Behavioral Analysis of Various Techniques of Model Order Reduction Used in the Reduction of Large Scale Control System

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Abstract: It is very important task to study the behavior of the processes occurring in the industry. To attain this task, the knowledge of the transfer function of the system should be there. When working in robust environment, these transfer functions becomes so tedious that it becomes very difficult to obtain these transfer functions and hence affects the study of the behavior of these system. Due to this, the requirement for reduction of these transfer function becomes a necessity to analyze the behavior of foresaid systems and it becomes easy to do the desired modifications in the system i.e addition of any feature, desired changes in the behavior etc., furthermore the thing to be kept in consideration while doing the reduction in transfer function that the behavior viz. peak overshoot, settling time, steady state error of the two systems (reduced and the original system) should be approximately same, so it is prime importance that the applied model order reduction technique should provide a more accurate approximation of original higher order system. The paper presents here the different categories of model order reduction techniques that can be applied to achieve the motto of model order reduction of higher order systems. The techniques presented are categorized into the four different categories to understand them and their merits and demerits and these will help in proper selection of the model order reduction technique to obtain the most accurate reduced order approximation of large scale system.
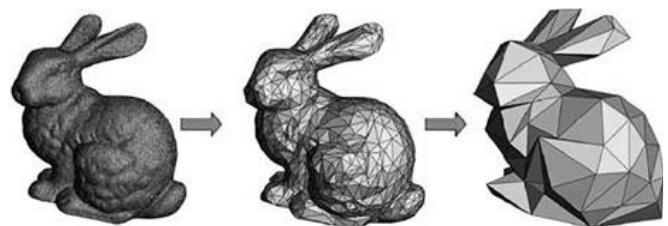
Keywords: model order reduction techniques, state space models, transfer function models, soft computing, mixed approaches

I. INTRODUCTION

A. Model Order Reduction

Most of the applications of Science and Engineering are expressed in the form of complex and large order models which are too difficult to analyze and designing of these models becomes a tedious task for the system designer. Reduction of the order of a transfer function is an essential task to analyze the behavior of the system viz. stability, output behavior, deviation in output etc. particularly when working in robust environment [1]. Instead of this, the small order systems are quite simpler to implement and their easy designing makes them more interesting than the large order models. Hence, there is a requirement of order reduction of large order systems so as to find a more suitable lower order model which can preserve the input output behavior as well as the properties of the original large order system. The main motive of Model Order Reduction is to approximate the large scale system into its corresponding reduced order equivalent

large form, so that the reduced order system behaves like a order system consisting all the major properties of the initial system even after reduction. The concept of model order reduction can be better understood with the graphical representation depicting that sometimes, nominal information is required to study the behavior of the system as in figure 1.

Figure 1: Graphical Representation of Model Order Reduction

The study of model order reduction is necessary because:
- Analysis and synthesis of higher order systems is quite difficult.
- Conventional computational approaches have an inefficient balance amongst computational time and accuracy.
- Modeling of large real time systems results in huge number of differential or difference equations.
- Large-scale nature of the models often leads to unmanageable demands on the computational resources.
- Large scale models are often computationally too expensive for real-time control which can act as a burden.

This paper constitutes the Literature review on the ground of developments occurred in the field of model order reduction. Then various model order reduction techniques are described and their comparative analysis is performed to obtain the results. The objective behind this paper is to cover some techniques of model order reduction so that the researcher can study them and do the necessary modifications in these techniques to achieve the more accurate reduced order system by applying these modified methods on large scale system.

II. LITERATURE REVIEW

Many researchers are working around the globe in the area of model order reduction of a large order system to approximate it into its reduced order equivalent system with minimizing the errors among them and hence making the obtained reduced order system more accurate. In achieving the task, some literature work has been carried out. B.C.

