On Rainbow Connection Number of Some Graphs

Shalini Rajendra Babu, N. Ramya

Abstract: The Rainbow connection number for the following graphs, two copies of Fan graph F_n by a path P_n, Arrow graph $A_n{}^2$ and $K_{1,m}$ Θ $K_{1,n}$, Jellyfish graph and Cycle Cactus graph have been described in this paper

Keywords: Rainbow Coloring, Fan Graph, Arrow Graph, Corona $K_{1,m}$ Θ $K_{1,n}$, Jellyfish graph, Cycle Cactus graph.

I. INTRODUCTION

Finite, undirected and simple graphs are considered. An edge colored graph G is rainbow edge connected, if any two vertices are connected by a path whose edges have distinct colors. Thus, the following natural parameters was defined by charted et al [1].

Let the rainbow connection of a connection graph G denoted by $r_c(G)$, be the smallest number of colors, that are needed in order to make rainbow edge connected. Let G be a nontrivial connected graph on which an edge coloring C: E(G) $\rightarrow \{1,2,\ldots n\}$, n∈N, is defined where adjacent edges may be colored the same. Rainbow connection has an interesting application for the secure transfer of classified information between agencies, while the information needs to be protected since it relates to national security, there must also be procedures that permit access between appropriate parties. [3]

Then we consider the rainbow coloring of the following graphs,

(i) Two copies of Fan graph by a Path P_n

(ii) Arrow graph $A_n{}^2$

(iii) $K_{1,m}$ Θ $K_{1,n}$

(iv) Jelly Fish graph

(v) Cycle-Cactus graph

(vi) 2-Tuple graph

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II. DEFINITION

A. *Fan graph*

A Fan graph $F_{m,n}$ is defined as the graph $\overline{k_m}$ + Pn, where $\overline{k_m}$ is the empty graph on *m* nodes and P_n is the path graph on n nodes, when *m* = 1, corresponds the usual Fan graph.

B. *Arrow graph*

An arrow graph $A_n{}^2$ with width 2 and length n is obtained by joining a vertex V with superior vertices of $p_2 \times p_n$ by 2 new edges from one end. [7]

C. *Corona* $K_{1,m}$ *Θ* $K_{1,n}$

$K_{1,m}$ Θ $K_{1,n}$ is a tree obtained by adding *n* pendant vertices of $K_{1,m}$.

D. *Jelly Fish graph*

The jelly fish graph $J(m,n)$ is obtained from a 4-cycle V_1, V_2, V_3, V_4 by joining V_1 and V_3 with an edge and appending m pendant edges to V_2 and n pendant edges to V_4.[6]

E. *Cactus*

A cactus is a connected graph in which any two simple cycles have at most even vertex in common

F. *2-Tuple graph*

Let $G = (V, E)$ be a simple graph and $G^0 = (V^0, E^0)$ be another copy of graph G. Join each vertex v of G to the corresponding vertex V^0 of G^0 by an edge. The new graph thus obtained we call 2-tuple graph of G. We denote 2-tuple graph of $G = (p,q)$ then $|V(T^2(G))| = 2p \; and \; |T^2(G)| = 2p + q$.[9]

III. MAIN RESULTS

Theorem 1:

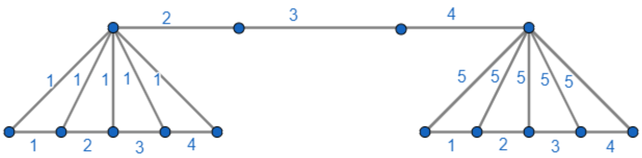
The graph G is obtained by joining two copies of fan graph F_n by a path of arbitrary, whose Rainbow coloring is $n-1$, "n" denotes number of vertices in the Fan graph.

Proof:

Let G be the graph obtained by joining two copies of Fan graph F_n by a path P_k of length $k-1$. [2]. Let us denote the successive vertices of first copy of Fan graph by $u_1, u_2, u_3 \ldots u_{n+1}$ and the successive vertices of second copy of fan graph by $w_1, w_2, w_3 \ldots w_{n+1}$. Let $v_1, v_2, u_3 \ldots v_k$ be the path P_k and $v_k = w_i$. For $n = 3, F_3$ is cycle C_3, its Rainbow coloring is discussed in [5], that the graph obtained by joining of P_n and K_2 admits a rainbow coloring with $2n-1$ colors. Here we consider the case for $n > 3$, we define a function $f: E(G) \to \{1,2,3,\ldots n-1\}$ as follows,
Coloring must be given,

$$f(u_i, u_{i+1}) = i, 1 \le i \le n-2 \quad (1)$$
$$f(v_i, u_i) = 1, for\ all\ i \quad (2)$$
$$f(w, w_{i+1}) = i, 1 \le i \le n-2 \quad (3)$$
$$f(v_i, v_{i+1}) = i+1, 1 \le i \le n-2 \quad (4)$$
$$f(v_i, w) = n-1, for\ all\ i \quad (5)$$

Illustrations
Rainbow coloring of the graph obtained by joining two copies of F_6 by a path P_4 shown in figure



Thus, the rainbow coloring of the above graph is 5.

