac{\partial N}{\partial x} + (\mu + k) \frac{\partial}{\partial x} \nabla^2 \psi \quad (9)$$

$$i\rho\omega jN = -2kN - k\nabla^2 \psi + \gamma \nabla^2 N \quad (10)$$

On differentiating to equation (8) w.r.t. y and to equation (9) w.r.t. x and then adding, the equations thus obtained we get:

$$i\rho\omega \nabla^2 \psi = k \nabla^2 N + (\mu + k) \nabla^4 \psi \quad (11)$$

multiply to equation (10) by k and to (11) by v and then subtracting to the equations thus obtained we get:

$$k(i\rho j\omega + 2k)N = -\gamma(\mu + k) \nabla^4 \psi + (i\rho\omega\gamma - k^2) \nabla^2 \psi \quad (12)$$

on operating ∇^2 on both sides of the equation (12) and then substituting the value of $\nabla^2 N$ in equation (11) we get:

$$\nabla^2 (\nabla^2 - a^2) (\nabla^2 - b^2) \psi = 0 \quad (13)$$

and

$$k(i\rho j\omega + 2k)N = -\gamma(\mu + k) \nabla^4 \psi + (i\rho\omega\gamma - k^2) \nabla^2 \psi \quad (14)$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian operator and

$$a^2 + b^2 = \frac{k(2\mu + k) + i\rho\omega(\gamma + j(\mu + k))}{\gamma(\mu + k)} \quad (15)$$

$$a^2 b^2 = \frac{i\rho\omega(2k + i\rho j\omega)}{\gamma(\mu + k)} \quad (16)$$

III. SOLUTION OF THE PROBLEM

The boundary conditions on the velocity components u, v and microrotation component N are :

$$\begin{aligned} y = 0 & \quad u = 0, \quad v = v_0, \quad N = 0 \\ y = h & \quad u = 0, \quad v = nv_0, \quad N = 0 \end{aligned} \quad (17)$$

Following Terrill [8], we introduce $f(\xi)$ and $g(\zeta)$ as:

$$\psi = h \left[\frac{U_0}{\alpha} - \frac{nv_0}{h} x \right] f(\xi), \quad N = \frac{1}{h} \left[\frac{U_0}{\alpha} - \frac{nv_0}{h} x \right] g(\zeta) \quad (18)$$

where $\zeta = \frac{y}{h}, \alpha = 1 - \frac{v_0}{nv_0}, 0 \leq |v_0| \leq |nv_0|$ and U_0 is

the average entrance velocity.

Using expressions (18) in equations (12) and (13) we get:

$$D^2(D^2 - a^2 h^2) (D^2 - b^2 h^2) f(\zeta) = 0, \quad (19)$$

$$\begin{aligned} kh^2(i\rho j\omega + 2k)g(\zeta) &= -\gamma(\mu + k)D^4 f(\zeta) \\ &+ h^2(i\rho\omega k^2)D^2 f(\zeta) \end{aligned} \quad (20)$$

where

$$D^2 = \frac{d^2}{d\xi^2}.$$

The boundary conditions on $f(\zeta)$ and $g(\zeta)$ become:

$$f(0) = 1 - \alpha = \frac{1}{n}, \quad f(1) = 1, \quad f'(0) = f'(1) = 0, \quad g(0) = g(1) = 0$$

(21)

$n = 2, 3, 4$

The solution of equation (19) is :

$$f(\zeta) = C_1 + C_2 \zeta + C_3 e^{ah\zeta} + C_4 e^{-ah\zeta} + C_5 e^{bh\zeta} + C_6 e^{-bh\zeta} \quad (22)$$

and hence

$$g(\zeta) = A(C_3 e^{ah\zeta} + C_4 e^{-ah\zeta}) + B(C_5 e^{bh\zeta} + C_6 e^{-bh\zeta}) \quad (23)$$

where

$$A = \frac{-h^2 \{ \gamma(\mu + k)a^4 + a^2 k^2 \} + i\gamma\rho\omega h^2 a^2}{(2k^2 + i\rho j\omega k)}$$

$$B = \frac{-h^2 \{ \gamma(\mu + k)b^4 + b^2 k^2 \} + i\gamma\rho\omega h^2 b^2}{(2k^2 + i\rho j\omega k)} \quad (24)$$

Using the dimensionless parameter $p_\alpha \left(= \frac{\mu + k}{k} \right)$;

frequency parameter $p_t \left(= \frac{\rho\omega h^2}{\mu + k} \right)$; micro-polarity

parameter $p_l \left(= \frac{k(2\mu + k)h^2}{\gamma(\mu + k)} \right)$; micro-rotation parameter

$p_j \left(= \frac{j(\mu + k)}{\gamma} \right)$ and the fluid parameter $K \left(= \frac{kh^2}{\gamma} \right)$

The values of $a^2 + b^2, a^2 b^2$ etc. can be obtain as follows :

$$a^2 + b^2 = \beta_1 + i\beta_2, \quad a^2 b^2 = \beta_3 + i\beta_4 \quad (25)$$

where

$$\beta_1 = \frac{1}{h^2} p_l, \quad \beta_2 = \frac{1}{h^2} p_t (1 + p_j), \quad \beta_3 = -\frac{1}{h^4} p_t^2 p_j \text{ and}$$

$$\beta_4 = \frac{2}{h^4} p_t K. \quad (26)$$

To calculate the value of a, b, $a^2 b^2 a^4$ and b^4 in terms of the dimensionless parameter p_α, p_t, p_l, p_j etc. We assume

$$\beta_5 = \beta_1^2 - \beta_2^2 - 4\beta_3, \quad \beta_6 = 2\beta_1\beta_2 - 4\beta_4$$

$$\beta_7 = \sqrt{\beta_5^2 + \beta_6^2}, \quad B_8 = \sqrt{\frac{(\beta_5 + \beta_7)}{2\beta_7}}$$

$$\beta_9 = \sqrt{\frac{(\beta_7 - \beta_5)}{2\beta_7}}, \quad \beta_{10} = \frac{(\beta_1 + \sqrt{\beta_7\beta_8})}{2}$$

$$\beta_{11} = \frac{(\beta_2 + \sqrt{\beta_7\beta_9})}{2}, \quad \beta_{12} = \frac{(\beta_1 - \sqrt{\beta_7\beta_8})}{2}$$

$$\beta_{13} = (\beta_2 - \sqrt{\beta_7 \beta_9}) / 2, \quad \beta_{14} = \sqrt{\beta_{10}^2 + \beta_{11}^2},$$

$$\beta_{15} = \sqrt{\frac{(\beta_{14} + \beta_{10})}{2\beta_{14}}}, \quad \beta_{16} = \sqrt{\frac{(\beta_{14} - \beta_{10})}{2\beta_{14}}},$$

$$\beta_{17} = \sqrt{\beta_{12}^2 + \beta_{13}^2}, \quad \beta_{18} = \sqrt{\frac{(\beta_{17} + \beta_{12})}{2\beta_{17}}},$$

and $\beta_{19} = \sqrt{\frac{(\beta_{17} - \beta_{12})}{2\beta_{17}}}$ (27)

