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Regular Graphs and Corona Graphs Based on Special Type of Labeling

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Abstract: ***Here we consider the special type of labeling as lucky edge labeling for Regular graphs and corona graphs.***

Keywords: ***Corona graph, Lucky edge labeling, Regular graph.***

I. INTRODUCTION

A Let G be a graph as follows,
(i) G is non-empty (ii) G is finite
(iii) edges and η(G) is maximum labels has been given in the graph.
If G is said to be regular graphs, each vertex have same neighbors.
Corona graph is obtained from two graphs, G of order P and H, taking one copy of G and P copies of H and joining by an edge the i^{th} vertex of G to every vertex in the i^{th} copy of H.

II. MAIN RESULTS.

A. *Theorem 3.1*

For every $n \geq 4$ where n is an even number, there exists a 3-regular $\left(n, \frac{3n}{2}\right)$ graph which holds Lucky edge labeling. [4]

Proof:

To prove that for 3-regular graph which admits Lucky edge labeling with lucky number is 4*i* + 3 where *i* = 1,2, 3… respectively.
Define the vertex labeling, for all n ≥ 4 (where *n* is an even number)

$$f(v_i) = i \; for \; all \; i \quad (1)$$

$$\text{Let } f: E \rightarrow \left\{1,2,3, \ldots \frac{3n}{2}\right\}$$

$$f(v_i, v_{i+1}) = 2j - 1, when \begin{cases} i = 1,2,3, \ldots \\ j = 2,3,4, \ldots \end{cases} respectively \quad (2)$$

$$f(v_n, v_1) = n + 1 \; when \; n = 4,6,8,.. \quad (3)$$

$$f(v_1, v_{n-k}) = n - k + 1, when \; k = 1,2,3, \ldots \quad (4)$$

$$f(v_{2+i}, v_{4+j}) = 6 + i + j when n = 4, for i = 0, j = 0 then edges of the form. \quad (5)$$

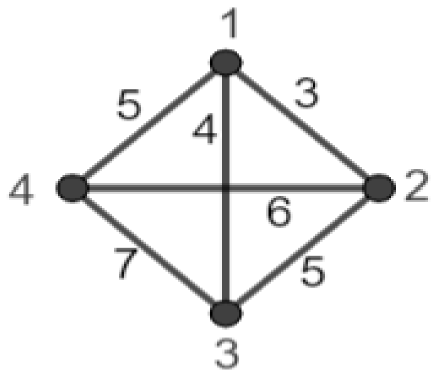
$$f(v_{2+i}, v_{4+j} = 6 + i + j \; when \; n = 6, for \; (i = 0, j = 1) and \; (i = 0, j = 1) and \; (i = 1, j = 2) \; then \; the \; edges \; of \; the \; form \quad (6)$$

$$f(v_{2+i}, v_{4+j}) = 6 + i + j \; when \; n = 8, for \; (i = 0, j = 2), (i = 1, j = 3) \; and \; (i = 2, j = 4) then \; edges \; of \; the \; form \quad (7)$$

The number of crossing edges, barring those crossing edges incident with v_1 are 1,2,3,4 respectively, that edges must be labeled as of the form
Similarly, n = 10, 12,14, …

n	i	j
4	0	0
6	(0,1)	(1,2)
8	(0,1,2)	(2,3,4)
10	(0,1,2,3)	(3,4,5,6)
12	(0,1,2,3,4)	(4,5,6,7)

Illustration: n = 4



Hence, a 3- regular (4,6) graph which admits Lucky edge labeling and its lucky number is 7.

Illustration: n = 10
Hence, a 3- regular (4,6) graph which admits Lucky edge labeling and its lucky number is 7.