Moore [2] used kalman's minimal realization theory which involves the controllability, observability subspaces keeping the thought that the small changes in these subspaces can reduce the dimension of the system. Keith Glover [3] proposed the hankel norm reduction technique for the model order reduction of linear multivariable systems using the reduction of hankel norm L∞ error among the original higher order system and obtained reduced order system. U. Desai et.al [4] presented a stochastic model order reduction technique while Christian de Villemagne [5] initially represented a method for the reduction of multi input multi output (MIMO) systems by using projection techniques. M.G. Safonov [6] designed a novel approach of model reduction of higher order state space model of LTI system which was a robust technique to find the relative error between the original and reduced order model system. S. Mukharjee [11] proposed the latest response matching technique for the model order reduction of an integer order higher order LTI system. Cheng [35] applied the clustering approach of model order reduction in power networks comprising of generators and loads. By using clustering approach, the dissimilarities in the various nodes of generator and load were found out by exploiting controllability grammian.

With the invention of mixed approaches which used two varied techniques for the reduction of numerator and denominator polynomial, many researchers worked on this concept. G. Parmar presented an approach for order reduction of integer order LTI systems by applying two different approaches: factor division algorithm and Eigen spectrum analysis [39] which combined the two previously designed order reduction techniques. C.B. Vishwakarma [20] proposed a mixed approach for reducing the order of a large scale integer order MIMO system by using modified pole clustering to diminish the denominator polynomial and Genetic algorithm was used to minimize the integral square error among the approximated reduced numerator polynomial and original system's numerator and hence adding the use of artificial intelligence in the model order reduction theory. By using the newly proposed optimization technique of Big Bang Big Crunch (BBBC), Philip [22] proposed an evolutionary computation based approach which was a mixture of two techniques: dominant pole method and big bang big crunch optimization method. Sikander [26] described a mixed approach which consisted of two methods: first was the reduction of numerator polynomial of the given transfer functions by factor division algorithm and second was the reduction of denominator polynomial by using stability equation method. Narwal [27] proposed a novel approach to reduce the order of high order integer order LTI systems by using both cuckoo search algorithm and routh approximation algorithms. Tiwari [37] presented an improvement in order reduction methods by combining the two techniques of reducing the order of numerator polynomial and denominator polynomial by two different methods. The numerator polynomial was reduced by factor division algorithm and denominator polynomial was reduced by using modified clustering method.

It is clear from the literature that the model order reduction is a necessity while dealing with large scale systems and due to this, it becomes a hot topic and attracts the researchers. The various theories were developed as studied in literature to reduce the order of a large scale system. The main aim of each technique is to reduce the error among the original large scale system and obtained reduced small order system and increase the accuracy between them. The invention of mixed approaches and genetic algorithm increases the interest in this area as the methods based on these approaches, generates the reduced order system with least amount of error. This motivates to work on these techniques so that some more newly developed technique can be applied for the reduction of large scale systems and more accurate reduced order approximation can be obtained with least amount of error.

III. METHODS OF MODEL ORDER REDUCTION

There are numerous techniques developed since 1970s. Some techniques can be applied to the state space model of the given system while others can be applied directly to the transfer function equation. After the evaluation of soft computing techniques, the approximations of the given system in the reduced form can be found directly through writing the equation for both the forms and then approximating them for the best possible result.

Some of the basic techniques used for the model order reduction are described as follows:

A. Order reduction of state space models

This category requires the conversion of given system into its state space representation refer to (1a & 1b) and thus obtaining its state matrices (A, B, C, D).

$$\bar{x} = Ax + Bu \tag{1a}$$

$$y = Cx + Du \tag{1b}$$

The reduction technique is then applied to reduce the order of state matrices (A_r, B_r, C_r, D_r). Using these reduced state matrices, the reduced order system can be obtained, refer to (2a & 2b):

$$\overline{x_r} = A_r x + B_r u \tag{2a}$$

$$y_r = C_r x + D_r u \tag{2b}$$

then these reduced order state matrices obtained, are converted back into the transfer function form, refer to (3)

$$G_r(s) = C_r(sI - A_r)^{-1}B_r + D_r \tag{3}$$

Some of the basic techniques of this category are defined as follows.