Then

$$a = \sqrt{\beta_{14} \beta_{15}} + i \sqrt{\beta_{14} \beta_{16}},$$

$$b = \sqrt{\beta_{17} \beta_{18}} + i \sqrt{\beta_{17} \beta_{19}},$$

$$a^2 = \beta_{10} + i \beta_{11}, \quad b^2 = \beta_{12} + i \beta_{13},$$

$$a^4 = \beta_{10}^2 - \beta_{11}^2 + i(2\beta_{10} \beta_{11}),$$

$$b^4 = \beta_{12}^2 - \beta_{13}^2 + i(2\beta_{12} \beta_{13}) \quad (28)$$

Now to calculate the value of A and B we assume

$$\beta_{20} = -p_\alpha h^4 (\beta_{10}^2 - \beta_{11}^2) - Kh^2 \beta_{10} - p_t p_\alpha h^2 \beta_{11}$$

$$\beta_{21} = p_t p_\alpha h^2 \beta_{10} - 2p_\alpha h^4 \beta_{10} \beta_{11} - Kh^2 \beta_{11}$$

$$\beta_{22} = (2k\beta_{20} + \beta_{21} p_t p_j) / (4k^2 + p_t^2 p_j^2)$$

$$\beta_{23} = (2K\beta_{21} - \beta_{20} p_t p_j) / (4K^2 + p_t^2 p_j^2)$$

$$\beta_{24} = -p_\alpha h^4 (\beta_{12}^2 - \beta_{13}^2) - Kh^2 \beta_{12} - p_t p_\alpha h^2 \beta_{13},$$

$$\beta_{25} = p + p_\alpha h^2 \beta_{12} - 2p_\alpha h^4 \beta_{12} \beta_{13} - Kh^2 \beta_{13},$$

$$\beta_{26} = (2K\beta_{24} + \beta_{25} p_t p_j) / (4K^2 + p_t^2 p_j^2),$$

$$\beta_{27} = (2K\beta_{25} - \beta_{24} p_t p_j) / (4K^2 + p_t^2 p_j^2) \quad (29)$$

$$\text{Then } A = \beta_{22} + i \beta_{23} \quad (30)$$

$$\text{and } B = \beta_{26} + i \beta_{27} \quad (31)$$

The values of the constants C₁, C₂, C₃, C₄, C₅ and C₆ are obtained from equations (22) and (23) subjected to the boundary conditions (21) for this from (21), (22) and (23) we get

$$\frac{1}{n} = C_1 + C_3 + C_4 + C_5 + C_6 \quad (32)$$

$$1 = C_1 + C_2 + C_3 e^{ah} + C_4 e^{-ah} + C_5 e^{bh} + C_6 e^{-bh} \quad (33)$$

$$0 = C_2 + C_3 ah - C_4 ah + C_5 bh - C_6 bh \quad (34)$$

$$0 = C_2 + C_3 ah e^{ah} - C_4 ah e^{-ah} + C_5 bh e^{bh} - C_6 bh e^{-bh} \quad (35)$$

$$0 = A (C_3 + C_4) + B (C_5 + C_6) \quad (36)$$

$$0 = A (C_3 e^{ah} + C_4 e^{-ah}) + B (C_5 e^{bh} + C_6 e^{-bh})$$

(37) From equations (32), (34) and (36) we get

$$C_1 = \frac{1}{n} + \left(\frac{B}{A} - 1\right) C_5 + \left(\frac{B}{A} - 1\right) C_6 \quad (38)$$

$$C_2 = 2ahC_4 + \left(\left(\frac{B}{A}\right)ah - bh\right)C_5 + \left(\left(\frac{B}{A}\right)ah + bh\right)C_6 \quad (39)$$

$$C_3 = -C_4 - \left(\frac{B}{A}\right)(C_5 + C_6) \quad (40)$$

on putting the values of C₁, C₂ and C₃ in equations no. (33), (35) and (37) we get;

$$1 - \frac{1}{n} = \alpha_1 C_4 + \alpha_2 C_5 + \alpha_3 C_6, \quad (41)$$

$$0 = \alpha_4 C_4 + \alpha_5 C_5 + \alpha_6 C_6, \quad (42)$$

$$0 = -\alpha_7 C_4 + \alpha_8 C_5 - \alpha_9 C_6, \quad (43)$$

where

$$\alpha_1 = 2ah - e^{ah} + e^{-ah},$$

$$\alpha_2 = \left(\frac{B}{A}\right) - 1 + \left(\frac{B}{A}\right)ah - bh - \left(\frac{B}{A}\right)e^{ah} + e^{bh},$$

$$\alpha_3 = \left(\frac{B}{A}\right) - 1 + \left(\frac{B}{A}\right)ah + bh - \left(\frac{B}{A}\right)e^{ah} + e^{-bh},$$

$$\alpha_4 = 2ah - ah e^{ah} - ah e^{-ah} = 2ah - 2ah \cosh(ah),$$

$$\alpha_5 = \left(\frac{B}{A}\right)ah - ah \left(\frac{B}{A}\right)e^{ah} - bh + bh e^{bh},$$

$$\alpha_6 = \left(\frac{B}{A}\right)ah - ah \left(\frac{B}{A}\right)e^{ah} + bh + bh e^{-bh}, \alpha_7 = 2A$$

$$\sinh(ah), \alpha_8 = B (e^{bh} - e^{ah}),$$

$$\alpha_9 = B (e^{ah} - e^{-bh}) \quad (44)$$

For calculating $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_9$ we assume

$$d_1 = h\sqrt{\beta_{14} \beta_{15}}, \quad d_2 = h_1 \sqrt{\beta_{14} \beta_{16}}, \quad d_3 = h\sqrt{\beta_{17} \beta_{18}} \quad \text{and}$$

$$d_4 = h\sqrt{\beta_{17} \beta_{19}} \quad \text{and}$$

$$\beta_{28} = 2d_1 - 2\cos d_2 \sinh d_1,$$

$$\beta_{29} = 2d_2 - 2\sin d_2 \cosh d_1$$

$$\beta_{30} = (\beta_{26} \beta_{22} + \beta_{27} \beta_{23}) / (\beta_{22}^2 + \beta_{23}^2)$$

$$\beta_{31} = (\beta_{27} \beta_{22} - \beta_{26} \beta_{23}) / (\beta_{22}^2 + \beta_{23}^2)$$

$$\beta_{32} = d_1 \beta_{30} - d_2 \beta_{31}, \quad \beta_{33} = d_2 \beta_{30} + d_1 \beta_{31}$$

$$\beta_{34} = \beta_{30} e^{d_1} \cos d_2 - \beta_{31} e^{d_1} \sin d_2$$

$$\beta_{35} = \beta_{30} e^{d_1} \sin d_2 + \beta_{31} e^{d_1} \cos d_2$$

$$\beta_{36} = e^{d_3} \cos d_4, \quad \beta_{37} = e^{d_3} \sin d_4$$

$$\beta_{38} = -1 + \beta_{30} + \beta_{32} - d_3 - \beta_{34} + \beta_{36}$$

$$\beta_{39} = \beta_{31} + \beta_{33} - d_4 - \beta_{35} + \beta_{37}$$

$$\beta_{40} = e^{-d_3} \cos d_4, \quad \beta_{41} = e^{-d_3} \sin d_4$$

$$\beta_{42} = -1 + \beta_{30} + \beta_{32} + d_3 - \beta_{34} + \beta_{40}$$

$$\beta_{43} = \beta_{31} + \beta_{33} + d_4 - \beta_{35} - \beta_{41}$$

$$\beta_{44} = 2d_1 - 2d_1 \cosh d_1 \cos d_2 + 2d_2 \sinh d_1 \sin d_2,$$

$$\beta_{45} = 2d_2 - 2d_2 \cosh d_1 \cos d_2 + 2d_1 \sinh d_1 \sin d_2,$$

$$\beta_{46} = \beta_{32} - d_1 \beta_{34} + d_2 \beta_{35} - d_3 + d_3 \beta_{36} - d_4 \beta_{37},$$