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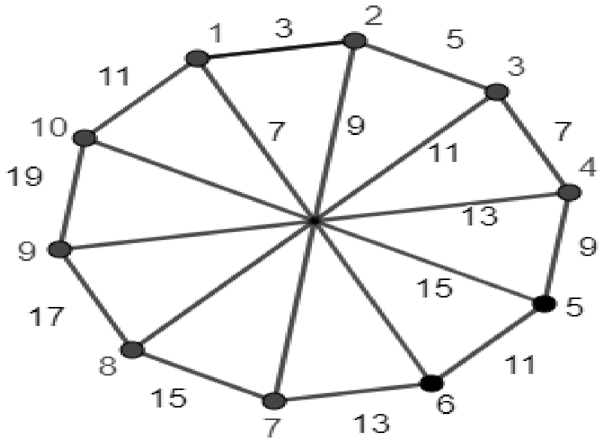


Hence, a 3- regular (10, 15) graph which admits Lucky edge labeling and its lucky number is 19.

B. *Theorem 3.2*

For every n $\geq$ 5 there exists a 4- regular (n, 2n) graph which admits Lucky edge labeling. [4].

Proof:

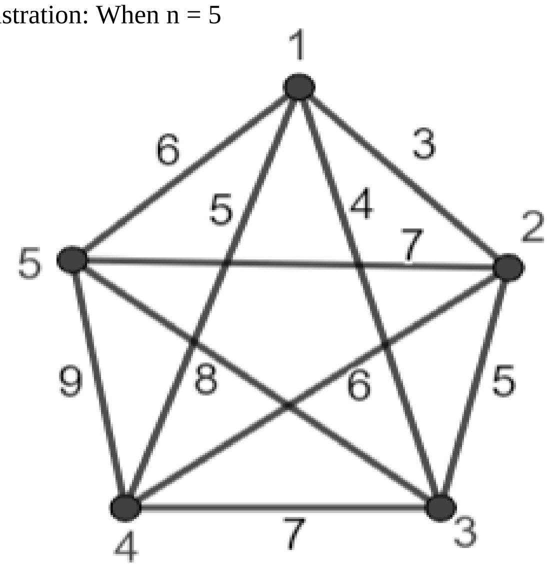
To prove that for 4-regular (n,2n) graph [4] its lucky number is 2n – 1.
Define the vertex labeling for all n $\geq$ 5.
$f(v_i) = i$ for all i
Let f: E $\rightarrow$ {1, 2, 3,…2n} such that

$$f(v_1, v_n) = n + 1 \quad (1)$$
$$f(v_i, v_{i+1}) = 2i + 1, where\ i = 1,2,3,\ldots \quad (2)$$

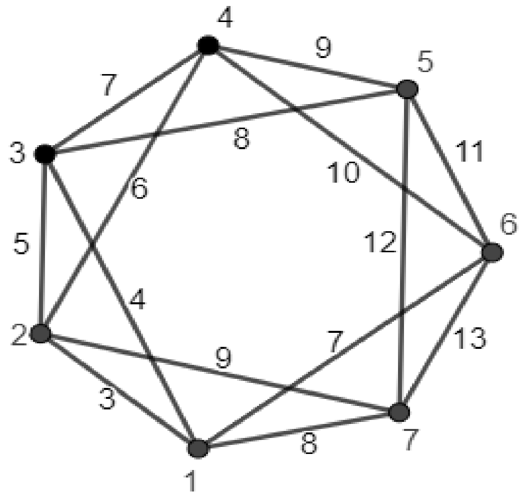
These all are the external edges; rest of the edges are crossing edges. It can be associated as,

$$f(v_1, v_{n-1}) = n \quad (3)$$
$$f(v_2, v_n) = n + 2 \quad (4)$$
$$f(v_i, v_{i+2}) = 2i + 2, where\ i = 1,2,3,\ldots \quad (5)$$

Illustration: When n = 5



Hence, a 4- regular (5, 10) graph which admits Lucky edge labeling and its lucky number is 9.
Illustration: When n = 7



Hence, a 4- regular (7, 14) graph which admits Lucky edge labeling and its lucky number is 13.

C. *Theorem 3.3*

The corona graph $P_n \odot K_2$ always contains a lucky edge labeling.

