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Intuitionistic Fuzzy Pseudo-Boolean Implicative Filters of Lattice Pseudo-Wajsberg Algebras

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Abstract: *In this paper, we introduce the notion of an intuitionistic fuzzy pseudo-Boolean implicative filter of lattice pseudo-Wajsberg algebra (LPWA) and to investigate some properties with illustrations.*

Keywords: *Pseudo-Boolean implicative filter; Fuzzy pseudo-Boolean implicative filter; Intuitionistic Fuzzy pseudo-Boolean implicative filter; Lattice pseudo-Wajsberg algebra(LPWA)*

I. INTRODUCTION

Pseudo-Wajsberg algebras were introduced by Ceterchi Rodica [2]. Pseudo-Wajsberg algebras are generalizations of Wajsberg algebras. Boolean algebra is the branch of algebra in which the values of the variables are the truth values true and false. Recently, the authors introduce the definition of pseudo-Boolean and fuzzy pseudo-Boolean implicative filter of lattice pseudo-Wajsberg algebra and obtain some related properties. The aim of this paper is to introduce the definition of an intuitionistic Fuzzy pseudo-Boolean implicative filter of LPWA and obtain some properties.

II. PRELIMINARIES

In this section, we recall some basic definitions and their properties which are helpful to develop the main results.

Definition 2.1[2]. An algebra $(\mathcal{A}, \rightarrow, \rightsquigarrow, ^-, ^\sim, 1)$ is called a LPWA if it satisfies the following axioms for all $x, y \in \mathcal{A}$,

(i) A partial ordering "$\leq$" on a LPWA $\mathcal{A}$, such that $x \leq y$ if and only if $x \rightarrow y = 1 \Leftrightarrow x \rightsquigarrow y = 1$.

(ii) $x \vee y = (x \rightarrow y) \rightsquigarrow y = (y \rightarrow x) \rightsquigarrow x$
$= (x \rightsquigarrow y) \rightarrow y = (y \rightsquigarrow x) \rightarrow x$

(iii) $x \wedge y = (x \rightsquigarrow (x \rightarrow y)^\sim)^- = \left((x \rightarrow y) \rightarrow x^-\right)^\sim$
$= (y \rightsquigarrow (y \rightarrow x)^\sim)^- = \left((y \rightarrow x) \rightarrow y^-\right)^\sim$
$= (y \rightarrow (y \rightsquigarrow x)^-)^\sim = \left((y \rightsquigarrow x) \rightsquigarrow y^\sim\right)^-$
$= (x \rightarrow (x \rightsquigarrow y)^-)^\sim = \left((x \rightsquigarrow y) \rightsquigarrow x^\sim\right)^-$.

Definition 2.2[2]. An algebra $(\mathcal{A}, \rightarrow, \rightsquigarrow, ^-, ^\sim, 1)$ with a binary operations "$\rightarrow$", "$\rightsquigarrow$" and quasi complements "$^-$", "$^\sim$" is called a pseudo-Wajsberg algebra if it satisfies the following axioms for all $x, y, z \in \mathcal{A}$,

(i) (a) $1 \rightarrow x = x$
(b) $1 \rightsquigarrow x = x$

(ii) $(x \rightsquigarrow y) \rightarrow y = (y \rightsquigarrow x) \rightarrow x$
$= (y \rightarrow x) \rightsquigarrow x = (x \rightarrow y) \rightsquigarrow y$

(iii) (a) $(x \rightarrow y) \rightarrow \left((y \rightarrow z) \rightsquigarrow (x \rightarrow z)\right) = 1$
(b) $(x \rightsquigarrow y) \rightsquigarrow \left((y \rightsquigarrow z) \rightarrow (x \rightsquigarrow z)\right) = 1$

(iv) $1^- = 1^\sim = 0$

(v) (a) $(x^- \rightsquigarrow y^-) \rightarrow (y \rightarrow x) = 1$
(b) $(x^\sim \rightarrow y^\sim) \rightsquigarrow (y \rightsquigarrow x) = 1$