$$\beta_{47} = \beta_{33} - d_1 \beta_{35} - d_2 \beta_{34} - d_4 + d_3 \beta_{37} + d_4 \beta_{36},$$

$$\beta_{48} = \beta_{32} - d_1 \beta_{34} + d_2 \beta_{35} + d_3 - (d_3 \beta_{36} + d_4 \beta_{37}) / (\beta_{36}^2 + \beta_{37}^2),$$

$$\beta_{49} = \beta_{33} - d_2 \beta_{34} - d_1 \beta_{35} + d_4 - (d_4 \beta_{36} - d_3 \beta_{37}) / (\beta_{36}^2 + \beta_{37}^2),$$

$$\beta_{50} = 2[\beta_{22} \cos d_2 \sinh d_1 - \beta_{23} \sin d_2 \cosh d_1]$$

$$\beta_{51} = 2[\beta_{23} \cos d_2 \sinh d_1 + \beta_{22} \sin d_2 \cosh d_1]$$

$$\beta_{52} = \beta_{26} (\beta_{36} - e^{d_1} \cos d_2) - \beta_{27} (\beta_{37} - e^{d_1} \sin d_2)$$

$$\beta_{53} = \beta_{26} (\beta_{37} - e^{d_1} \sin d_2) + \beta_{27} (\beta_{36} - e^{d_1} \cos d_2)$$

$$\beta_{54} = \beta_{26} (e^{d_1} \cos d_2 - e^{-d_3} \cos d_4) - \beta_{27} (e^{d_1} \sin d_2 + e^{-d_3} \sin d_4)$$

$$\beta_{55} = \beta_{27} (e^{d_1} \cos d_2 - e^{-d_3} \cos d_4) + \beta_{26} (e^{d_1} \sin d_2 + e^{-d_3} \sin d_4) \quad (45)$$

Hence

$$\alpha_1 = \beta_{28} + i \beta_{29}, \quad \alpha_2 = \beta_{38} + i \beta_{39},$$

$$\alpha_3 = \beta_{42} + i \beta_{43}, \quad \alpha_4 = \beta_{44} + i \beta_{45},$$

$$\alpha_5 = \beta_{46} + i \beta_{47}, \quad \alpha_6 = \beta_{48} + i \beta_{49},$$

$$\alpha_7 = \beta_{50} + i \beta_{51}, \quad \alpha_8 = \beta_{52} + i \beta_{53},$$

$$\alpha_9 = \beta_{54} + i \beta_{55}, \quad (46)$$

On solving equations (41), (42), (43) we get the values of C₄, C₅ and C₆ as

$$C_4 = \frac{\left(1 - \frac{1}{n}\right) \alpha_{13}}{(\alpha_{10} \alpha_{13} - \alpha_{11} \alpha_{12})} = \alpha_{14} \text{ (say)}$$

$$C_5 = \frac{-\alpha_{12} \alpha_{14}}{\alpha_{13}} = \alpha_{15} \text{ (say)}$$

$$C_6 = \frac{(\alpha_8 \alpha_{15} - \alpha_7 \alpha_{14})}{\alpha_9} = \alpha_{16} \text{ (say)} \quad (47)$$

Where

$$\alpha_{10} = \frac{(\alpha_1 \alpha_9 - \alpha_3 \alpha_7)}{\alpha_9}, \quad \alpha_{11} = \frac{(\alpha_2 \alpha_9 + \alpha_3 \alpha_8)}{\alpha_9}$$

$$\alpha_{12} = \frac{(\alpha_4 \alpha_9 - \alpha_6 \alpha_7)}{\alpha_9}, \quad \alpha_{13} = \frac{(\alpha_6 \alpha_8 + \alpha_5 \alpha_9)}{\alpha_9} \quad (48)$$

The values of $\alpha_{10}, \alpha_{11}, \dots, \alpha_{16}$ are calculated in terms of $\beta_{56}, \beta_{57}, \dots, \beta_{69}$, let us take

$$d_5 = \beta_{28} \beta_{54} - \beta_{29} \beta_{55} - \beta_{42} \beta_{50} + \beta_{43} \beta_{51}$$

$$d_6 = \beta_{28} \beta_{55} + \beta_{29} \beta_{54} - \beta_{42} \beta_{51} - \beta_{43} \beta_{50}$$

$$d_7 = \beta_{54}^2 + \beta_{55}^2$$

$$d_8 = \beta_{38} \beta_{54} - \beta_{39} \beta_{35} + \beta_{42} \beta_{52} - \beta_{43} \beta_{53}$$

$$d_9 = \beta_{38} \beta_{55} + \beta_{39} \beta_{54} + \beta_{42} \beta_{53} + \beta_{43} \beta_{52}$$

$$d_{10} = \beta_{44} \beta_{54} - \beta_{45} \beta_{55} - \beta_{48} \beta_{50} + \beta_{49} \beta_{51}$$

$$d_{11} = \beta_{44} \beta_{55} + \beta_{45} \beta_{54} - \beta_{48} \beta_{51} - \beta_{49} \beta_{50}$$

$$d_{12} = \beta_{48} \beta_{52} - \beta_{49} \beta_{53} + \beta_{46} \beta_{54} - \beta_{47} \beta_{55}$$

$$d_{13} = \beta_{49} \beta_{52} + \beta_{48} \beta_{53} + \beta_{46} \beta_{55} + \beta_{47} \beta_{54} \quad (49)$$

Then

$$\beta_{56} = (\beta_{54} d_5 + \beta_{55} d_6) / d_7$$

$$\beta_{57} = (\beta_{54} d_6 - \beta_{55} d_5) / d_7$$

$$\alpha_{10} = \beta_{56} + i \beta_{57}$$

$$\beta_{58} = (\beta_{54} d_8 + \beta_{55} d_9) / d_7$$

$$\beta_{59} = (\beta_{54} d_9 - \beta_{55} d_8) / d_7$$

$$\alpha_{11} = \beta_{58} + i \beta_{59}$$

$$\beta_{60} = (\beta_{54} \beta_{10} + \beta_{55} \beta_{11}) / d_7$$

$$\beta_{61} = (\beta_{54} \beta_{11} + \beta_{55} \beta_{10}) / d_7$$

$$\alpha_{12} = \beta_{60} + i \beta_{61}$$

$$\beta_{62} = (\beta_{54} d_{12} + \beta_{55} d_{13}) / d_7$$

$$\beta_{63} = (\beta_{54} d_{13} - \beta_{55} d_{12}) / d_7 \quad (50)$$

$$\alpha_{13} = \beta_{62} + i \beta_{63}$$

Now if take

$$d_{14} = \beta_{56} \beta_{62} - \beta_{57} \beta_{63} - \beta_{58} \beta_{60} + \beta_{59} \beta_{61}$$

$$d_{15} = \beta_{57} \beta_{62} + \beta_{56} \beta_{63} - \beta_{58} \beta_{61} - \beta_{59} \beta_{60}$$

$$\beta_{64} = \left(1 - \frac{1}{n}\right) (\beta_{62} d_{14} + \beta_{63} d_{15}) / (d_{14}^2 + d_{15}^2)$$

$$\beta_{65} = \left(1 - \frac{1}{n}\right) (\beta_{63} d_{14} - \beta_{62} d_{15}) / (d_{14}^2 + d_{15}^2)$$

