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Graphical Robust PID Tuning of TITO Processes

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***Abstract**: In this paper, PID controller for Decentralized TITO process is tuned using graphical approach. Ideal decoupler is used to decouple TITO system and then each decoupled system is approximated to FOPDT model. The Graphical method uses boundaries determined by the loci of stability and frequency domain characteristics such as Gain, and Phase margin etc. common region of these loci gives the values of the PID controller coefficients. The results are validated using MATLAB and the simulated results are presented for Coupled tank system.*

***Keywords** : PID control, Decentralized, TITO, Coupled Tank system, Graphical method of PID tuning.*

I. INTRODUCTION

Chemical processes are mostly Multi-input Multi-output (MIMO) systems. As multivariable processes have interactions, directionality plays a major role in these systems[1]. which makes it difficult to design a controller. Proportional Integral Derivative (PID) controller can give good performance, but the problem is with the structure of the controller to be selected. Most of the multivariable processes use either centralized or decentralized control structure the latter is simple, but the performance is not accurate. Chemical process industry uses PID control structure at a low level with optimal control strategy at a high level, but the designing and implementing of these controllers is difficult. In order to achieve acceptable performance, feedback control is required instead of just feed-forward control in existence of uncertainty.

In Decentralized control, system is decoupled into independent subsystems (SISO structure). Decentralized controllers have specific structures, which are not suitable for some standard synthesis controllers like $H_2$and H_∞. A lot of research is done on the decentralized control with a focus on stability using time domain[2], as well as frequency domain analysis techniques[3].

Due to the simple design methodology, the development of Decentralized controllers received a good choice. Many types of methods are proposed for designing these controllers. Luyben et.al[4]proposed Biggest Log Tuning (BLT) method based on detuning approach which uses RGA for the measurement of degree of interactions. In this method first ZN settings are calculated for every individual loop and then detuned by some factor whose value varies between 2 and 5.

A similar method is proposed by Chien et al.[5], Stability of system and loop performance are not clearly addressed in these methods. Using solution of minimum variance control problem, a multivariable PID controller was designed by R. Yusof et al.[6]This method requires ARMAX model and hence increasing the complexity of computations. Lee and Edgar proposed a method where dominant poles are placed on desired positions to improve system performance[7], though these methods do not reduce significant effect of interactions present in system.Hovd et al.[3], and Shiu et al.[8], developed sequential method where the fastest loop controller was first tuned and then following the sequence. In this method, the performance depends on the sequence consider so it doesn't guarantee better performance. Xu and Ho[9] proposed a method based on the direct Nyquist array given by Rosenbrock, this method uses Gain and Phase margin specifications to tune the controllers. This method is one of the auto tuning methods used for decentralized controllers in TITO processes. When there are significant interactions then this method often gives poor performances. If the detailed model of the plant is not available, then the model is derived from historical data.Maciejowski method, Davison method, kovio and penttinen method, Relay feedback method are some of the non-parametric methods[10]. Maciejowski method decouples the system at cut-off frequency for this, frequency response of the system is calculated at a single point[11].The graphical method can be solved using D-decomposition method and Kharitanov method. Kharitanov method is comparatively less complex than D-decomposition method which is mathematically complex to divide the variables.

II. GRAPHICAL METHOD

D decomposition technique is used to determine coefficients of the PID controller Graphically[3]. Parametric space is divided into two regions using the loci of (1) Stability, (2) Gain margin, (3) Phase margin, (4) Reference to Disturbance Ratio and (5) Dominant pole.

A. Stability margin

Consider a generalized unity feedback system with Gain and Phase margin tester (GPMT) as shown in Figure 1.

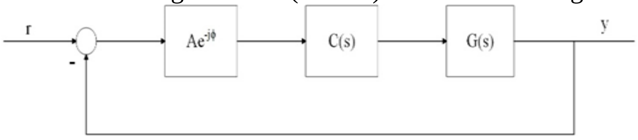

Fig. 1 General unity feedback system with GPMT and controller.