(vi) $(x \rightarrow y^-)^\sim = (y \rightsquigarrow x^\sim)^-$

Proposition 2.3[2]. A LPWA $(\mathcal{A}, \rightarrow, \rightsquigarrow, ^-, ^\sim, 1)$ satisfies the following axioms for all $x, y \in \mathcal{A}$,

(i) $x \rightarrow x = 1, x \rightsquigarrow x = 1$
(ii) $x \rightarrow (y \vee z) = (x \rightarrow y) \vee (x \rightarrow z)$
(iii) $x \rightsquigarrow (y \vee z) = (x \rightsquigarrow y) \vee (x \rightsquigarrow z)$
(iv) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (x \rightarrow z)$
(v) $(x \vee y) \rightsquigarrow z = (x \rightsquigarrow z) \wedge (x \rightsquigarrow z)$
(vi) $x^- \rightarrow x = x;\ x^\sim \rightsquigarrow x = x$
(vii) $x \rightarrow x^- = x^-;\ x \rightsquigarrow x^\sim = x^\sim$
(viii) $(x^- \rightsquigarrow 0) \rightarrow x = 1;\ (x^\sim \rightarrow 0) \rightsquigarrow x = 1$
(ix) $x \rightarrow 0 = x^-;\ x \rightsquigarrow 0 = x^\sim$
(x) $0 \rightarrow x = 1;\ 0 \rightsquigarrow x = 1$
(x) $(x \rightarrow x^-)^\sim = x;\ (x \rightsquigarrow x^\sim)^- = x$
(xi) $(x^\sim)^- = (x^-)^\sim = x$

Definition 2.4[4]. Let $\mathcal{A}$ be LPWA. An intuitionistic fuzzy set $T = (\mu_T, \gamma_t)$ of $\mathcal{A}$ is called an intuitionistic fuzzy implicative filter of $\mathcal{A}$ if it satisfies the following axioms for all $x, y \in \mathcal{A}$,

(i) $\mu_T(1) \geq \mu_T(x)\ ;\ \gamma_T(1) \leq \gamma_T(x)$
(ii) $\mu_T(y) \geq \min\{\mu_T(x \rightarrow y), \mu_T(x)\}$;
$\gamma_T(y) \leq \max\{\gamma_T(x \rightarrow y), \gamma_T(x)\}$
(iii) $\mu_T(y) \geq \min\{\mu_T(x \rightsquigarrow y), \mu_T(x)\}$;
$\gamma_T(y) \leq \max\{\gamma_T(x \rightsquigarrow y), \gamma_T(x)\}$

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Definition 2.5[6]. Let $\mathcal{A}$ be LPWA. An intuitionistic fuzzy implicative filter $T = (\mu_T, \gamma_T)$ of $\mathcal{A}$ is called an intuitionistic fuzzy prime implicative filter of $\mathcal{A}$ if it satisfies the following axioms for all $x, y \in \mathcal{A}$,

(i) $\mu_T(x \vee y) \geq \min\{\mu_T(x), \mu_T(y)\}$

(ii) $\gamma_T(x \vee y) \leq \max\{\gamma_T(x), \gamma_T(y)\}$

III. MAIN RESULTS

3.1. Intuitionistic Fuzzy Pseudo-Boolean implicative filter of Lattice Pseudo-Wajsberg Algebra

In this section, we define Intuitionistic fuzzy pseudo-Boolean implicative filter of LPWA, and obtain some results with illustrations.