$$d_{16} = \beta_{61} \beta_{65} - \beta_{60} \beta_{64}$$

$$d_{17} = \beta_{60} \beta_{65} + \beta_{61} \beta_{64}$$

$$d_{18} = \beta_{62}^2 + \beta_{63}^2$$

$$\beta_{66} = (\beta_{62} d_{16} - \beta_{63} d_{17}) / d_{18}$$

$$\beta_{67} = -(\beta_{63} d_{16} + \beta_{62} d_{17}) / d_{18}$$

$$d_{19} = \beta_{52} \beta_{66} - \beta_{53} \beta_{67} - \beta_{50} \beta_{64} + \beta_{51} \beta_{65}$$

$$d_{20} = \beta_{52} \beta_{67} + \beta_{53} \beta_{66} - \beta_{50} \beta_{65} - \beta_{51} \beta_{64}$$

$$\beta_{68} = (\beta_{54} d_{19} + \beta_{55} d_{20}) / d_7$$

$$\beta_{69} = (\beta_{54} d_{20} - \beta_{55} d_{19}) / d_7$$

$$d_{21} = \beta_{65} \beta_{23} - \beta_{64} \beta_{22} - \beta_{26} (\beta_{66} + \beta_{68}) + \beta_{27} (\beta_{67} + \beta_{69})$$

$$d_{22} = \beta_{65} \beta_{22} + \beta_{64} \beta_{23} + \beta_{27} (\beta_{66} + \beta_{68}) + \beta_{26} (\beta_{67} + \beta_{69})$$

$$d_{23} = \beta_{22}^2 + \beta_{23}^2$$

$$\beta_{70} = (\beta_{22} d_{21} - \beta_{23} d_{22}) / d_{23}$$

$$\beta_{71} = -(\beta_{22} d_{22} + \beta_{23} d_{21}) / d_{23}$$

$$\beta_{72} = \frac{1}{n} (-\beta_{70} - \beta_{64} - \beta_{66} - \beta_{68})$$

$$\beta_{73} = \beta_{71} + \beta_{65} + \beta_{67} + \beta_{69}$$

$$\beta_{74} = 2(d_1 \beta_{64} - d_2 \beta_{65})$$

$$\beta_{75} = 2(d_1 \beta_{65} + d_2 \beta_{64})$$

$$\beta_{76} = d_3 (\beta_{68} - \beta_{66}) + d_4 (\beta_{67} - \beta_{69})$$

$$\beta_{77} = d_4 (\beta_{68} - \beta_{66}) + d_3 (\beta_{69} - \beta_{67})$$

$$\beta_{78} = (d_1 \beta_{26} - d_2 \beta_{27}) / d_{23}$$

$$\beta_{79} = (d_1 \beta_{27} + d_2 \beta_{26}) / d_{23}$$

$$\beta_{80} = \beta_{22} (\beta_{66} + \beta_{68}) + \beta_{23} (\beta_{67} + \beta_{69})$$

$$\beta_{81} = \beta_{22} (\beta_{67} + \beta_{69}) - \beta_{23} (\beta_{66} + \beta_{68})$$

$$\beta_{82} = \beta_{74} + \beta_{76} + (\beta_{78} \beta_{80} - \beta_{79} \beta_{81})$$

$$\beta_{83} = \beta_{75} + \beta_{77} + (\beta_{79} \beta_{80} + \beta_{78} \beta_{81}) \quad (51)$$

Then

$$C_4 = \alpha_{14} = \beta_{64} + i \beta_{65}$$

$$C_5 = \alpha_{15} = \beta_{66} + i \beta_{67}$$

$$C_6 = \alpha_{16} = \beta_{68} + i \beta_{69} \quad (52)$$

On putting the values of C₄, C₅, C₆ in equations (38), (39) and (40) we get

$$C_3 = \beta_{70} + i \beta_{71},$$

$$C_1 = \beta_{72} - i \beta_{73},$$

$$C_2 = \beta_{82} + i \beta_{83}, \quad (53)$$

On substituting the values of the constants C₁, C₂, C₃, C₄, C₅, C₆ in equations (22); (23) and thereafter using the values of f(ζ) and g(ζ) thus obtained in the expressions (18) we get the value of the stream function ψ and the micro-rotation N

in terms of the variable ζ and $\xi = \left[\frac{U_0}{\alpha} - \frac{nv_0 x}{h} \right]$. On taking

h=1, ξ=1 the expressions (6) and (7) provides us

$$U = \text{Real part of } u(x, y) e^{i\omega t}$$

$$= [\beta_{82} + e^{d_1 \zeta} \{ \beta_{70} (d_1 \cos(d_2 \zeta) - d_2 \sin(d_2 \zeta)) - \beta_{71} (d_1 \sin(d_2 \zeta) + d_2 \cos(d_2 \zeta)) \}] -$$

$$e^{-d_1 \zeta} \{ \beta_{64} (d_1 \cos(d_2 \zeta) + d_2 \sin(d_2 \zeta)) - \beta_{65} (d_2 \cos(d_2 \zeta) - d_1 \sin(d_2 \zeta)) \} + e^{d_3 \zeta} \{ \beta_{66} (d_3 \cos(d_4 \zeta) - d_4 \sin(d_4 \zeta)) -$$

$$\begin{aligned} & \beta_{67}(d_3\sin(d_4\zeta)+d_4\cos(d_4\zeta))\}- \\ & e^{-d_3\zeta} \{ \beta_{68}(d_3\cos(d_4\zeta)+d_4\sin(d_4\zeta))-\beta_{69}(d_4\cos(d_4\zeta)- \\ & d_3\sin(d_4\zeta))\}. \text{cost-}[\beta_{83}+ e^{d_1\zeta} \{ \beta_{71}(d_1\cos(d_2\zeta)- \\ & d_2\sin(d_2\zeta))+\beta_{70}(d_1\sin(d_2\zeta)+d_2\cos(d_2\zeta))\}- \\ & e^{-d_1\zeta} \{ \beta_{65}(d_1\cos(d_2\zeta)+d_2\sin(d_2\zeta))+\beta_{64}(d_2\cos(d_2\zeta)- \\ & d_1\sin(d_2\zeta))\} + e^{d_3\zeta} \{ \beta_{67}(d_3\cos(d_4\zeta)- \\ & d_4\sin(d_4\zeta))+\beta_{66}(d_3\sin(d_4\zeta)+d_4\cos(d_4\zeta))\}- \\ & e^{-d_3\zeta} \{ \beta_{69}(d_3\cos(d_4\zeta)+d_4\sin(d_4\zeta))+\beta_{68}(d_4\cos(d_4\zeta)- \\ & d_3\sin(d_4\zeta))\}]. \text{sint} \quad (54) \end{aligned}$$

$$\begin{aligned} \bar{V} &= \text{Real part of } v(x, y)e^{iwt} \\ &= [(\beta_{72}+\beta_{82}\zeta)+ e^{d_1\zeta} \{ \beta_{70}\cos(d_2\zeta)-\beta_{71}\sin(d_2\zeta)\} + \\ & e^{-d_1\zeta} \{ \beta_{64}\cos(d_2\zeta)+\beta_{65}\sin(d_2\zeta)\} + e^{d_3\zeta} \{ \beta_{66}\cos(d_4\zeta)- \\ & \beta_{67}\sin(d_4\zeta)\} + e^{-d_3\zeta} \{ \beta_{68}\cos(d_4\zeta)+\beta_{69}\sin(d_4\zeta)\}]. \text{cost-} \\ & [(\beta_{83}\zeta-\beta_{73})+ e^{d_1\zeta} \{ \beta_{71}\cos(d_2\zeta)+\beta_{70}\sin(d_2\zeta)\} + \\ & e^{-d_1\zeta} \{ \beta_{65}\cos(d_2\zeta)- \\ & \beta_{64}\sin(d_2\zeta)\} + e^{d_3\zeta} \{ \beta_{67}\cos(d_4\zeta)+\beta_{66}\sin(d_4\zeta)\} + \\ & e^{-d_3\zeta} \{ \beta_{69}\cos(d_4\zeta)-\beta_{68}\sin(d_4\zeta)\}]. \text{sint} \quad (55) \end{aligned}$$