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Let the system be given as

$$G(s) = \frac{K}{Ts+1}e^{-sL} \quad (1)$$

Where, K, T, L are static gain, time constant and dead time respectively. Controller C(s) with integral gain K_i, derivative gain K_d and Proportional gain K_p is given by

$$C(s) = K_p + \frac{K_i}{s} + K_d s \quad (2)$$

From Fig. 1, The closed loop characteristic polynomial is given by

$$\delta(s) = Ts^2 + s + (KK_p s + KK_i + KK_d s^2)e^{-sL} \quad (3)$$

Now, multiply the equation with e^{sL}, Then we have

$$\delta(s) = (Ts^2 + s)e^{sL} + KK_p s + KK_i + KK_d s^2 \quad (4)$$

Let $= j\omega$, then (4) can be given as

$$\delta(j\omega) = (-T\omega^2 + j\omega)e^{j\omega L} + jKK_p\omega + KK_i - KK_d\omega^2 \quad (5)$$

Splitting up (5) into real and imaginary parts then we have
$\delta(j\omega) = \delta_r(\omega) + j\delta_i(\omega)$

$$\delta_r(\omega) = KK_i - T\omega^2\cos(\omega L) - \omega\sin(\omega L) - KK_d\omega^2 \quad (6)$$

$$\delta_i(\omega) = \omega\cos(\omega L) - T\omega^2\sin(\omega L) + KK_p\omega \quad (7)$$

By using (6) & (7), solve for K_p and K_i

$$K_p = \frac{1}{K\omega}[T\omega^2\sin(\omega L) - \omega\cos(\omega L)] \quad (8)$$

$$K_i = \frac{1}{K}[T\omega^2\cos(\omega L) + \omega\sin(\omega L) + KK_d\omega^2] \quad (9)$$

Stability margin can be determined using the Jacobean matrix *J*, given by

$$J = \begin{pmatrix} \frac{\partial\delta_r(\omega)}{\partial K_p} & \frac{\partial\delta_r(\omega)}{\partial K_i} \\ \frac{\partial\delta_i(\omega)}{\partial K_p} & \frac{\partial\delta_i(\omega)}{\partial K_i} \end{pmatrix}$$

According to S Srivastava and VS Pandit.[12] and D.-J wang[13] "If det *J* <0 then right side region of curve is stable in increasing order of ω.If det *J* >0 then left side region of curve is stable in increasing order of ω."

B. Gain and Phase margin loci

ConsiderGPMT, $Ae^{-j\phi}$ discussed by DJ Wang[14]. GPMT is used for simulation purpose only and it doesn't affect the original system. From Fig. 1 closed loop characteristic polynomial is given by

$$\delta(s) = Ts^2 + s + (KK_p s + KK_i + KK_d s^2)e^{-sL}e^{-j\phi} \quad (10)$$

By using the procedure mentioned in finding the stability margin we get

$$K_p = \frac{1}{AK\omega}[T\omega^2\sin(\omega L) - \omega\cos(\omega L)] \quad (11)$$

$$K_i = \frac{1}{AK}[T\omega^2\cos(\omega L) + \omega\sin(\omega L) + AKK_d\omega^2] \quad (12)$$

In the (11) & (12) with $A = 1$ we get phase margin locus with desired φ. By taking $\varphi = 0$ we get gain margin locus with desired *A*. Existence of uncertainty in the range of crossover frequency may lead to poor performance.

C. Stability margin guaranteeing set point tracking

Let $s = -\zeta\omega \pm j\omega\sqrt{1-\zeta^2} = \sigma \pm j\omega$

Where, ζ is damping ratio, from(4) we get

$$\begin{aligned}\delta_r &= e^{\sigma L}(x_1\cos(\omega L) - x_2\sin(\omega L)) \\ &+ KK_i + KK_p + KK_d(\sigma^2 - \omega^2)\end{aligned} \quad (13)$$

$$\begin{aligned}\delta_i &= e^{\sigma L}(x_1\sin(\omega L) + x_2\cos(\omega L)) \\ &+ 2KK_d\sigma\omega + KK_p\omega\end{aligned} \quad (14)$$

Where $x_1$and $x_2$are given by

$$x_1 = T(\sigma^2 - \omega^2) + \sigma$$
$$x_2 = 2\sigma\omega T + \omega$$

From (13) & (14) we get

$$K_p = \frac{-1}{K\omega}(e^{\sigma L}(x_1\sin(\omega L) + x_2\cos(\omega L)) + 2KK_d\sigma\omega) \quad (15)$$

$$\begin{aligned}K_i = \frac{-1}{K}[&e^{\sigma L}(x_1\cos(\omega L) - x_2\sin(\omega L)) \\ &+ KK_p + KK_d(\sigma^2 - \omega^2)]\end{aligned} \quad (16)$$