Proof:

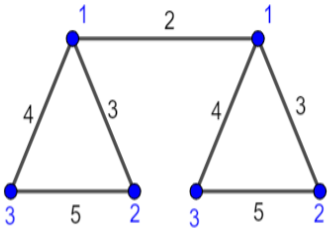
In $G = P_n \odot K_2$, construction of vertex set, and edge as follows.
Let $v(G) = v(P_n) \cup v(nk_2)$
Where $v(P_n) = \{u, u_2, \ldots u_{n-1}, u_n\}$ and
$v(nK_2) = \{v_1, v_2, \ldots v_{2n-1}, v_{2n}\}$.
$E(G) = \{u_i, u_{i+1};\ 1 \leq i \leq n-1\} \cup \{v_{2i-1}v;\ 1 \leq i \leq n\} \cup \{u_i v_i;\ 1 \leq i \leq n\} \cup \{u_i v_{2i};\ 1 \leq i \leq n\}$
be the vertex set and edge set of G respectively. [6]
Now $|v(G)| = 3n$ and $|E(G)| = 4n - 1$.
We will classify the edges of corona of $P_n \odot K_2$ in three cases.
i)Path edges
ii)K_2 edges
iii)Edges joining from K_2 with the path.
Vertex set defined as,

$$f(u_i) = 1 \quad (1)$$
$$f(u_{i+1}) = 4, when\ i = 2,3,6,7,\ldots \quad (2)$$
$$f(v_i) = 2, when\ i = 1,2,3,4,\ldots \quad (3)$$
$$f(w_i) = 3, when\ i = 1,2,3,4,\ldots \quad (4)$$

Edge Set defined as,

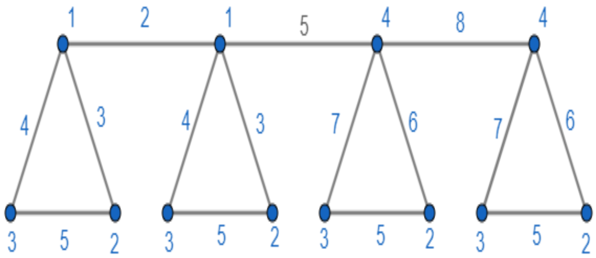
$$f(u_i u_{i+1}) = 2, when\ i = 1,5,9,\ldots \quad (1)$$
$$f(u_i u_{i+1}) = 5, when\ i\ is\ an\ even \quad (2)$$
$$f(u_i u_{i+1}) = 8, when\ i = 3,7,11,\ldots \quad (3)$$
$$f(u_i, v_i) = 3, when\ i = 1,2,5,6,9,10,\ldots \quad (4)$$
$$f(w_i u_i) = 4, when\ i = 1,2,5,6,9,10,\ldots \quad (5)$$
$$f(u_i, v_i) = 6, when\ i = 3,4,7,8,\ldots \quad (6)$$
$$f(v_i, w_i) = 5, when\ i = 3,4,7,8,\ldots \quad (7)$$
$$f(w_i u_i) = 7, when\ i = 3,4,7,8,\ldots \quad (8)$$

Illustration: $P_2 \odot K_2$



The Lucky number of $P_2 \odot K_2$ is 5.

Illustration: $P_4 \odot K_2$



The Lucky number of $P_4 \odot K_2$ is 8.

D. *Theorem 3.4*

The corona graph $P_n \odot C_4$ always admits a lucky edge labeling.

Proof:

In a graph $G = P_n \odot C_4$, construction of V(G) and E(G) as follows,

Let $v(G) = v(P_n) \cup (C_4{}^1) \cup v(C_4{}^2) \cup \ldots \cup v(C_4{}^n)$

$where\ v(P_n) = \{u_1, u_{2,\ldots} u_n\}\ and\ v(C_4{}^i) = \{v_i, w_i, x_i, y_i : 1 \le i \le n\}$

and $C_4{}^i$ is the i^{th} copy of C_4, be the vertex set and edge set of G respectively. [6]

The corona of $P_4 \odot C_4$ is given below.