$$\begin{aligned} \bar{N} &= \text{Real part of } N(x, y)e^{iwt} = \text{Real part of } N(\text{cost} + i \text{ sint}) \\ &= \{ \beta_{22}(\beta_{84}+\beta_{85})-\beta_{23}(\beta_{88}+\beta_{89})+\beta_{26}(\beta_{86}+\beta_{87})- \\ & \beta_{27}(\beta_{90}+\beta_{91})\} \text{cost} \\ &- \{ \beta_{23}(\beta_{84}+\beta_{85})+\beta_{22}(\beta_{88}+\beta_{89})+ \beta_{27}(\beta_{86}+\beta_{87})+\beta_{26}(\beta_{90}+\beta_{91})\} \text{sint} \quad (56) \end{aligned}$$

where $wt = \tau$ is dimensionless phase parameter.

3.1 Pressure Distribution

From equations (8), (9) and (18), we get

$$\begin{aligned} \left[\frac{U_0}{\alpha} - \frac{nv_0x}{h} \right] i\rho\omega f'(\zeta) &= -\frac{\partial p}{\partial x} + \frac{k}{h^2} \left[\frac{U_0}{\alpha} - \frac{nv_0x}{h} \right] g'(\zeta) \\ &+ \frac{u+k}{h^2} \left[\frac{U_0}{\alpha} - \frac{nv_0x}{h} \right] f'''(\zeta) \quad (57) \end{aligned}$$

$$\begin{aligned} i\rho\omega nv_0 f(\zeta) &= -\frac{1}{h} \frac{\partial p}{\partial \zeta} + \frac{knv_0}{h^2} g(\zeta) \\ &+ \frac{u+k}{h^2} nv_0 f''(\zeta) \quad (58) \end{aligned}$$

From equation (57), we get

$$\begin{aligned} \frac{\partial p}{\partial x} &= \left[\frac{U_0}{\alpha} - \frac{nv_0x}{h} \right] \left[\frac{k}{h^2} g'(\zeta) + \frac{u+k}{h^2} f'''(\zeta) - i\rho\omega f'(\zeta) \right] \\ &= \left[\frac{U_0}{\alpha} - \frac{nv_0x}{h} \right] f(\zeta) \quad (59) \end{aligned}$$

where

$$\frac{k}{h^2} g'(\zeta) + \frac{u+k}{h^2} f'''(\zeta) - i\rho\omega f'(\zeta) = F(\zeta)$$

Then at $\zeta = 0$ we get

$$F(0) = C_7 = \frac{k}{h^2} g'(0) + \frac{u+k}{h^2} f'''(0) \quad (60)$$

On integrating to equation (58) with respect ζ and taking the limit from $\zeta = 0$ to ζ we obtain :

$$\begin{aligned} \frac{1}{h} [p(x, \zeta) - p(x, 0)] &= \frac{u+k}{h^2} nv_0 f'(\zeta) \\ &+ nv_0 \int_0^\zeta \left(\frac{k}{h^2} g(\zeta) - i\rho\omega f(\zeta) \right) d\zeta \quad (61) \end{aligned}$$

on integrating (59) w.r.t.x, we get

$$\begin{aligned} p(x, \zeta) - p(0, \zeta) &= \left(\frac{U_0}{\alpha} - \frac{nv_0x}{2h} \right) x \\ \left[\frac{k}{h^2} g'(\zeta) + \frac{u+k}{h^2} f'''(\zeta) - i\rho\omega f'(\zeta) \right] & \quad (62) \end{aligned}$$

On putting $\zeta = 0$ in both sides of the equation (62), we get

$$\begin{aligned} p(x, 0) - p(0, 0) &= \left(\frac{U_0}{\alpha} - \frac{nv_0x}{2h} \right) x C_7 \\ p(x, 0) &= p(0, 0) + \left(\frac{U_0}{\alpha} - \frac{nv_0x}{2h} \right) x C_7 \quad (63) \end{aligned}$$

On substituting the value of p (x,0) in equation (61) we get

$$\begin{aligned} p(x, \zeta) - p(0, 0) &= \left(\frac{U_0}{\alpha} - \frac{nv_0x}{2h} \right) x C_7 + \frac{u+k}{h} nv_0 f'(\zeta) \\ &+ hnv_0 \int_0^\zeta \left[\frac{k}{h^2} g(\zeta) - i\rho\omega f(\zeta) \right] d\zeta \quad (64) \end{aligned}$$

3.2 Skin Friction

Taking $i = 1, j = 2$ in equation (4) and converting into physical components x and y the shearing stress τ_{yx} becomes :

$$\tau_{yx} = \frac{k}{h} \left[\frac{U_0}{\alpha} - \frac{nv_0x}{h} \right] \left[\frac{u+k}{h^2} f''(\zeta) + g(\zeta) \right] \quad (65)$$

Hence the coefficient of skin friction on the lower and upper plates is given by

$$C_f = \frac{2\tau_{yx}}{\rho u_0^2} \text{ at } \zeta = 0 \text{ and } \zeta = 1.$$

IV. RESULTS AND DISCUSSION

The variation of the velocity U with ζ at $n=2.0, h=1.0, K=0.5, p_j=5.0, p_\alpha=4.0, p_\tau=3.0, \xi=1$ for different values $p_1=1, 5, 10$ in case of $\tau=0, \pi/3$ is represented through Fig. (2.1). It is clear from the figure and numerical values that the velocity U decreases in the region $0 \leq \zeta \leq 0.2$ and $0.8 \leq \zeta \leq 1$ and increases in the region $0.3 \leq \zeta \leq 0.7$ with an increase in the micro-polarity parameter p_1 in case of $\tau=0$ whenever the behaviour of the velocity U in case of $\tau=\pi/3$ is just reversed to its behaviour in case of $\tau=0$. The velocity is zero at the wall of the channel. Fig. (2.2) represent the variation of the dimensionless velocity U with ζ at $n=2, h=1, K=0.5, p_1=2, p_\tau=3, \xi=1$ for different values of $p_j=1, 5, 10$ in case of $\tau=0, \pi/3$. It is seen from this figure and obtained numerical values that there is no much difference in the values of U w.r.t. different values of p_j in case of $\tau=0, \pi/3$ but the values and branches of graph of U have significant difference in case of $\tau=0, \pi/3$.

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The branch of the velocity U in case of $\tau=0$ is lying above the branch of U in case of $\tau=\pi/3$ throughout the gaplength. All the branches in the graphs of U are being overlapped in case of $\tau=0$ and $\pi/3$ separately. The velocity attains its maxima in the middle of the gaplength. Fig. (2.3) represents the variation of U with ζ for different values of $p_i=1,5,10$. The behaviour of the velocity U in this figure is similar to its behaviour seen in fig 2.2. Here the velocity U increase with an increase in p_i in the regions $0 \leq \zeta \leq 0.2$; $0.8 \leq \zeta \leq 1$ and decreases in the region $0.3 \leq \zeta \leq 0.7$ in case $\tau=0$. The behaviour U with p_i in case $\tau=\pi/3$ is just reversed to that of its behaviour with p_i in case $\tau=0$.