D. Reference to Disturbance Ratio

BB Alagoz, FN Deniz, C Keles and N Tan[15], [16] proposed Reference to Disturbance Ratio (RDR) to evaluate the disturbance rejection capabilities of the system in parameter space. Ratio between the forward path with reference signal and disturbance path with disturbance signal is termed as RDR. RDR is given by

$$RDR = \frac{|Q_r(j\omega)|^2}{|Q_d(j\omega)|^2} \quad (17)$$

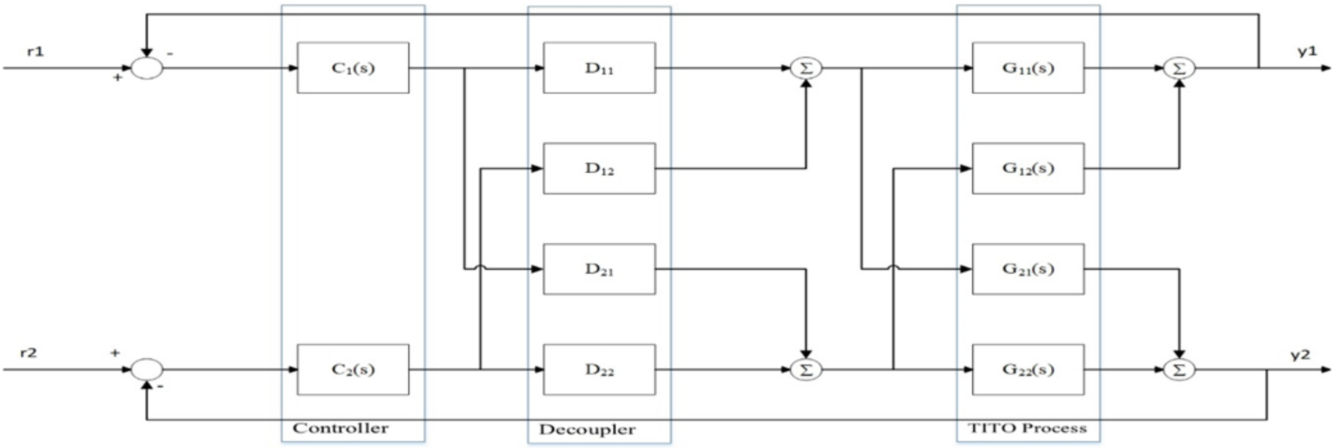

Fig. 2 Generalized TITO system with decentralized controller and ideal decoupler

To evaluate RDR performance analysis assume that the energy spectrum of disturbance signal and reference signals are equal such that $|r(j\omega)|^2 = |d(j\omega)|^2$, then RDR is reduced to

$$RDR = |C(j\omega)|^2 \qquad (18)$$

From (2) And (18) we have

$$RDR = \left(\frac{K_i}{\omega} - K_d\omega\right)^2 + K_p^2 \qquad (19)$$

Since the proportional gain K_P has been independent of frequency, it is employed for setting the minimum value of the RDR. RDR performance is limited by maximum of RDR stability boundary, given by

$$RDR(\omega) \le max\{RDR_{sbl}(\omega)\} = max\{|(G(j\omega))^{-1}|^2\} \quad (20)$$

The Lower RDR boundary with minimal RDR_{db} can be given by

$$K_p(\omega) \ge \sqrt{10^{0.1RDR_{db}} - \left(\frac{K_i(\omega)}{\omega} - \omega K_d\right)^2} \qquad (21)$$

Large Bandwidth and tight control are required when a large disturbance occurs in the plant.

III. SYSTEM MODELING AND ANALYSIS

A general two input two output system with decoupler is given in Fig. 2.
Here G(s) is the system transfer function and D(s) is the decoupling matrix. The G(s) and D(s) are given by

$$G(s) = \begin{pmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{pmatrix} \qquad (22)$$

$$D(s) = \begin{pmatrix} D_{11}(s) & D_{12}(s) \\ D_{21}(s) & D_{22}(s) \end{pmatrix} \qquad (23)$$

The equivalent Transfer function of the system with diagonal controllers is given by Q(s)

$$Q(s) = \begin{pmatrix} Q_{11}(s) & 0 \\ 0 & Q_{22}(s) \end{pmatrix} \qquad (24)$$

