International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249-8958 (Online), Volume-9 Issue-2, December, 2019

Pseudo Abelian Ternary Semihypergroups

Seetha Mani P, Sarala Y, Jaya Lalitha G

ABSTRACT---In this article we introduce the notion of left pseudo, lateral pseudo, right pseudo, pseudo, quasi abelian Ternarysemihypergroups and characterize them by utilizing various hyperideals of Ternarysemihypergroup extending those from Semigroups.

Keywords: Left Pseudo, Right Pseudo, Lateral Pseudo, Pseudo, Quasi abelian Ternarysemihypergroups.

I. INTRODUCTION AND PRELIMINARIES

The analytical concept of Semigroups was extensively investigated by Clifford and Preston[2]. A. Anjaneyulu[1] was the first who examined the theory of Pseudosymmetric ideals and PseudosymmetricSemigroups. In [6] Sarala studied the Pseudosymmetric ideals in Ternarysemigroups. Hyperstructure concept was presented by Davvaz in [5]; although characterized Hypergroups dependent on the idea of hyper operation and evaluate their features. In an ancient algebraic formation, the distribution of two components is a component, while in an analytical hyperstructure, the distribution of two components is a set. Davvaz etc., in [5] examined a family of algebraic hyperstructure which illustrate to a generalization of Semigroups, Hypersemigroups and n -arySemigroups.

In this article we introduced the notion of left pseudo, lateral pseudo, right pseudo, quasi abelianTernarysemihypergroups and characterize them by utilizing varioushyperideals of Ternarysemihypergroups extending those for Semigroups.

II. PSEUDO ABELIAN TERNARY

SEMIHYPERGROUPS:

Definition2.1: A Ternarysemihypergroup (T, g) is said to be abelian provided

g(a, b, c) = g(b, c, a) = g(c, a, b) = g(b, a,c) = g(c, b,a) = g(a,c, b) for a, b, c ∈T.

Definition2.2: A Ternarysemihypergroup (T,g) is said to be quasi abelian if for each p, q, r ∈T, there exist an odd natural number ‘n'such that g(a, b, c) = g(b^n, a, c) = g(b, c, a) = g(c^n, b, a) = g(c, a, b) = g(a^n,c, b)

Theorem2.3: If (T, g) is an abelian Ternarysemihypergroup then (T, g) is a quasi abelian Ternarysemihypergroup.

Proof: Suppose (T, g) is an abelian Ternarysemihypergroup. Put p, q, r ∈T.

Consider g(p, q, r) = g(q, r, p) = g(r,p, q)

g(p, q, r) = g(q, r, p) = g(r, p, q)g(p, q, r) =

g(q^1 p, r) = g(q, r, p) = g(r^1 , p, q)=g(r,p,q) =g(p^1,r,q).Hence (T, g) is a quasi abelianTernarysemihypergroup.

Definition2.4: A Ternarysemihypergroup (T, g) is said to be normal if g(a, b, T) = g(T,a,b) ∀a,b∈ T.

Theorem2.5: If (T, g) is a quasi abelian Ternarysemihypergroup then (T, g) is a normal Ternarysemihypergroup.

Proof: Put p,q∈T. If y∈g(p,q,T). Then y∈g(p,q,r) where r ∈ T. Since (T,g) is quasi abelian Tenarysemihypergroup. g(p,q,r)=g(r^n,p,q)⊆g(T,p,q). Therefore g(p,q,T)⊆g(T,p,q). Similarly g(T,p,q)⊆g(p,q,T). Then g(p,q,T) = g(T,p,q) ∀p,q∈T.

Corollary2.6: Every abelian Ternarysemihypergroup is a normal Ternarysemihypergroup.

Proof: Put (T,g) be an abelian Ternarysemihypergroup. By theorem2.3, (T,g) is a quasi abelian Ternarysemihypergroup. By theorem 2.5, (T,g) is a normal Ternarysemihypergroup. Hence, every abelian ternarysemihypergroup is a normal Ternarysemihypergroup.

Definition2.7: A Ternarysemihypergroup (T, g) is said to be a left pseudo abelian

if g(g(p,q,r),s, t) = g(g(q, r,p), s,t) = g(g(r, p, q), s, t) = g(g(q, p, r), s, t) = g(g(r,q, p), s, t) =

g(g(p, r, q), s, t)∀p, q, r,s, t ∈T.

Theorem2.8: If (T, g) is an abelian Ternarysemihypergroup, then (T, g) is a left pseudoabelian Ternarysemihypergroup.

Proof: Suppose (T, g) is an abelian Ternarysemihypergroup i.e.,

g(p, q, r) = g(q, r, p) = g(r, p, q) = g(q, p, r) = g(r, q, p) = g(p, r, q) ∀p,q,r∈ T.

Then considering (g(p, q, r), s, t) = g(g(q, r, p), s, t) = g(g(r, p, q), s, t) = g(g(q, p, r),s, t) =

g(g(r, q, p), s, t) = g(g(p, r,q), s, t)∀s, t ∈T.