The variation of the velocity U with ζ at $h=1$ $K=0.5$, $p_j=5$, $p_\alpha=4$, $p_t=3$, $p_l=2$ for different values of $n=2,5,8$ in case of $\tau=0, \pi/3$ is represented through Fig. 4. It is clear from this figure that in case of $\tau=0$, value of U is maximum than its values in case $\tau=\pi/3$ for same n throughout the gaplength. It is also clear from obtained numerical values that the velocity U is increasing with an increase in n throughout the gaplength with its maximum value at the middle point of the gaplength.

Fig.5 exhibits the behaviour of the velocity V with ζ at $n=2$, $h=1$, $K=0.5$, $p_j=5$, $p_\alpha=4$, $p_t=3$, for different values of $p_i=1,5,10$ in case of $\tau=0, \pi/3$. Two separated curve are seeing in this figure, the above one is the curve of the case $\tau=0$ (formed from the overlapped branches of $p_i=1,5,10$) and the lower one is of the case $\tau=\pi/3$ (for $p_i=1,5,10$). It is also observed that the velocity V is decreasing with an increase in p_i up to the middle and start increasing thereafter upto the wall $\zeta=1$ in case of $\tau=0, \pi/3$ both. The velocity is minimum at the lower wall and maximum at the upper wall. In fig.6 the curves of the velocity V with different p_j are similar to the curves of V with p_l (fig.5) in case of $\tau=0, \pi/3$. The variation of V with p_j has no definite trend in case of $\tau=0$ whenever in case of $\tau=\pi/3$ the velocity V increases with an increase in p_j up to the middle with its reversed behaviour thereafter. In fig.7 the velocity increases with an increase p_t with up to the middle and decreases thereafter up to the wall $\zeta=1$ in both the cases $\tau=0$ and $\pi/3$. In fig.8 the curves of the velocity V for different values of $n=2, 5, 8$ are similar in shape as it's curves in fig. 5,6,7. The velocity is being decrease with an increase in n throughout the gaplength in both the cases $\tau=0, \pi/3$.

Fig.9 represents the behaviour of the micro-rotation velocity function \bar{N} at $n=2$, $h=1$, $K=0.5$ $p_j=5$, $p_\alpha=4$, $p_t=3$ for different values of $p_i=1,5,10$ in case of $\tau=0$ and $\pi/3$. It is evident from this figure that \bar{N} is zero at $\zeta=0, 0.5$ and 1.0 in both the cases $\tau=0$ and $\pi/3$. It is also evident from the figure that the micro-rotation function \bar{N} increases with an increase in p_i in the first half of the gaplength and start decreasing in the second half in both the case $\tau=0$ and $\pi/3$. The value of the function \bar{N} is maximum in the middle of the first half and minimum in the middle of the second half of the gaplength in both the cases $\tau=0$ (except $p_i=1$) and $\tau=\pi/3$ (except $p_i=1$). The result of maxima and minima at $p_i=1$ is reverse to that if $p_i=5,10$ in both the cases.

Fig.10 depicts the behaviour of the micro-rotation velocity \bar{N} with ζ at $n=2$, $h=1$, $\xi=1$, $K=0.5$, $p_t=2$, $p_l=3$ for different values of $p_j=1,5,10$. It is evident from this figure that the velocity is zero at lower ($\zeta=0$), middle ($\zeta=0.5$) and

upper ($\zeta=1$) wall with it's maximum in the middle of the first half and minimum in the middle of the second half of the gaplength. It is also observed that the micro rotation \bar{N} decreases with an increase in p_j in the first half and increases in the second half of the second half of the gaplength in both the cases $\tau=0, \tau=\pi/3$. Fig.11 shows that the behaviour of microrotation velocity \bar{N} with p_t at $\tau=0$ is similar to it's behaviour with p_j in fig.10. In case of $\tau=\pi/3$ no clear trend of micro-rotation is detected as it increases from $p_t=1$ to 5 and decrease at $p_t=10$ in the first half and shows reversed behaviour in the second half of the gaplength. The variation of micro-rotation velocity \bar{N} with ζ for different values of n is represented through fig.12. The shape of the curves of \bar{N} is similar approximately to the shape of figure.9, 10, 11. The micro-rotation velocity \bar{N} increases with an increase in n in the first half and decreases in the second half of the gaplength in case of $\tau=0, \pi/3$ both.

V. CONCLUSION

It is evident from the graphs of the dimensionless velocity U that the velocity U is maximum at the middle point of the gap length and vanishes at the surface of the plates. The velocity curves are parabolic with vertex upward. The values of the velocity at the phase $\tau=0$ is maximum than its values at phase $\tau=\pi/3$. It is also concluded thus the dimensionless velocity V is also maximum at phase $\tau=0$ than its value at $\tau=\pi/3$. The difference in the values of V w.r.t. different values of p_l , p_t and p_j respectively is not significant at both the phases separately. Hence the branches of the velocity V w.r.t. p_t , p_l and p_j at the separate phase $\tau=0$ and $\pi/3$ are being overlapped whenever in case of the variation of V with n the branches are not being overlapped.

It is also observed that there is no microrotation at the surface of the plates and in the middle of the gaplength whenever the microrotation is maximum in the middle of the first half and maximum in magnitude in the middle of the second half of the gap length.

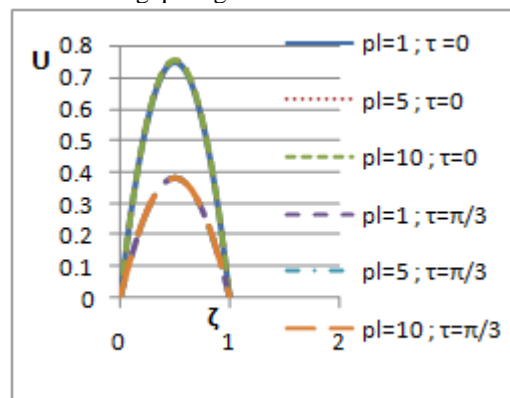


Fig.1: Variation of U with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of p_i .

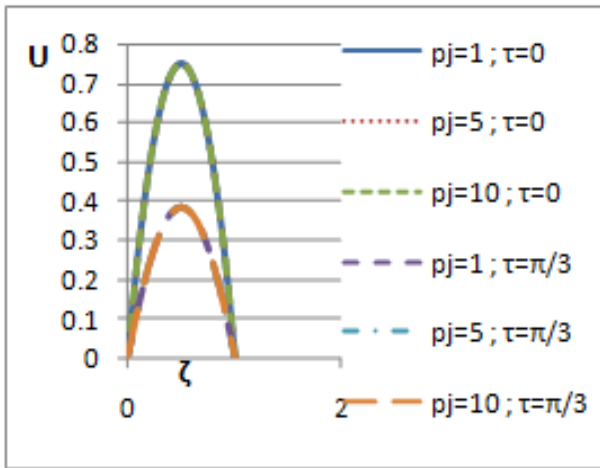


Fig.2: Variation of U with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of p_j .

