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Computational Analysis of Kinematics of 3 – Links Articulated Robotic Manipulator

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Abstract: ***The Computational Analysis of Kinematics of 3 – Links Articulated Robotic Manipulator has been presented in this. The design of robot manipulators requires accurate computational analysis, involving the geometric position of the linking arms. The method of Forward Kinematics and Inverse Kinematics were employed in estimating the robotic arm's position with respect to link lengths and angle, in which the angle required to move the end effector to a desired position is estimated and determined. A three link robotic arm with a rigid rotational base was also illustrated using free body diagrams, and computational estimation of the required parameters. The outcomes of the forward kinematics reveals that the robot end effector position can be estimated using the values of x, y, and z coordinates thereby providing a better means of controlling or adapting robot's arm/motion to its environment.***

Keywords: ***Articulated Robot; End effector; Forward Kinematics; Free Body Diagram; Inverse Kinematics; Planar Robot; Robotic; Rigid Body; Robot Arm.***

I. INTRODUCTION

Robots are built with several linkages otherwise known as manipulators. These manipulators are electronically controlled to enable them interact with their environments [3]. Kinematic is a term that is used in the field of robotics to explain the geometry [1] of the robot motion. The study and design of robotic manipulators requires careful computation of the kinematics to be able to determine the position of the end effector at every instance of motion. This gives rise to the need for accurate computational analysis and design of robotic manipulators. The two important points considered during this phase of design remains the Forward kinematic (FK) and Inverse Kinematic (IK). While the FK describes the position and orientation of the robot arm end effector with the joint angles and link lengths given, the IK describes the computation of the angle required to move the end effector to a desired point [2, 4, 6, 7]. Sırma in [4] discusses geometry and matrix transformation approaches for the computation of FK, and Analytic and Inverse Jacobean methods for IK. Ben-Ari in [5] had applied the geometric method in the computation of FK and the analytic method in the computation of IK.

This paper presents the computational analysis of Forward and Inverse Kinematics of 3 – links planer robotic arm using the geometric and analytic methods respectively as described by Sırma in [4].

II. METHODOLOGY

A. Geometric Analysis of the Free Body of the proposed Model

The free body diagram of figure 1 represents the physical system. The model shows that the conceived robotic system is a three joint revolute serial-link robot arm and was chosen for this specific application and implementation. The free body diagram is shown below:

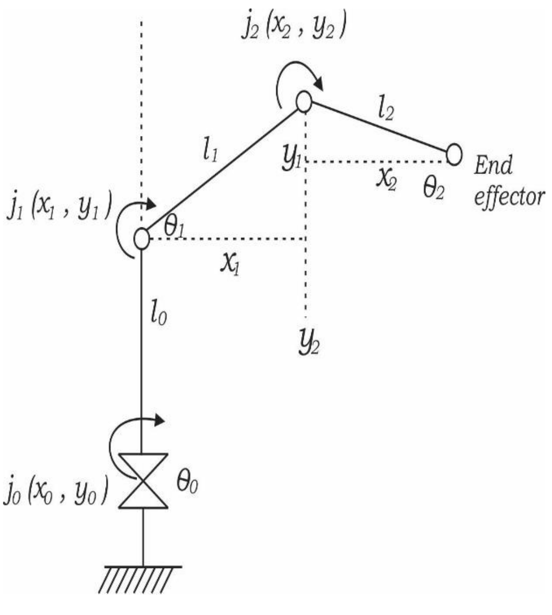

Figure 1: Free Body diagram of 3-link robot

Figure 1 presents three joints, j_0, j_1 and j_2, and three links, l_0, l_1 and l_2. x_0 and y_0 are the coordinates of the joint j_1, x_1 and y_1. The coordinates of joint j_2 are x_2 and y_2. The mass of each link would be represented as m. These parameters were used to derive the mathematical models that describe this system. The following models would be derived:

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1. Forward kinematic model
2. Inverse Kinematic Model

B. Computational Analysis of the Forward Kinematics (FK)

At Joint J_0 (with x = 0)

$$x_0 = 0$$
$$y_0 = l_0$$

At Joint J_1:

$$\cos(\theta_1) = \frac{x_1}{l_1}, \therefore\ x_1 = l_1 \cos(\theta_1)$$

$$\sin(\theta_1) = \frac{y_1}{l_1}, \therefore\ y_1 = l_1 \sin(\theta_1)$$

At Joint J_2:

$$x_1 = l_1 \cos(\theta_1) + l_2 \cos(\theta_2)$$
$$y_1 = l_1 \sin(\theta_1) + l_2 \sin(\theta_2)$$

Now, the forward kinematic equations are:

$$x = l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) \quad \text{eq. 1}$$
$$y = l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) \quad \text{eq. 2}$$
$$z = \theta_0 \quad \text{eq. 3}$$

Eq. 1, 2 and 3 are used to determine the location of the end effector within the robot region of operation.