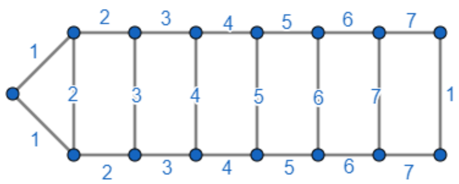
Theorem 2:

An Arrow graph $A_n{}^2$ which admits a Rainbow coloring, and whose Rainbow connection number is n.

Proof:

Let $A_n{}^2$ be an Arrow graph combined by a vertex v_0 with superior vertices of $p_2 \times p_n$ by 2 new edges.
Let us denote v_0 be the starting vertex.$v_1, v_2, \ldots v_n$ be the upper vertices of Arrow graph, which has to be connected with v_0. Similarly, $w_1, w_2, \ldots w_n$ be the lower vertices of an Arrow graph, which has to be connected with v_0. Here we consider the case for $n \ge 3$, we define the function $f: E(G) \to \{1,2,3,\ldots n\}$ as follows, coloring has to be given
Rainbow coloring of $A_7{}^2$

$$f(v_0, v_i) = 1 \quad (2)$$
$$f(v_i, v_{i+1}) = i+1 for\ i \le 1 \le n \quad (2)$$
$$f(v_0, w_1) = 1 \quad (3)$$
$$f(w_i, w_{i+1}) = i+1, for\ 1 \le i \le n \quad (4)$$
$$f(w_n, v_n) = 1 \quad (5)$$



Thus, the rainbow coloring of the above graph is 7.

Theorem 3:

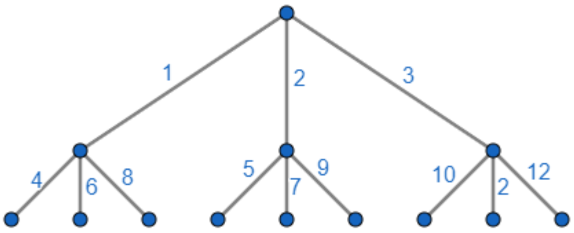
The graph $K_{1,m} \Theta K_{1,n}$ for all m, n which admits Rainbow coloring, whose rainbow connection number n. Where 'n' represents number of edges.

Proof:

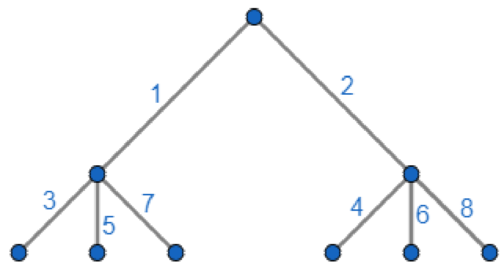
Let v_0 be the root of the tree. Let.$v_1, v_2, \ldots v_m$ be the children of the root. Each subtree $v_i, 1 \le i \le m$, will have 'n' number of vertices which have $v_{i1}, v_{i2}, \ldots v_{in}$
leaves [4]. The vertices that acts as a leaf of the graph $K_{1,m} \Theta K_{1,n}$ are colored as follows

Illustration

Case (i) when $m = n$



Rainbow coloring of $K_{1,3} \Theta K_{1,3}$ is 12.
Case (ii) when $m \neq n$



$K_{1,2} \Theta K_{1,3}$

Thus, the rainbow coloring of the above graph is 8.

Theorem: 4

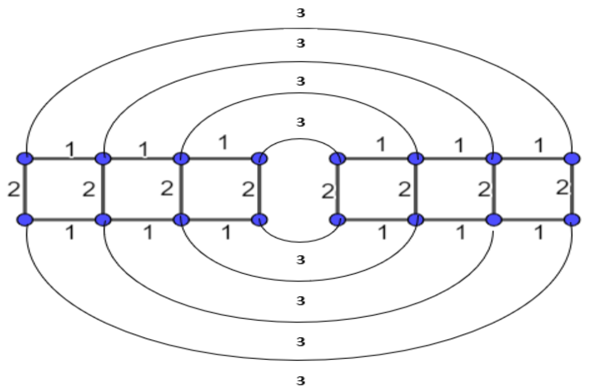
Given the graph G as $T^2(P_n \times P_2)$ $n \ge 2$ then rainbow connection number $rc(G)$ is exactly 3.