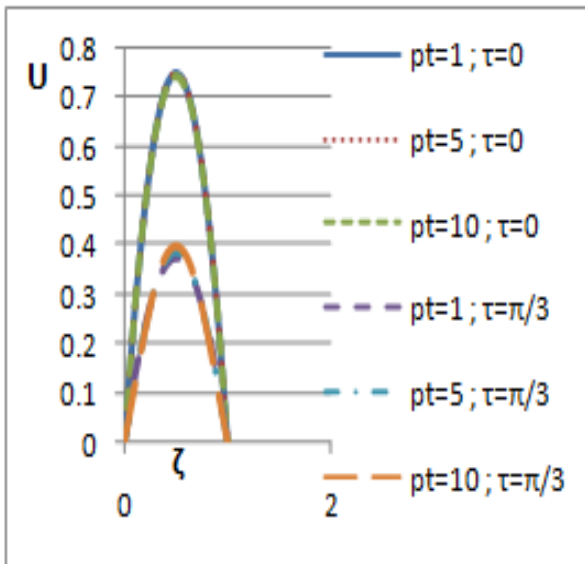


Fig.3: Variation of U with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of p_t .

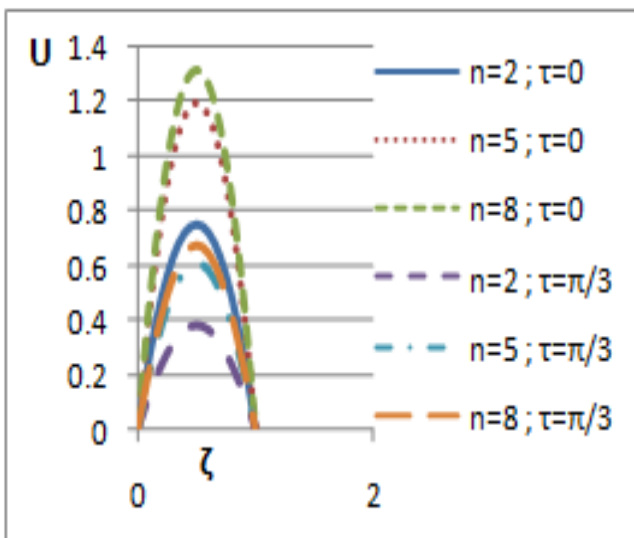


Fig.4: Variation of U with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of n .

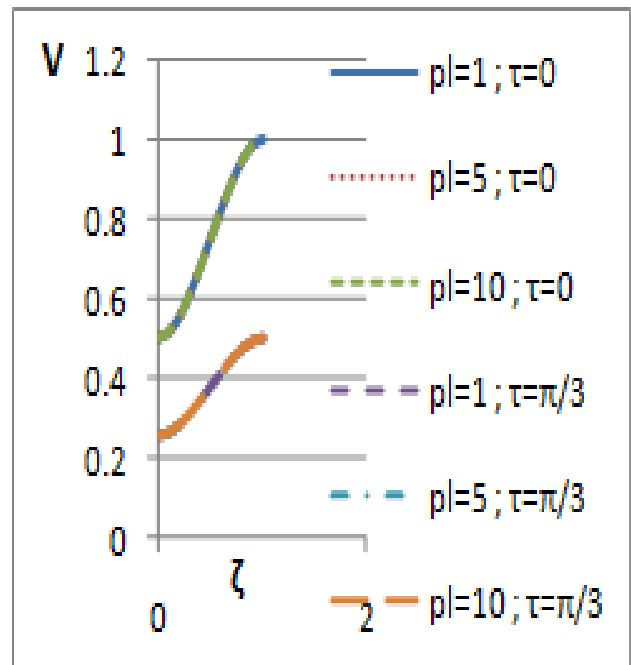


Fig.5: Variation of V with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of p_l .

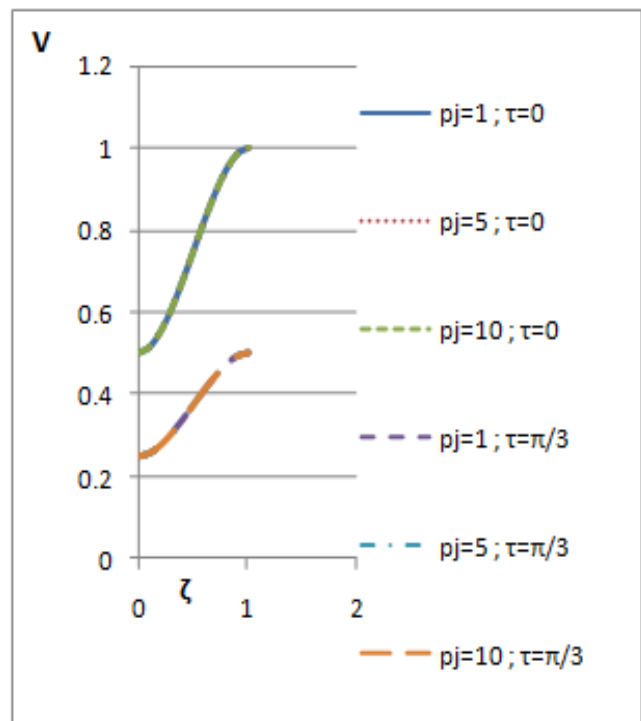


Fig.6: Variation of V with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of p_j .

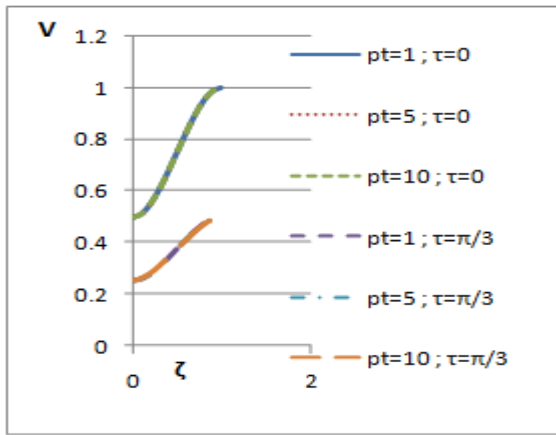


Fig.7: Variation of V with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of p_i .

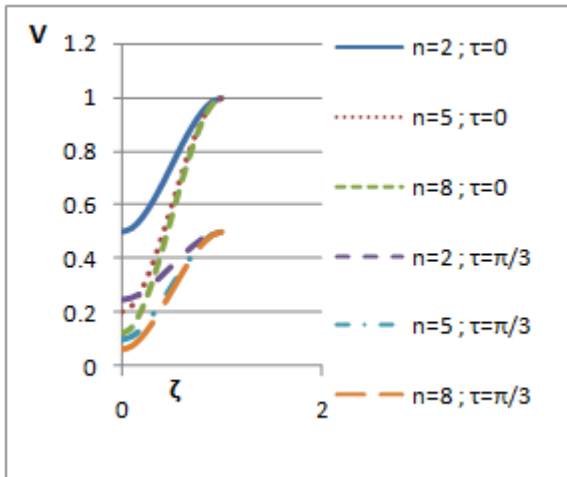


Fig.8: Variation of V with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of n .

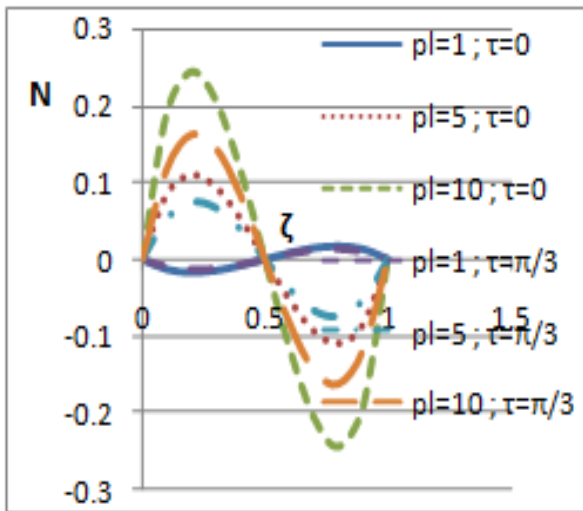


Fig.9: Variation of \bar{N} with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of p_i .

