Abstract: Here we consider the special type of labeling as lucky edge labeling for Regular graphs and corona graphs.

Keywords: Corona graph, Lucky edge labeling, Regular graph.

I. INTRODUCTION

A. Let G be a graph as follows,
(i) G is non-empty (ii) G is finite
(iii) edges and η(G) is maximum labels has been given in the graph.
If G is said to be regular graphs, each vertex have same neighbors.
Corona graph is obtained from two graphs, G of order P and H, taking one copy of G and P copies of H and joining by an edge the i{sup}th vertex of G to every vertex in the i{sup}th copy of H.

II. MAIN RESULTS.

A. Theorem 3.1

For every n ≥ 4 where n is an even number, there exists a 3-regular \( \left( \frac{2n}{3} \right) \) graph which holds Lucky edge labeling. [4]

Proof:

To prove that for 3-regular graph which admits Lucky edge labeling with lucky number is 4i + 3 where i = 1, 2, 3… respectively.
Define the vertex labeling, for all n ≥ 4 (where n is an even number)

\[ f(v_i) = i \text{ for all } i \]  \hspace{1cm} (1)
\[ f : E \rightarrow \left\{ 1, 2, 3, \ldots, \frac{n}{2} \right\} \]

\[ f(v_i, v_{i+j}) = 2j - 1, \text{ when } \left\{ \begin{array}{l} i = 1, 2, 3, \ldots, \frac{n}{2} \end{array} \right\} \]  \hspace{1cm} (2)

\[ f(v_{i+k}, v_{i+k}) = n - k + 1, \text{ when } k = 1, 2, 3, \ldots \]  \hspace{1cm} (3)

\[ f(v_{i+j}, v_{i+j+k}) = 6 + i + j \text{ when } n = 4, \text{ for } i = 0, j = 1, 2 \]  \hspace{1cm} (4)

Illustration: n = 4

Hence, a 3- regular (4,6) graph which admits Lucky edge labeling and its lucky number is 7.

Illustration: n = 10

Hence, a 3- regular (4,6) graph which admits Lucky edge labeling and its lucky number is 7.
B. **Theorem 3.2**

For every \( n \geq 5 \) there exists a 4-regular \((n, 2n)\) graph which admits Lucky edge labeling. [4].

**Proof:**

To prove that for 4-regular \((n, 2n)\) graph [4] its lucky number is \(2n - 1\).

Define the vertex labeling for all \( n \geq 5 \).

Let \( f: E \rightarrow \{1, 2, 3, \ldots, 2n\} \) such that

\[
\begin{align*}
    f(v_i, v_{i+1}) &= n + 1 \\
    f(v_i, v_{i+1}) &= 2i + 1, \text{where } i = 1, 2, 3, \ldots \\
    f(v_i, v_{i+1}) &= n + 2 \\
    f(v_i, v_{i+1}) &= 2i + 2, \text{where } i = 1, 2, 3, \ldots
\end{align*}
\]

(1) (2) (3) (4) (5)

Illustration: When \( n = 5 \)

Hence, a 4- regular \((5, 10)\) graph which admits Lucky edge labeling and its lucky number is 9.

Illustration: When \( n = 7 \)

Hence, a 4- regular \((7, 14)\) graph which admits Lucky edge labeling and its lucky number is 13.

C. **Theorem 3.3**

The corona graph \(P_n \circ K_2\) always contains a lucky edge labeling.

**Proof:**

In \( G = P_n \circ K_2 \), construction of vertex set, and edge as follows.

Let \( v(G) = v(P_n) \cup v(K_2) \) and

\[
\begin{align*}
    v(P_n) &= \{u_1, u_2, \ldots, u_{n-1}, u_n\} \\
    v(K_2) &= \{v_1, v_2, \ldots, v_{2n-1}, v_{2n}\}
\end{align*}
\]

\[
E(G) = \{u_i, u_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_{2i-1}, v_{2i} : 1 \leq i \leq n\} \cup \{u_i, v_i : 1 \leq i \leq n\}
\]

be the vertex set and edge set of \( G \) respectively. [6]

Now \( |v(G)| = 3n \) and \( |E(G)| = 4n - 1 \).