Proof:

Let $\{xi, yi, xi'yi'; 1 \le i \le n\}$ be the vertices $\{ai, ai', bi, bi'; 1 \le i \le n-1, ci, ci'; 1 \le i \le n, di; i \le i \le 2n\}$
be the edges.
Construct the characteristics of G to integers as follows
Define

$$f^*E: \to \{1,2,3\} \quad (A)$$
$$f^*(ai) = 1\ for\ all\ i \quad (1)$$
$$f^*(ai') = 1\ for\ all\ i \quad (2)$$
$$f^*(bi) = 1\ for\ all\ i \quad (3)$$
$$f^*(bi') = 1\ for\ all\ i \quad (4)$$
$$f^*(ci) = 2\ for\ all\ i \quad (5)$$
$$f^*(di) = 3\ for\ all\ i \quad (6)$$

Illustration: $T^2(P_4 \times P_2)$



Thus the Rainbow connection number $rc(G)$ is exactly 3 as seen in (A).
It clearly shows the $rc(G)$ is the minimum number of color needed to edge coloring at least one of paths in G.

Theorem:5

For every $m \geq 1, n \geq 1$ there exists a Jelly fish graph which admits a rainbow coloring with rainbow connection number is $m + n + 2$.

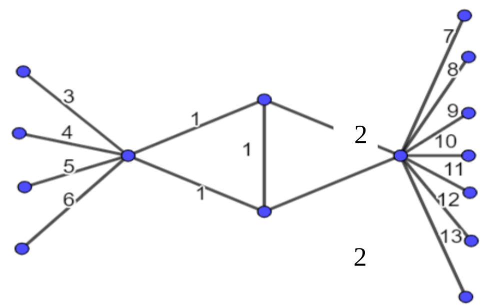
Proof:

Let G be the graph with $m + n + 4$ vertices and $m + n + 5$ edges where m represents number of pendent edges in left hand side, n represents its number of pendent edges in the right hand side and $E(G) = E_1 \cup E_2$ where
$E_1 = \{xu, uy, yv, vx, xy\}$
$E_2 = \{uui, vvj, 1 \leq i \leq m, 1 \leq j \leq m\}$
ui's are from left pandant edges
vi's are from right pandant edges
Labeling has to be defined as,
$f^*: E(G) \rightarrow \{1,2, \ldots m + n + 2\}$

$$f^*(xy) = 1 \quad (1)$$
$$f^*(xu) = 1 \quad (2)$$
$$f^*(yu) = 1 \quad (3)$$
$$f^*(xv) = 2 \quad (4)$$
$$f^*(vy) = 2 \quad (5)$$
$$f^*(uui) = 2 + i, for\ i = 1,2, \ldots m \quad (6)$$
$$f^*(vvj) = 3 + m + j, for\ j = 0,1,2, \ldots n - 1 \quad (7)$$

Claim: f^* is rainbow coloring
Proof: From (i) and (vii) equations found above establishes the rainbow connection number as
$m + n + 2$

Illustration: $J(4,7) - Jelly\ fish$



In the above graph $J(4,7)$ graph has Rainbow connection number as 13.

Theorem:6

A cycle-cactus $C_k^{(n)}$, consist of n copies of cycle $C_k, k \geq 3$, concatenated at exactly one vertex, which holds Rainbow connection number 'k' where 'k' is the number of vertices of the cycle.