a) Balanced truncation method

The balanced truncation technique [2] is the basic and foremost technique of model order reduction as most of the reduction techniques rely on it for obtaining the balanced form of the system. In this technique the A, B, C and D state matrices are transformed by using a non-singular matrix T into a balanced system such that

$$(A_r, B_r, C_r, D_r) = (T^{-1}AT, T^{-1}B, CT, D) \tag{4}$$

The balanced system obtained (A_r, B_r, C_r, D_r) is the reduced order approximation of (A, B, C, D)

b) Hankel Norm Approximation

Hankel norm approximation [3] finds the approximation $G_r(s)$ of the higher order system $G(s)$ such that the norm of error

$$\|G(s) - G_r(s)\|_H$$ is minimum. Initially, the controllability (P) and observability (Q) matrices were found out which satisfied the lyapunov equations, refer to (5 & 6)

$$AP + PA^T = -BB^T \qquad (5)$$

$$A^T Q + QA = -C^T C \qquad (6)$$

The hankel norm of the G(s) can be found out, refer to (7)

$$\|G(s)\|_H = \lambda_{max}^{1/2}(PQ) \qquad (7)$$

Where $\lambda_{max}(PQ)$ denoted the largest eigen value of matrix PQ and provides the most controllable or observable state. Then after finding the hankel norm, the least relevant states were discarded and remaining states formed the reduced order system.

c) Balanced stochastic method

Balanced stochastic realization [4] is the representation of the system with state covariance matrix which is equal to the canonical correlation coefficient matrix $\sum$ for the output process. The following steps are employed to obtain the balanced stochastic realization:

(i) Obtain the forward innovations representation and backward innovations representation for the given system.

(ii) Using a set of Ricatti equations, the FIR and BIR obtained from step (i) is converted into the correlation coefficient matrix $\sum$

(iii) Apply the transformation to the obtained equation to get the forward and backward BSR.

The resultant BSRs are the reduced form of the original higher order system by using the balanced stochastic technique.

d) Projection Technique

The process involved in this technique is usually a projection of higher order system on the lower order system. The reduced order state matrices in this technique can be written as referred in (8)

$$(A_r, B_r, C_r, D_r) = (LAT, LB, CT) \qquad (8)$$

Where L and T represents the projector matrices.

In this technique, the frequency response and a selected number of derivatives of frequency response are matched at selected frequencies. Also the power spectral density and its derivatives are matched at selected frequencies. The selected frequencies include the range of low frequencies, medium frequencies and higher frequencies. The frequency moments and power moments are calculated at this range of frequencies and hence reduced order system is obtained [5].

e) Schur Decomposition Technique

This technique [6] makes use of controllability (P) and observability (Q) matrices for the reduction process but without using any balancing operation. This technique involves the process of dividing the matrix PQ in the right and left eigen spaces associated with big eigen values and then singular value decomposition of these two matrices is found out, consequently by using a set of equations, the reduced order model state matrices are computed and hence the transfer function for the state model is designed. This technique can be better described by the following execution steps:

(i) Firstly the matrix PQ is converted into schur equivalent form by using $VPQV^T$. Where V is an upper triangular matrix.

(ii) Then real transformation of V into ascending and descending order are computed, refer to (9 & 10)

$$V_A^T PQ V_A = \begin{bmatrix} \lambda_1 & \cdots & \cdots & \cdots & \cdots \\ 0 & \lambda_2 & \cdots & \cdots & \cdots \\ \vdots & & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \lambda_n \end{bmatrix} \qquad (9)$$

$$V_D^T PQ V_D = \begin{bmatrix} \lambda_n & \cdots & \cdots & \cdots & \cdots \\ 0 & \lambda_{n-1} & \cdots & \cdots & \cdots \\ \vdots & & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \lambda_1 \end{bmatrix} \qquad (10)$$

(iii) Now V_A and V_D are portioned in the form

$$V_A = \left\{ \overbrace{V_{R(small)}}^{n-k} \middle| \overbrace{V_{L(big)}}^{k} \right\} \qquad (11)$$

$$V_D = \left\{ \overbrace{V_{R(big)}}^{k} \middle| \overbrace{V_{L(small)}}^{n-k} \right\} \qquad (12)$$

(iv) Then k big values ($V_{L(big)}$ & $V_{R(big)}$) are used to find out the reduced system.

So these five techniques discussed can be applied when the given system is in state space form or by converting the transfer function form to state space form. The main limitation of these techniques is that these techniques uses balancing of state equations and hence are computationally complex, so these cannot be applied for much higher systems.