The decoupler matrix D(s) can be calculated using the method proposed by Nordfeldt et.al[17] and Hajare et al.[18]

$$D(s) = \text{adj}[G(s)]K(s) \qquad (25)$$

The decoupler transfer functions are of higher order so approximate these transfer functions to FOPDT models. Let the FOPDT model is given by $\hat{G}_{ii}(s)$

$$\hat{G}_{ii}(s) = \frac{K_{ii}}{T_{ii}s+1}e^{-L_{ii}s} \qquad (26)$$

By using frequency response fitting at phase crossover frequency (ω_{cii}) and zero frequency[14], [18] we get

$$\hat{G}_{ii}(0) = Q_{ii}(0)$$

$$|\hat{G}_{ii}(j\omega_{cii})| = |Q_{ii}(j\omega_{cii})|$$

$$\angle\hat{G}_{ii}(j\omega_{cii}) = \angle Q_{ii}(j\omega_{cii})$$

The process gain (K_{ii}), Time constant (T_{ii}) and Delay time (L_{ii}) are obtained as follows

$$K_{ii} = Q_{ii}(0) \qquad (27)$$

$$T_{ii} = \sqrt{\frac{K_{ii}^2 - |Q_{ii}(j\omega_{cii})|^2}{|Q_{ii}(j\omega_{cii})|^2\ \omega_{cii}^2}} \qquad (28)$$

$$L_{ii} = \frac{\pi + \arctan(-\omega_{cii}T_{ii})}{\omega_{cii}T_{ii}} \qquad (29)$$

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IV. MATHSIMULATION EXAMPLE

Let us consider a coupled tank system given by Hazare et al.[18].

$$G(s)=\begin{bmatrix}\frac{0.43}{29s+1}e^{-5s} & \frac{0.145}{40s+1}e^{-10s}\\ \frac{0.172}{35s+1}e^{-10s} & \frac{0.37}{27s+1}e^{-5s}\end{bmatrix} \quad (30)$$

Using (20) calculate decoupler matrix *D(s)*

$$D(s)=\begin{bmatrix}\frac{2.55}{27s+1} & \frac{-1}{40s+1}e^{-5s}\\ \frac{-1}{35s+1}e^{-5s} & \frac{2.5}{29s+1}\end{bmatrix} \quad (31)$$

Equivalent SISO loop transfer functions calculated using (27-29) are given as

$$G_{11}=\frac{0.9245}{113.2s+1}e^{-12.7s} \quad (32)$$

$$G_{22}=\frac{0.78}{113.2s+1}e^{-12.7s} \quad (33)$$

Using graphical method, parameters of PID controller are calculated. The range of K_p and K_i while keeping K_d as constant is given in Fig. (3)

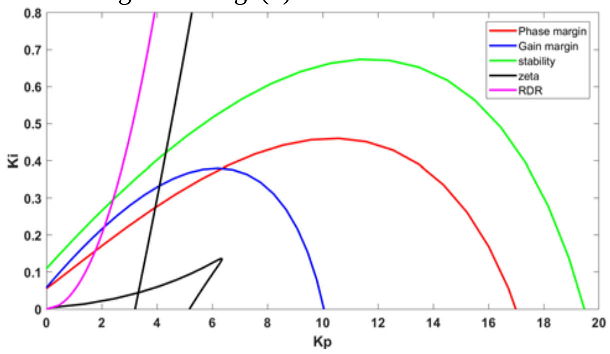

Fig. 3 Range of K_p, K_i with Gain and Phase margin, RDR, stability and ζ_{cl} boundaries for Y_1

Calculating Jacobian for eq (6) and eq (7), we get det $J<0$, which means that right side of locus is stable for increasing order of the ω. Now consider few points in the parametric space viz., **a** (18,0.7), **b** (14,0.1), **c** (10,0.1), **d** (1,0.06), **e** (2,0.05), **f** (4.0.07). Considering K_d = 15, The time response characteristics of these points are given in Fig (4) and Fig (5). From Fig (4) we can observe that the system is unstable for point **a,** which is outside of the intersection region. From fig (5), observe that moving away from RDR boundary, gives good disturbance rejection performance.

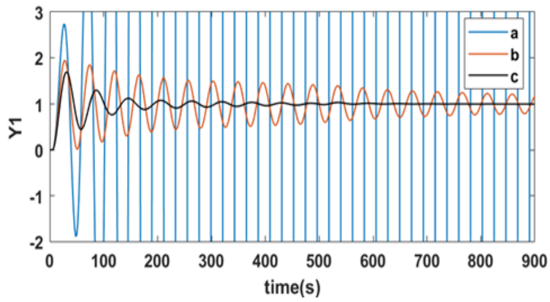

Fig. 4 Time response characteristics for points a, b, c of Y_1

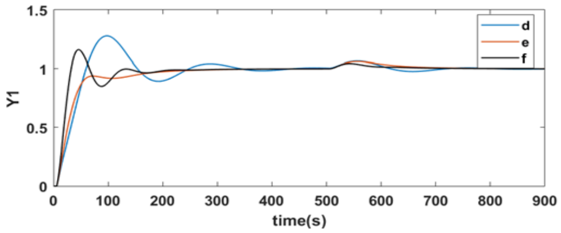

Fig. 5 Time response characteristics for points d, e, f of Y_1

Now, consider the other loop with the system G_{22} given by eq. (33). Parametric space with constant K_d =15, K_p, K_i is given in Fig (6).