Definition 3.1. Let $\mathcal{A}$ be LPWA. An intuitionistic fuzzy set $T = (\mu_T, \gamma_T)$ of $\mathcal{A}$ is called an intuitionistic fuzzy Pseudo-Boolean implicative filter of $\mathcal{A}$ if it satisfies the following axioms for all $x \in \mathcal{A}$,

(i) $\mu_T(x \vee x^-) = \mu_T(1)$; $\gamma_T(x \vee x^-) = \gamma_T(1)$

(ii) $\mu_T(x \vee x^\sim) = \mu_T(1)$; $\gamma_T(x \vee x^\sim) = \gamma_T(1)$

Example 3.2. Consider a set $\mathcal{A} = \{0, l, m, 1\}$. Define a partial ordering "$\leq$" on A, such that $0 \leq l \leq m \leq 1$ and the binary operations $" \to "$, $" \rightsquigarrow "$ and quasi complements $" - "$, $" \sim "$ given by the following tables 3.1., 3.2., 3.3., and 3.4.

x	x^-
0	1
l	m
m	l
1	0

Table:3.1. Complement

$\to$	0	l	m	1
0	1	1	1	1
l	m	1	1	1
m	l	m	1	1
1	0	l	m	1

Table:3.2. Implication

x	$x^\sim$
0	1
l	m
m	l
1	0

Table:3.3. Complement

$\rightsquigarrow$	0	l	m	1
0	1	1	1	1
l	m	1	1	1
m	l	l	1	1
1	0	l	m	1

Table:3.4. Implication

Consider an intuitionistic fuzzy subset $T = (\mu_T, \gamma_T)$ on $\mathcal{A}$

$$\mu_T(x) = \begin{cases} 0.7 & if \;\; x \in \{0,1\} \quad \text{for all } x \in \mathcal{A} \\ 0.3 & otherwise \quad \text{for all } x \in \mathcal{A} \end{cases}$$

$$\gamma_T(x) = \begin{cases} 0.1 & if \;\; x \in \{0,1\} \quad \text{for all } x \in \mathcal{A} \\ 0.6 & otherwise \quad \text{for all } x \in \mathcal{A} \end{cases}$$

Then T is an intuitionistic fuzzy pseudo-Boolean implicative filter of $\mathcal{A}$.

In the same Example 3.2, let us consider an intuitionistic fuzzy subset $T = (\mu_T, \gamma_T)$ on $\mathcal{A}$ as,

$$\mu_T(x) = \begin{cases} 0.51 & if \;\; x \in \{0, l\} \quad \text{for all } x \in \mathcal{A} \\ 0.13 & otherwise \quad \text{for all } x \in \mathcal{A} \end{cases}$$

$$\gamma_T(x) = \begin{cases} 0.41 & if \;\; x \in \{0, l\} \quad \text{for all } x \in \mathcal{A} \\ 0.83 & otherwise \quad \text{for all } x \in \mathcal{A} \end{cases}$$

Then T is not an intuitionistic fuzzy pseudo-Boolean implicative filter of $\mathcal{A}$.

Since

$$\mu_T(l \vee m) = \mu_T((l \to m) \rightsquigarrow m) = \mu_T((m \to l) \rightsquigarrow l)$$
$$= \mu_T((l \rightsquigarrow m) \to m) = \mu_T((m \rightsquigarrow l) \to l)$$
$$\mu_T(l \vee m) = \mu_T(l) = \mu_T(1) = \mu_T(m) = \mu_T(1)$$
$$\mu_T(l \vee m) \neq 0.5 = 0.2 = 0.2 = 0.2$$
$$\gamma_T(l \vee m) = \gamma_T((l \to m) \rightsquigarrow m) = \gamma_T((m \to l) \rightsquigarrow l)$$
$$= \gamma_T((l \rightsquigarrow m) \to m) = \gamma_T((m \rightsquigarrow l) \to l)$$
$$\gamma_T(l) = \gamma_T(1) = \gamma_T(m) = \gamma_T(1)$$
$$\gamma_T(l \vee m) = 0.4 \neq 0.8 = 0.8 = 0.8$$

Proposition 3.3. Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy implicative filter of LPWA $\mathcal{A}$, then any intuitionistic fuzzy implicative filter of $\mathcal{A}$ is an intuitionistic fuzzy Pseudo-Boolean implicative filter of $\mathcal{A}$.