Proof:

Let $G_1, G_2, \ldots G_n$ be the copies of cycles C_k, all concatenated at exactly one vertex namely x_0.
Let $x_0, x_{11}, x_{12}, x_{13}, \ldots x_{1k}$ be the vertices of G_1 . $x_0, x_{21}, x_{22}, x_{23}, \ldots x_{2k}$ be the vertices of G_2, finally let $x_0, x_{n1}, x_{n2}, x_{n3}, \ldots x_{nk}$ be the vertices of G_n.
Let $f^*: E(G) \rightarrow \{1,2,3, \ldots k\}$, then the coloring has been given as follows

$$f(x_0, x_{11}) = 1 \quad (1)$$
$$f(x_0, x_{21}) = 1$$
.
.
.
.
.
$f(x_0, x_{n1}) = 1$, for all n≥ 1
$$f(x_{11}, x_{12}) = 2 \quad (2)$$
$$f(x_{21}, x_{22}) = 2$$
.
.
.
$f(x_{n1}, x_{n2}) = 2$, for all n≥ 1
$$f(x_{12}, x_{13}) = 3 \quad (3)$$
$$f(x_{22}, x_{23}) = 3$$
.
.
.
$f(x_{n2}, x_{n3}) = 3$, for all n≥ 1
$$f(x_{1,k-1}, x_0) = k \quad (4)$$
$$f(x_{2,k-1}, x_0) = k$$
.
.
$f(x_{n,k-1}, x_0) = k$, for all n≥ 1

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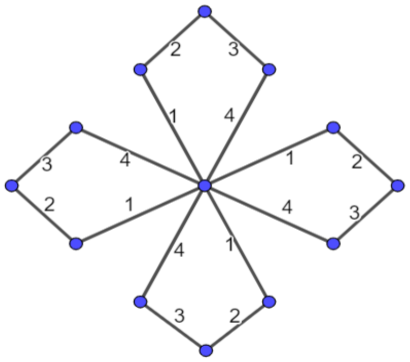
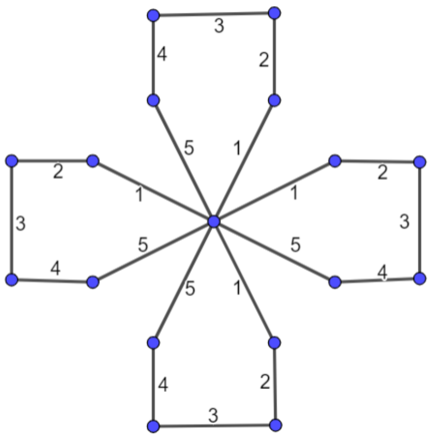
Illustration: Cycle-Cactus $C_4^{(4)}$



In the above graph the rainbow connection number is 4.

Cycle-Cactus $C_5^{(4)}$



In the above graph the rainbow connection number is 5.

Findings:

S.no	Graph	Rainbow Connection number
1	Two copies of Fan graph by a path	n-1 n-number of vertices
2	Arrow graph A_n^2	n n-number of vertices
3	Corona graph $K_{1,m} \Theta K_{1,n}$	n n-number of vertices
4	Jelly Fish graph	m + n + 2 m - Pendent edges in LHS n - Pendent edges in RHS
5	Cycle-Cactus graph	k k – number of Vertices
6	2-Tuple graph	3

IV. CONCLUSION:

It is of interest to study the connection number for various classes, after than what has been found in the literature.

REFERENCES

1. G. Chartrand, G.L.Johns, K.A. Mc Keon, P.Zhang. " Rainbow connection in graphs" Math Bohem 133 (2008), 85-98.
2. G.V.Ghodasasa, J.P Jena " Prime Cordial Labeling of some Special Graph families" Int. Journal of Mathematics and soft Computing, Vol 4 (2), 2014.
3. R.Prabha and Indra Rajasingh " Rainbow Coloring of Crown Graphs", J.Comp & Math.Sci.Vol.3(3), 390-394(2012).
4. J.Baskar Babujee and L.Shobana, " Prime and Prime Coloring labeling for Some Special Graphs", Int. Journal of Contemp .Math Sciences, Vol.5,2010, no.47,2347-2356.
5. N.Ramya, K.Rangarajan and R.Sattanathan, " On Rainbow Coloring of Some Classes of Graphs", Int. Journal of Computer Applications, Vol.46.no 18 May 2012.
6. "Pair sum labeling of some special graphs" K.Manimekalai, K. Thirusangu International Journal of Computer Applications Vol.69.May 2013
7. "Colorful Labeling of arrow graphs and double arrow graphs".V.J.Kaneria, M.M. Jariya and H.M.Makadia Malaya Journal Of Mathematik 3(4).2015.
8. Sharon Philomena.V and K.Thirusangu " Square and Cube difference Labeling of Cycle Cactus, Special Tree and a New Key Graphs" Annals of Pure and Applied Mathematics Vol.8.2014
9. "Some Graph Operations of Even Vertex Odd Mean Labeling Graphs" M.Kannan, R.Vikrama Prasad and R.Gobi International Journal of Applied Engg. Research Vol.12.2017.

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