Thus (T,g) is a left pseudo abelian Ternarysemihypergroup.

Note2.9: The reverse of the above theorem is not true. i.e., (T, g) is a left pseudo abelianTernarysemihypergroup then (T, g) need not be an abelian Ternarysemihypergroup.

Revised Manuscript Received on December 30, 2019.

* Correspondence Author

Seetha Mani P, Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India (Email: mani.srinu26@gmail.com)

Sarala Y, Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India

Jaya Lalitha G. Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India

Example2.10: Put T = {p, q, r, s ,t}. Define a ternary operation [ ] on T as [pqr] =

g(p,q, r) where the binary operation ' . ' is defined as

.	p	q	r	s	t
p	p	p	p	p	p
q	q	p	p	p	p
r	p	p	p	p	p
s	p	p	p	p	p
t	p	p	r	s	T

Clearly (T, g) is a Ternarysemihypergroup.

Now (T,g) is a left pseudo abelianTernarysemihypergroup. But (T, g) is not abelianTernarysemihypergroup.

Definition2.11: A Ternarysemihypergroup (T, g) is said to be a lateral pseudo abelian Ternarysemihypergroup if g(p, g(q,r,s),t) = g(p, g(r,s,q), t) = g(p, g(s, q, r),t) = g(p, g(r, q,s), t) = g(p, g(s, r, q), t) = g(p, g(q, s, r), t)∀p,q, r, s, t ∈T.

Theorem2.12: If (T, g) is an abelian Ternarysemihypergroup then (T, g) is lateral pseudo abelian Ternarysemihypergroup.

Proof: Suppose (T, g) is an abelian Ternarysemihypergroup. i.e., g(a, b, c) =

g(b, c, a) = g(c, a, b) = g(b, a, c) = g(c, b, a) = g(a, c,b) ∀a, b,c∈T. Consider g(a, g(b, c, d), e) = g(a, g(c, d, b), e) = g(a, g(d, b, c), e) = g(a, g(c, b, d), e) = g(a, g(d, c,b), e) = g(a, g(b, d, c), e),∀a, b,c, d,e∈T. Thus (T,g) is a lateral pseudo abelian Ternarysemihypergroup.

Note2.13: The reverse of theorem 2.12 is not true.

Example2.14: In example 2.10; (T; g) is a lateral pseudo abelian Ternarysemihypergroup. But (T, g) is not an abelian Ternarysemihypergroup.

Definition2.15: A ternary semi hypergroup (T; g) is known as right pseudo commutative Ternarysemihypergroup provided

g(a, b, g(c,d, e)) = g(a, b, g(d, e, c)) = g(a, b, g(e, c, d)) = g(a, b, g(d, c,e))

= g(a, b,g(e,d, c)) = g(a, b, g(c, e, d)) ∀a, b, c, d, e ∈T

Theorem2.16: If (T, g) is an abelian Ternarysemihypergroup then (T, g) is

a right pseudo abelian Ternarysemihypergroup.

Proof: Similar to the proof of 2.8

Example2.17: Consider the Ternarysemi hypergroup (T, g). In example 2.10, (T, g)

is a right pseudo abelian, but (T, g) is not an abelian Ternarysemihypergroup.

Definition2.18: A Ternarysemihypergroup (T, g) is known to be pseudo abelian if (T, g) is a left pseudo abelian, right pseudo abelian and lateral pseudo abelian Ternarysemihypergroup.

Theorem2.19: If (T, g) is an abelian Ternary semihypergroup then (T, g) is a pseudo cabelian Ternarysemihypergroup.

Proof: Suppose that (T; g) is a abelian Ternary semi hypergroup. By theorem 2.8, (T, g) is a left pseudo abelian Ternarysemihypergroup.

By Theorem 2.12,(T, g) is a lateral pseudo abelian Ternarysemihypergroup. By Theorem 2.16, (T, g) is a right pseudo abelian Ternarysemihypergroup. Thus (T, g) is a pseudo abelian Ternarysemihypergroup.

Remark2.20: If (T, g) is a pseudo abelian Ternary semi hypergroup then

(T, g) need not be an abelian Ternarysemi hypergroup.

Example2.21: In Example2.12., (T, g) is a pseudo abelian. But (T, g) is not an abelian Ternarysemihypergroup.

Theorem2.22: If (T,g) is a quasi abelian ternarysemihypergroup then

(T, g) is pseudo abelian ternarysemihypergroup.

Proof: Suppose (T, g) is a quasi abelian Ternarysemihypergroup.

i.e., g(a, b, c) = g(b^n , a, c) = g(b, c,a) = g(c^n, b a) = g(c, a, b) = g(a^n , c, b),∀a, b, c ∈T and n is a natural odd number. Consider g((a, b, c), d, e) = g(g(b^1, a, c), d,e) = g(g(b,c, a), d,e) = g(g(c^1 , b, a), d, e) = g(g(c, a, b), d, e) = g(g(a^1 , c, b),d, e). Thus (T,g) is a left pseudo abelian

Ternarysemihypergroup. Similarly (T; g) is a right and lateral pseudo abelian Ternarysemihypergroup. Hence (T, g) is a pseudo abelian Ternarysemihypergroup.