B. Order reduction of transfer function

This type of reduction techniques does not require any conversion of transfer function into state space equivalent model as these can be directly applied to the transfer function in numerator & denominator form. The techniques using this concept are described as:

a) Continued fraction expansion method

In this technique, firstly the numerator equation is divided by denominator equation and the result is represented in the form of continued fraction expansion in cauer form. Then the first k (order of reduced system) factors are kept and higher order factors are truncated. The expansion is left for only upper k factors. Then the inverse process is executed and converted equation is the continued fraction expansion in the numerator and denominator form and the resultant k order reduced equation is the approximation of n order higher order system (k<n) by continued fraction expansion method [7].

b) PADE's approximation method

The PADE's approximant is a rational function represented with the combination of two polynomials $[p_m(x)]/[Q_n(x)]$ of degree m and n respectively. So this rational function is said to be PADE's approximant of a function f(x) if and only if the power series expansion of rational function $[p_m(x)]/[Q_n(x)]$ is same as that of f(x).

The technique of PADE's approximation works in the steps that firstly the PADE's approximant of the rational higher order transfer function $[p_m(x)]/[Q_n(x)]$ is approximated into its PADE's approximant f(x),

then a reduced rational order transfer function $[p_{m'}(x)]/[Q_{n'}(x)]$ is to be found out which is the PADE's approximant of f(x). Hence the PADE's approximant of given $[p_m(x)]/[Q_n(x)]$ higher order and reduced rational order transfer function $[p_{m'}(x)]/[Q_{n'}(x)]$ are same, therefore the reduced order system is said to be as best approximation of higher order system [8].

c) Stability equation method

The stability equations of the given transfer function in numerator and denominator form is obtained and then these equations are reduced to maintain the stability of the system [9].

Consider a system given by its transfer function refer to (13)

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \quad (13)$$

Where n>m

Equation (13) can be represented in the even and odd equations of numerator and denominator, refer to (14)

$$G(s) = \frac{N_e(s) + N_o(s)}{D_e(s) + D_o(s)} \quad (14)$$

The equations of numerator and denominator in even and odd form are known as stability equations and they are truncated directly to form the reduced order equations and then combined to form the complete transfer function of reduced system.

d) Factor division method

This technique of factor division [10] can be simply understood by its name. The poles are represented in the factored form and the numerator is divided by the factor and reduced order system is obtained. Consider the original system is represented by its transfer function form:

$$G(s) = \frac{b_0 + b_1 s + b_2 s^2 + \cdots + b_{n-1} s^{n-1}}{(s+p_1)(s+p_2)\cdots(s+p_{n-1})(s+p_n)} \quad (15)$$

If the numerator is divided by the factor $(s+p_n)$, then the order of the system is reduced to (n-1) from 'n'. This method can be extended further for obtaining desired order of reduction.

e) Response matching or pole retention technique

This technique is based on simply matching the response of the reduced system with that of original higher order system by taking some poles from the total number of poles. Suppose a system possess twelve poles and it is desired to design a system with 3 poles, then by random selection take the three poles out of twelve and match the response of the systems to check which combination is better in reduced system [11].

The entire transfer function can have three types of poles: real poles, complex poles and repeated poles. On the basis of these three type of poles, the different cases are presented to select the poles. Such as: (i) all of the poles are real (ii) some poles are real and some are complex conjugate pair (iii) some poles are real and some are repeated poles (iv) all poles are repeated poles. Depending on these combinations, the reduced system can be compared with the corresponding higher order system.

The Techniques discussed here can be applied when the large scale system is represented by transfer function form in numerator and denominator form. These techniques involves large mathematical calculations so these becomes complex for higher system (Order>50). Also these techniques do not guarantee the stability of the reduced system even when the original higher order system is stable.

C. Order Reduction on the basis of soft computing methods

The soft computing techniques or Artificial Intelligence [12] also proves to be helpful in model order reduction of higher order systems. The soft computing techniques involved in the reduction process are genetic algorithm, fuzzy logic, artificial neural network (ANN), firefly algorithm, invasive weed optimization, particle swarm optimization, big bang big crunch algorithm, cuckoo search algorithm and more. The concept of using these optimization or soft computing techniques is that, firstly a reduced approximation of the higher order system is generated then this approximated system is optimized on the basis of parameters like integral square error, integral absolute error, integral time absolute error, peak overshoot and settling time. Then optimized system evolved is known as reduced order system.