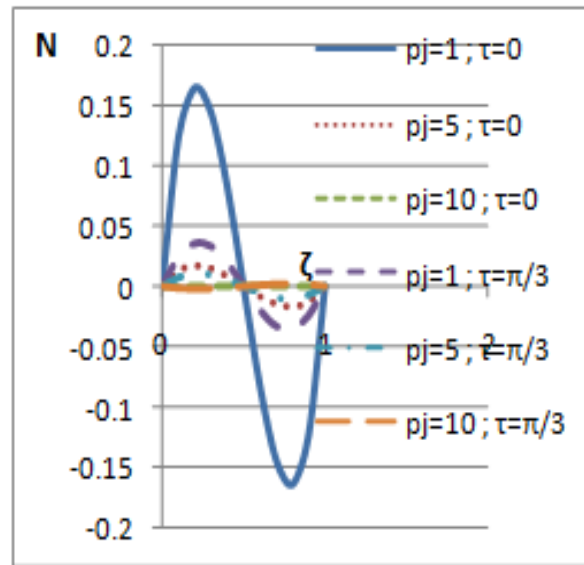


Fig.10: Variation of \bar{N} with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of p_j .

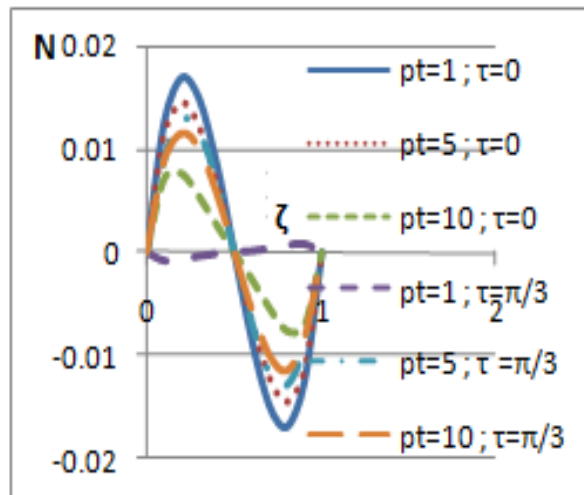


Fig.11: Variation of \bar{N} with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of p_i .

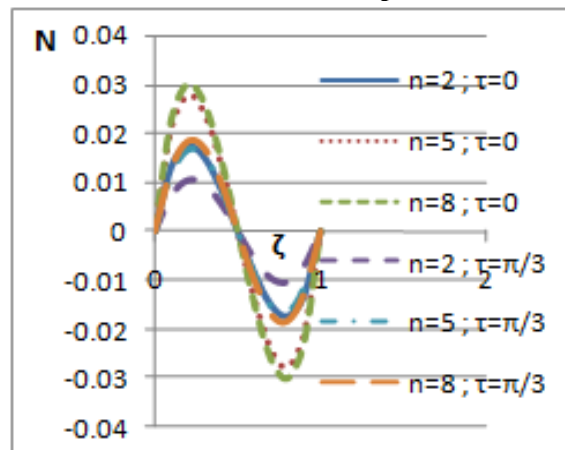


Fig.12: Variation of \bar{N} with ζ at $\tau=0$ and $\tau=\pi/3$ for different values of n .

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- ❖ Ph.D. on “Study of some flow and heat transfer problems in non-Newtonian Reiner-Rivlin fluid” from department of mathematics; Meerut College, Meerut affiliated to C.C.S. University Meerut in 2011.
- ❖ M.Sc. Mathematics from Meerut College, Meerut (C.C.S. University Meerut) in 2004 with 1st division.

Research Papers:

- ❖ Flow of a Non-Newtonian Reiner-Rivlin Fluid Over an Enclosed Torsionally Oscillating Disc with Uniform Suction and Injection. K.R. Singh, Ajit Kumar Journal of Reflection des ERA-JMS, Vol. 4, Issue 1, (2009) P: 41-64.
- ❖ Flow of a Non-Newtonian Reiner-Rivlin Fluid Between Two Enclosed Torsionally Oscillating Discs. K.R. Singh, Ajit Kumar Journal of The Mathematics Education, Vol. XLIII, No.1, March (2009) P: 32.
- ❖ Heat Transfer in the Forced Flow of a Non-Newtonian Reiner-Rivlin Fluid Between Two Porous Discs of Different Permeability. K.R. Singh, Shubhi Singhal, Ajit Kumar International Journal of Fluid Mechanics, Vol. 1 (2), July-December (2009), P: 167-180.
- ❖ Heat Transfer in the Flow of Reiner-Rivlin Fluid Between Two Enclosed Counter Rotating Discs. K.R. Singh, Ajit Kumar

Work Experience:

- ❖ I have four years teaching experience, worked as tutor in Meerut College, Meerut, from 31-10-2007 to 05-02-2011 till.
- ❖ I have also one year teaching experience, worked as Assistant Professor in S.V.S. Engineering College, Mawana, Meerut from 01-08-2011 to 18-08-2012 till.
- ❖ Presently working as Assistant Professor in Teerthanker Mahaveer University Moradabad, from 24-08-2012 to till date.



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Educational Qualification:

- ❖ **Ph.D.** awarded from Meerut College, Meerut affiliated to Chaudhary Charan Singh University, Meerut on entitled “Computational Techniques for flows of a micropolar fluid”.
- ❖ **M.Sc.** (Mathematics) (2004), from Meerut College, Meerut affiliated to Chaudhary Charan Singh University, Meerut in first division with 63.7%.

Experiences: 13 years of teaching

- ❖ Worked as Lecturer at RGEC, Meerut from 28-08-2006 to 21-07-2008.
- ❖ Worked as Lecturer at IEC Engg. College, Greater Noida from 31-08-2008 to 17-11-2009.
- ❖ Worked as Senior Lecturer at Sunderdeep Engg College, Ghaziabad from 18-11-2009 to 15-07-2012.
- ❖ Worked as Assistant Professor at IIMT Engg. College, Meerut Since 18 July 2012 to 25 January 2019.
- ❖ Working as Assistant Professor at Noida Institute Of Engineering of Technology Greater Noida from 28 January 2019 to till date.



Flow of Micropolar Fluid Between Two Parallel Plates with Different Periodic Suction and Injection

Journal Published

- ❖ Preeti ,Kavi Raj Singh,Ajit kumar “Thin film flow of an electrically conducting micropolar fluid down an inclined plane”, International Journal of mathematics and computing applications.(ISSN :0976-6790). IJMCA,Vol 4 ,No1 January-June 2012 (page 1-12) Published.
 - ❖ Preeti ,Kavi Raj Singh,Ajit kumar “Flow of an electrically conducting micropolar fluid flow on a moving belt”. International Journal of mathematics and applied statistics”(ISSN:0973-5739). IJMAS, Vol 3 ,No1(2012) Published.
 - ❖ Preeti, Kavi Raj Singh,Ajit Kumar “Flow of a micropolar fluid between two porous discs with uniform suction. " International Journal of Emerging Research in Management & Technology. (ISSN :2278-9359), Volume-5, Issue-4, April 2016. (Published)
- 4 . Ajit Kumar, Preeti , Kavi Raj Singh" Flow of an electrically conducting micropolar fluid down a vertical cylinder. "International Research Journal of mathematics, Engineering and IT, Vol.3, Issue 4, IF-3.563, (ISSN: 2349-0322), April 2016. (Published). Associated Asia Research Foundation (AARF)."