III. PSEUDO INTEGRAL TERNARY SEMIHYPERGROUPS& RESULTS

Now we introduce the concept of Pseudo integral in Ternarysemihypergroups. These results give an important characterization of Ternarysemihypergroup.

Definition3.1: A hyperideal P of a Ternary semihypergroup (T,g) is termed as a

Pseudosymmetric hyperideal, if x, y, z ∈T,

g(x, y, z) ⊆ P ⇒ g(x, s, y, t, z) ⊆ P

∀s, t ∈ T .

Definition3.2: A Ternarysemihypergroup (T, g) is termed as a Pseudosymmetric, if its every hyperideal is Pseudosymmetric.

Definition3.3: The intersection of all hyperideals of a Ternarysemihypergroup

(T, g) is termed as kernel of T .

It is denoted by K.

Definition3.4: A Ternarysemihypergroup (T, g) with nonempty kernel K is said to be Pseudo integral Ternarysemihypergroup provided K is a pseudo symmetric

hyperideal.

Note3.5: The nonempty intersection of any family of Pseudosymmetric hyperideals of a Ternarysemihypergroup (T,g) is a Pseudosymmetric hyperideal of T.

Theorem3.6: Every Pseudosymmetric Ternarysemihypergroup (T,g) with nonempty kernel is a Pseudo integral Ternarysemihypergroup.

Proof: Let (T,g) is a Pseudosymmetric Ternarysemihypergroup with kernel K. Then every hyperideal of T is a Pseudosymmetric hyperideal. Since kernel is the intersection of all hyperideals, by note 3.5 ; kernel is a Pseudosymmetric hyperideal. Hence (T,g) is a Pseudo integral Ternarysemihypergroup.

Retrieval Number: B5121129219/2019©BEIESP
DOI: 10.35940/ijeat.B5121.129219
Journal Website: www.ijeat.org

Published By:
Blue Eyes Intelligence Engineering
& Sciences Publication

IV. CONCLUSION

In this article we introduced the notion of left pseudo,lateral pseudo, right pseudo, quasi abelian Ternarysemihypergroups and characterize them by utilizing various hyperideals of Ternarysemihypergroups extending those from semigroups and studied analogus results for Ternarysemihypergroups. These are useful in order to analyse a new class of Ternarysemihypergroups. Some further work can be done in the future characterizing different classes of Ternarysemihypergroup through Pseudoabelian Ternary hyperideals . Concrete examples

have been constructed in support of our discussion. In particular we discussed different properties of Pseudo abelian Ternarysemihypergroups

REFERENCES

1. Anjaneyulu.A, Semigroups in which Prime ideals are maximal, Semigroup forum, 22, (1981), 151-158.
2. Clifford.A.H., and G.B. Preston,The algebraic theory of Semigroups, American MathematicalSociety, Providence Vol.I, (1961).
3. Davvaz.B, W.A.Dudek and T. Vougiouklis, A generalisation of n-ary algebraic systems, Com-mun. Algebra 37 (2009) 1248-1263
4. Jaya lalitha, G. Sarala, Y., "On ternary semigroups which are unions of a finite number of principal ideals", Global journal of Pure and Applied Mathematics, Vol. 11, No.2, (2015), 1079-1086.
5. Seetha mani.P, Jaya lalitha, G. Sarala, Y., "Ideal theory in A ternary semigroups", IJCIET, Vol.9, No.3, (2018), 170- 177.
6. Sarala.Y, Anjaneyulu.A and MadhusudanaRao.D; Pseudo Symmetric ideals in ternary semigroups, International refered journal of Engineering and science, vol-1; Issue-4,(2012), 33-43

AUTHOR PROFILE

Seetha Mani P, Faculty in,working as JL in SDMS kalasala since 20 years. MSc,MPhil, research scholar in Department of Mathematics, Koneru Lakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India.She is the author of several research papers published in reputed journals. She is a member in International Association of Engineers (IAENG).

Dr .Y.Sarala is a Assistant Professor, Department of Mathematics, K L University, Guntur, India. She obtained Ph.D degree from Acharya Nagarjuna University in 2013. She is the author of several research papers published in reputed journals. She is a member in International Association of Engineers (IAENG). She is published a book in Lambert Academic Publishing entitled " Algebraic Structures in Ternary Semigroups".

Dr G. Jaya Lalitha is a Assistant Professor, Department of Mathematics, K L University, Guntur, India. She obtained Ph.D degree from K L University in 2017. She is the author of several research papers published in reputed journals. She is a member in International Association of Engineers (IAENG), Associate member in Institute of Research Engineers and Doctors and Life Member in Andhra Pradesh Society for mathematical Sciences.