We will classify the edges of corona of \( P_n \circ K_2 \) in three cases.

i) Path edges

ii) \( K_2 \) edges

iii) Edges joining from \( K_2 \) with the path.

Vertex set defined as,

\[
\begin{align*}
    f(u_i) &= 1 \\
    f(u_{i+1}) &= 4, \text{when } i = 2, 3, 6, 7, \ldots \\
    f(v_i) &= 2, \text{when } i = 1, 2, 3, 4, \ldots \\
    f(v_i) &= 3, \text{when } i = 1, 2, 3, \ldots \\
    f(w_i) &= 4, \text{when } i = 1, 2, 3, 5, 6, 9, 10, \ldots \\
    f(w_i) &= 5, \text{when } i = 1, 2, 3, 4, 7, 8, \ldots \\
    f(w_i) &= 6, \text{when } i = 3, 4, 7, 8, \ldots
\end{align*}
\]

(6) (7) (8)

Illustration: \( P_3 \circ K_2 \)

The Lucky number of \( P_3 \circ K_2 \) is 5.

Illustration: \( P_4 \circ K_5 \)

The Lucky number of \( P_4 \circ K_5 \) is 8.
D. Theorem 3.4

The corona graph $P_n \odot C_d$ always admits a lucky edge labeling.

Proof:
In a graph $G = P_n \odot C_d$, construction of $V(G)$ and $E(G)$ as follows,
Let $\nu(G) = \nu(P_n) \cup (C_d^1) \cup \nu(C_d^2) \cup \ldots \cup \nu(C_d^n)$
where $\nu(P_n) = \{u_1, u_2, u_n\}$ and $\nu(C_d^i) = \{v_i, w_i, x_i, y_i; 1 \leq i \leq n\}$
and $C_d^i$ is the $i^{th}$ copy of $C_d$ be the vertex set and edge set of $G$ respectively. [6]

The corona of $P_n \odot C_d$ is given below.

$|\nu(G)| = 5n$ and $|E(G)| = 9n - 1$.

The vertex set can be defined as follows

\[ f(u_1) = 1, \text{when } i = 1, 2, 5, 6, \ldots \quad (1) \]
\[ f(u_2) = 6, \text{when } i = 3, 4, 7, 8, \ldots \quad (2) \]
\[ f(v_1) = 2, \text{when } i = 1, 2, 3, \ldots \quad (3) \]
\[ f(w_1) = 3, \text{when } i = 1, 2, 3, \ldots \quad (4) \]
\[ f(x_1) = 4, \text{when } i = 1, 2, 3, \ldots \quad (5) \]
\[ f(y_1) = 5, \text{when } i = 1, 2, 3, \ldots \quad (6) \]

Edge set can be defined as
\[ f(u_1, v_1) = 2, \text{when } i = 1, 2, 5, \ldots \quad (7) \]
\[ f(u_2, v_1) = 7, \text{when } i \text{ is an even } \quad (8) \]
\[ f(u_1, w_1) = 12, \text{when } i = 3, 7, \ldots \quad (9) \]
\[ f(v_1, w_1) = 5, \text{when } i = 1, 2, 3, \ldots \quad (10) \]
\[ f(x_1, y_1) = 6, \text{when } i = 1, 2, 5, \ldots \quad (11) \]
\[ f(u_1, x_1) = 11, \text{when } i = 3, 4, 7, \ldots \quad (12) \]
\[ f(v_1, x_1) = 5, \text{when } i = 1, 2, 5, \ldots \quad (13) \]
\[ f(u_1, y_1) = 10, \text{when } i = 3, 4, 7, \ldots \quad (14) \]
\[ f(x_1, y_1) = 9, \text{when } i = 1, 2, 3, \ldots \quad (15) \]

Illustration: $P_3 \odot C_4$

The Lucky number of $P_3 \odot C_4$ is $9$.

Illustration: $P_4 \odot C_4$

The Lucky number of $P_4 \odot C_4$ is $12$.

III. CONCLUSION

Here we establish the fact the Lucky edge labeling based on special type of graphs that is $P_n \odot C_d, P_n \odot K_3, 3$-regular and $4$-regular graphs. This can be extended for generalized Corona and Regular graphs.

REFERENCES


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