C. Computational Analysis of the Inverse Kinematics (IK)

In this computation, only l_1 and l_2 were used to compute the inverse kinematics. The free body diagram for the inverse kinematics is shown in Figure 2

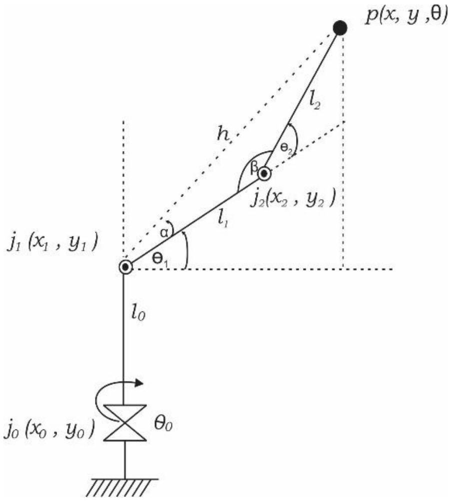

Figure 2: Inverse Kinematic

From Figure 2, Pythagoras theorem holds that

$$h^2 = {x_p}^2 + {y_p}^2 \quad \text{eq. 4}$$

Let

$$\Theta = \alpha + \theta_1$$

and

$$\theta_1 = \Theta - \alpha \quad \text{eq. 5}$$

Also from Pythagoras theorem,

$$\tan\Theta = \frac{y_p}{x_p},$$

Hence

$$\therefore\ \theta_1 = \left(\tan^{-1}(y_p/x_p)\right) - \alpha \quad \text{eq. 6}$$

Now, from cosine law,

$$a^2 = b^2 + c^2 - 2bc\cos A \quad \text{eq. 7}$$

Where a = h, b = l_1, c = l_2, A = β.

But

β = 180 - θ_2 (angles on a straight line) eq. 8

Substituting l_1, l_2, and β in equation 7,

$$h^2 = {l_1}^2 + {l_2}^2 - 2\, l_1 l_2 \cos(180 - \theta_2)$$

$$\cos(180 - \theta_2) = \frac{h^2 - {l_1}^2 - {l_2}^2}{-2\, l_1 l_2} = \frac{{l_1}^2 + {l_2}^2 - h^2}{2\, l_1 l_2}$$

eq. 9

$$\therefore\ \theta_2 = 180 + \left(\cos^{-1}\frac{{l_1}^2 + {l_2}^2 - h^2}{2\, l_1 l_2}\right) \quad \text{eq. 10}$$

Using sin rule,

$$\frac{\sin\alpha}{l_2} = \frac{\sin\beta}{h} \quad \text{eq. 11}$$

Inserting equations 4 and 8, into 11

$$\alpha = \sin^{-1}\frac{l_2 \sin 180 - \theta_2}{\sqrt{{x_p}^2 + {y_p}^2}}$$

Putting α into eq. 6

$$\theta_1 = \left(\tan^{-1}(y_p/x_p)\right) - \sin^{-1}\frac{l_2 \sin 180 - \theta_2}{\sqrt{{x_p}^2 + {y_p}^2}} \text{eq. 12}$$

Eq. 11 and eq. 12 show the angular displacement required to move the end effector to a desired location.

In summary, tables I and II below shows the results obtained after the computations:

Table- I: Summary Results of the FK at the joints

Joints	*x-axis*	*y-axis*
J_0	0	l_0
J_1	$l_1\cos(\theta_1)$	$l_1\sin(\theta_1)$
J_2	$l_1\cos(\theta_1) + l_2\cos(\theta_2)$	$l_1\sin(\theta_1) + l_2\sin(\theta_2)$

Table I shows that at the revolute joint, J_0, the value of the FK along x-axis was 0, and along y-axis was equal to the value of l_0. This computation was done by visual inspection. At j_2, FK along x-axis is the product of l_1 by cosine of the angle at that joint. On the other hand, the value of FK along y-axis was computed as the product of the length of link l_1 by the sine of the angle at that joint. At J_2, the result show that the motion at this point depends on the motion at joint J_1. The overall end effector coordinate by this computation are given as eq. 1 to eq. 3 above. Hence the final values of FK coordinates are:

$$x = l_1\cos(\theta_1) + l_2\cos(\theta_1 + \theta_2)$$
$$x = l_1\cos(\theta_1) + l_2\cos(\theta_1 + \theta_2)$$
$$z = \theta_0$$

Similarly, table 2 below presents the summary of the computation of the IK as was computed.

Table- II: Summary Result of the IK Computation

Joint	**Angle**	**Value**
J_1	θ_1	$(\tan^{-1}(y_p/x_p)) - \sin^{-1}\frac{l_2\sin 180 - \theta_2}{\sqrt{{x_p}^2 + {y_p}^2}}$
J_2	θ_2	$180 + \left(\cos^{-1}\frac{{l_1}^2 + {l_2}^2 - h^2}{2\, l_1 l_2}\right)$

The computation of IK gave rise to table II, which presents the summary result of the computation. The result show that at J_1, θ_1 is the angle required to move the end effector to a desired position. The computational value of the angle is given as eq. 12, and depends on the values of l_2, x_p and y_p, where p represents the end effector.

On the other hand, the result also shows that the value of the angle θ_2 at joint J_2 depends on the following parameters shown in table 3.

Table- III: Parameters of the angle at J2

Parameters	**Description**
l_1	Link between J_1 and J_2
l_2	Link between J_2 and end effector
h	Value of the hypotenuse formed during the computation

III. CONCLUSION

The geometric analysis presents the mathematical derivation that described the free body system. The computational analysis absolutely resolved the forward and inverse kinematics that helped determined the location and angular movement of the end effector within the robotic region.

Geometric approach was adopted for the computation of the Forward Kinematics and the analytical method was adopted for the computation of Inverse Kinematics. The following can be deduced from the analysis:

1. The results of the forward kinematics reveals that the robot end effector position can be estimated using the values of x, y, and z coordinates.
2. The inverse kinematics would be determined by x and y coordinates only.
3. This approach has simplified the process robotic design problems with respect to finding the end effector's position at every instance of the robot operations.

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