D. Order reduction on the basis of mixed approaches

The concept of mixed approach involves the use of two model order reduction techniques on the same system. One technique is applied on the numerator polynomial while other technique is focussed on the denominator polynomial of the higher order system keeping in consideration that the order of reduced polynomial equation must be lower than that of denominator polynomial. The advantage of using the mixed approach is that the errors in the reduced system get reduced with the application of two techniques and this combination of techniques has always proven out that it is better to apply mixed approach than single technique for minimizing error between higher order and the corresponding reduced order system.

The techniques of model order reduction are categorized here into four categories. Among these soft computing and mixed approaches are of more interest as these techniques guarantees the more accurate reduced system as compared to other techniques. Also techniques under these categories provide the zero steady state error in reduced system along with decreased Integral square error.

IV. COMPARATIVE ANALYSIS

The model order reduction has become a very hot topic in the recent years due to the development of more advanced reduction techniques and with introduction of artificial intelligence in the area of model order reduction for

controlling the processes. Some more modifications have taken place from time to time.

The invention of first technique which involved balancing of the equations resulted in an error of 0.05988 with the reduction of a 3rd order system into its 2nd order corresponding system.

This error was very large and was not acceptable. This developed the interest of researchers in the field of order reduction techniques. With the further developments in the techniques of model order reduction like Hankel norm, projection, stochastic based balancing techniques, the relative error amongst them was reduced more and then by using schur decomposition technique, the relative error was reduced to 1.87×10^{-3} which was a great achievement. The use of artificial intelligence reduced the steady state error to zero but dynamic error had some value which could be further reduced with the invention of mixed approaches which helped in the reduction of integral square error to the value of 1.62×10^{-2} and steady state error equal to zero with the reduction of a 9th order system into its 2nd order equivalent form which made the reduced system more accurate approximation of the original large scale system.

All the research work was carried out on the reduction of error among the original large order system and the reduced approximated system. The amount of error was reduced from a high value to a very small value that hardly made any difference between the original and reduced approximation. Therefore, table I describes the amount of error occurring in the reduced system obtained using different reduction techniques.

I. Comparison of various Techniques on the basis of amount of error

Reduction Methods	Amount of error (ISE)
Mukharjee [11]	5.6897×10^{-02}
Shamash [8]	27.92×10^{-02}
Chen [9]	7.2101
Lucas [10]	27.92×10^{-02}
Parmar [39]	4.809×10^{-02}
Vishwakarma [20]	1.406×10^{-02}
Phillip [22]	2.82×10^{-02}
Sikander [26]	0.07261×10^{-02}
Sharad [29]	0.544×10^{-02}
Sikander [33]	1.390×10^{-02}
Sharad [37]	1.62×10^{-02}

The comparison in I shows that the error between the original large order system and reduced approximated small order system decreases with invention of new approaches. Hence the invention of more techniques can give more reduction in error, which generates the interest of researchers in the development of new techniques of model order reduction.

V.CONCLUSION

The study of model order reduction techniques depicts that there are different techniques to reduce the order of any large scale system which renders help to the control engineer to study the behavior of the system on which he is working.

The model order reduction techniques are divided into four major categories in this article. The first two techniques describe the type of system required for the reduction process. First category describes the techniques which can be applied when the system is represented in the form of state space equations. Second category describes the techniques which can be applied to the system with a given in the form of transfer function. These techniques are very simple to use but with the passage of time, these techniques have become obsolete due to the amount of error they possess in the reduced order system. The error leads to less accurate reduced order approximation, therfore this error is not desirable for better approximation of large scale system.

With further modifications, third and fourth category of model order reduction techniques is developed. Third category describes the use of soft computing techniques, which gives the better approximation of large scale system by providing very less amount of error among the original system and its corresponding reduced system. The fourth category uses the combination of two techniques to reduce one system. By using this approach, all the merits of previously defined techniques can be used for the reduction process and flaws can be eliminated. The researcher can use any technique from the three categories to reduce the large scale system and form its reduced order approximation with least error and high accuracy.

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