$|v(G)| = 5n\ and\ |E(G)| = 9n - 1.$

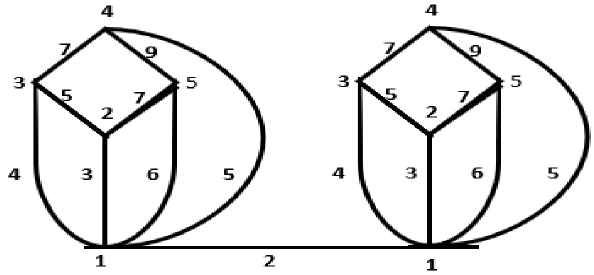
Vertex set can be defined as follows

$$f(u_i) = 1, when\ i = 1,2,5,6,\ldots \quad (1)$$
$$f(u_i) = 6, when\ i = 3,4,7,8\ldots \quad (2)$$
$$f(v_i) = 2, when\ i = 1,2,3,4,\ldots \quad (3)$$
$$f(w_i) = 3, when\ i = 1,2,3,4,\ldots \quad (4)$$
$$f(x) = 4, when\ i = 1,2,3,4,\ldots \quad (5)$$
$$f(y_i) = 5, when\ i = 1,2,3,4,\ldots \quad (6)$$

Edge Set can be defined as

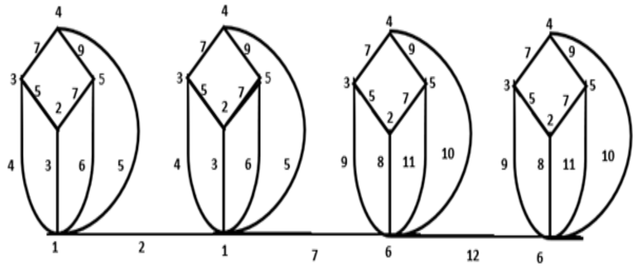
$$f(u_i u_{i+1}) = 2, when\ i = 1,5,9,\ldots \quad (1)$$
$$f(u_i u_{i+1}) = 7, when\ i\ is\ an\ even \quad (2)$$
$$f(u_i u_{i+1}) = 12, when\ i = 3,7,11,\ldots \quad (3)$$
$$f(v_i, w_i) = 5, when\ i = 1,2,3,\ldots \quad (4)$$
$$f(v_i y_i) = 7, when\ i = 1,2,3,\ldots \quad (5)$$
$$f(u_i y_i) = 6, when\ i = 1,2,5,6,\ldots \quad (6)$$
$$f(u_i y_i) = 11, when\ i = 3,4,7,8,\ldots \quad (7)$$
$$f(u_i x_i) = 5, when\ i = 1,2,5,6,\ldots \quad (8)$$
$$f(u_i x_i) = 10, when\ i = 3,4,7,8,\ldots \quad (9)$$
$$f(x_i y_i) = 9, when\ i = 1,2,3,4,\ldots \quad (10)$$
$$f(w_i x_i) = 7, when\ i = 1,2,3,4,\ldots \quad (11)$$
$$f(u_i w_i) = 4, when\ i = 1,2,5,6\ldots \quad (12)$$
$$f(u_i w_i) = 9, when\ i = 3,4,7,8,\ldots \quad (13)$$
$$f(u_i v_i) = 3, when\ i = 1,2,5,6\ldots \quad (14)$$
$$f(u_i v_i) = 8, when\ i = 3,4,7,8\ldots \quad (15)$$

Illustration: $P_2 \odot C_4$



The Lucky number of $P_2 \odot C_4$ is 9.

Illustration: $P_4 \odot C_4$



The Lucky number of $P_4 \odot C_4$ is 12.

III. CONCLUSION

Here we establish the fact the Lucky edge labeling based on special type of graphs that is $P_n \odot C_4$, $P_n \odot K_2$, 3-regular and 4-regular graphs. This can be extended for generalized Corona and Regular graphs.

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