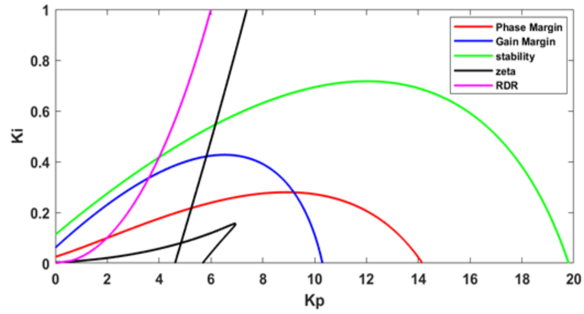

Fig. 6 Range of K_p, K_i with Gain and Phase margin, RDR, stability and ζ_{cl} boundaries for Y_2

Now consider few points in the parametric space viz., **a**(16,0.9), **b** (14,0.1), **c** (12,0.2), **d** (1,0.06), **e** (2,0.04), **f**(5.0.07). Considering K_d = 15, The time response characteristics of these points are given in fig (7) and fig (8). From fig (7) we can observe that the system is unstable for point **a,** which is outside of the intersection region. From fig (8), point **f** has more disturbance rejection capability compared to point **d**. The set of values on the linear region of the ζ_{cl} curve gives time response characteristics near to specified value,values on negative slope of locus gives poor performance.

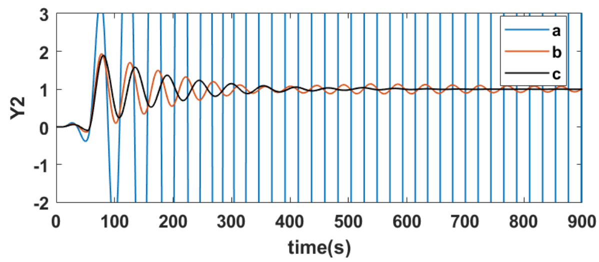

Fig. 7 Time response characteristics for points a, b, c of Y_2

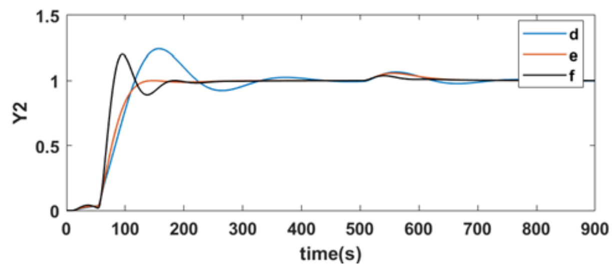

Fig. 8 Time response characteristics for points d, e, f of Y_2

Table I Time response characteristics for Y_1 with K_d=15

Point	K_p	K_i	Gain Margin(dB)	Phase Margin(degree)	Peak overshoot(%)	Rise time(sec.)	Settling time(sec.)
d	1	0.06	18.2	29.1	27.7	42.325	322
e	2	0.03	17.2	51.6	8.7	33.628	106
f	4	0.05	13.1	58.7	23.4	18.4	143

Table II Time response characteristics for Y_2 with K_d=15

Point	K_p	K_i	Gain Margin(dB)	Phase Margin(degree)	Peak overshoot(%)	Rise time(sec.)	Settling time(sec.)
d	1	0.06	19.7	30.3	24.1	49.739	395
e	2	0.04	18.6	58.7	0	49.25	128
f	5	0.07	12.6	62.3	19.3	18.936	169

V. CONCLUSION

In this study, PID controller is designed for TITO system using Graphical method. Ideal decoupler is used to reduce interactions anddecoupled systems are approximated to FOPDT models. GPMT is used for robust analysis of each subsystem. The simulation results for coupled tank system show that the system is within the specified gain and phase margin limits. The disturbance rejection limits are identified using RDR. The gain values K_p and K_i are dependent on the preset value of K_d. The graphical method will give optimum gain values of the PID controller satisfying stability, gain and phase margin, disturbance rejection and specified time response characteristics.

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