Proof. Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy implicative filter of LPWA $\mathcal{A}$.

We have $\mu_T(x \vee x^-) \geq \min\{\mu_T(x \to (x \vee x^-)), \mu_T(x)\}$

[From (ii) of definition 2.4]

$= \min\{\mu_T((x \to x) \vee (x \to x^-)), \mu_T(x)\}$

[From (ii) of proposition 2.3]

$= \min\{\mu_T(1 \vee (x \to x^-)), \mu_T(x)\}$

[From (i) of proposition 2.3]

$= \min\{\mu_T(x \to x^-), \mu_T(x)\} \leq \mu_T(1)$

[From (i) of definition 2.4]

Thus $\mu_T(x \vee x^-) = \mu_T(1)$ for all $x \in \mathcal{A}$ and also

$\mu_T(x \vee x^\sim) \geq \min\{\mu_T(x \rightsquigarrow (x \vee x^\sim)), \mu_T(x)\}$

[From (iii) of definition 2.4]

$= \min\{\mu_T((x \rightsquigarrow x) \vee (x \rightsquigarrow x^\sim)), \mu_T(x)\}$

[From (iii) of proposition 2.3]

$= \min\{\mu_T(1 \vee (x \rightsquigarrow x^\sim)), \mu_T(x)\}$

[From (i) of proposition 2.3]

$= \min\{\mu_T(x \rightsquigarrow x^\sim), \mu_T(x)\} \leq \mu_T(1)$

[From (i) of definition 2.4]

Therefore, $\mu_T(x \vee x^\sim) = \mu_T(1)$ for all $x \in \mathcal{A}$

Similarly $\gamma_T(x \vee x^-) \leq \max\{\gamma_T(x \to (x \vee x^-)), \gamma_T(x)\}$

[From (ii) of definition 2.4]

$= \max\{\gamma_T((x \to x) \vee (x \to x^-)), \gamma_T(x)\}$

[From (ii) of proposition 2.3]

$= \max\{\gamma_T(1 \vee (x \to x^-)), \gamma_T(x)\}$

[From (i) of proposition 2.3]

$\gamma_T(x \vee x^-) = \max\{\gamma_T(x \to x^-), \gamma_T(x)\} \geq \gamma_T(1)$

for all $x \in \mathcal{A}$ [From (i) of definition 2.4]

Therefore, $\gamma_T(x \vee x^-) = \gamma_T(1)$ and also

$\gamma_T(x \vee x^\sim) \le \max\{\gamma_T(x \rightsquigarrow (x \vee x^\sim)), \gamma_T(x)\}$

[From (iii) of definition 2.4]

$= \max\{\gamma_T((x \rightsquigarrow x) \vee (x \rightsquigarrow x^\sim)), \gamma_T(x)\}$

[From (iii) of proposition 2.3]

$= \max\{\gamma_T(1 \vee (x \rightsquigarrow x^\sim)), \gamma_T(x)\}$

[From (i) of proposition 2.3]

$\gamma_T(x \vee x^\sim) = \max\{\gamma_T(x \rightsquigarrow x^\sim), \gamma_T(x)\} \ge \gamma_T(1)$

for all $x \in \mathcal{A}$ [From (i) of definition 2.4]

Thus, $\gamma_T(x \vee x^\sim) = \gamma_T(1)$ for all $x \in \mathcal{A}$

Hence, any intuitionistic fuzzy implicative filter of $\mathcal{A}$ is an intuitionistic fuzzy Pseudo-Boolean implicative filter of $\mathcal{A}$.

Proposition 3.4.

Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$, then which satisfies the following inequalities

(i) $\mu_T(x^-) = \mu_T(x \to x^-)$; $\gamma_T(x^-) = \gamma_T(x \to x^-)$

(ii) $\mu_T(x^\sim) = \mu_T(x \rightsquigarrow x^\sim)$; $\gamma_T(x^\sim) = \gamma_T(x \rightsquigarrow x^\sim)$

for all $x \in \mathcal{A}$.

Proof. Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$.

Let $\mu_T(x^-) = \mu_T(1 \to x^-)$ [From (i)(a) of definition 2.2]

$= \mu_T(1 \to (x \to x^-))$

[From (vii) of proposition 2.3]

$= \mu_T(x \to x^-)$ for all $x \in \mathcal{A}$

[From (i)(a) of definition 2.2]

$\gamma_T(x^-) = \gamma_T(1 \to x^-)$ [From (i)(a) of definition 2.2]

$= \gamma_T(1 \to (x \to x^-))$

[From (vii) of proposition 2.3]

$= \gamma_T(x \to x^-)$ for all $x \in \mathcal{A}$

[From (i)(a) of definition 2.2]

Similarly, we prove $\mu_T(x^\sim) = \mu_T(x \rightsquigarrow x^\sim)$ and $\gamma_T(x^\sim) = \gamma_T(x \rightsquigarrow x^\sim)$ for all $x \in \mathcal{A}$.

Proposition 3.5. Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$, then which satisfies the following inequalities

(i) $\mu_T((x^-)^\sim) = \mu_T(x)$; $\gamma_T((x^-)^\sim) = \gamma_T(x)$

(ii) $\mu_T((x^\sim)^-) = \mu_T(x)$; $\gamma_T((x^\sim)^-) = \gamma_T(x)$

for all $x \in \mathcal{A}$.

Proof. Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$.

(i) Let $\mu_T(x) = \mu_T(x^- \to x)$ [From (vi) of proposition 2.3]

$= \mu_T(x^- \to (x^\sim)^-)$

[From (xi) of proposition 2.3]

$= \mu_T(x^- \to (x^\sim \to 0)) = \mu_T(x^- \rightsquigarrow 0)$

[From (ix) of proposition 2.3]

$= \mu_T((x^-)^\sim)$ [From (ix) of proposition 2.3]

Similarly, we prove that $\gamma_T((x^-)^\sim) = \gamma_T(x)$

(ii) Let $\mu_T(x) = \mu_T(x^\sim \rightsquigarrow x)$

[From (vi) of proposition 2.3]

$= \mu_T(x^\sim \to (x^-)^\sim)$

[From (xi) of proposition 2.3]

$= \mu_T(x^\sim \to (x^- \to 0)) = \mu_T(x^\sim \to 0)$

[From (ix) of proposition 2.3]

$= \mu_T((x^\sim)^-)$

[From (ix) of proposition 2.3]

Similarly, we prove that $\gamma_T((x^\sim)^-) = \gamma_T(x)$ for all $x \in \mathcal{A}$.

Proposition 3.6. Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$. Let T_1 and T_2 be two intuitionistic fuzzy implicative filters of $\mathcal{A}$, T_1 is a subset of T_2 and $\mu_{T_1}(1) = \mu_{T_2}(1)$; $\gamma_{T_1}(1) = \gamma_{T_2}(1)$. If T_1 is an intuitionistic fuzzy Pseudo-Boolean implicative filter of $\mathcal{A}$ then T_2 an intuitionistic fuzzy Pseudo-Boolean implicative filter $\mathcal{A}$.

Proof. Let T_1 be an intuitionistic fuzzy Pseudo-Boolean implicative filter of $\mathcal{A}$.

$\mu_{T_1}(x \vee x^-) = \mu_{T_1}(1)$; $\gamma_{T_1}(x \vee x^-) = \gamma_{T_1}(1)$ and

$\mu_{T_1}(x \vee x^\sim) = \mu_{T_1}(1)$; $\gamma_{T_1}(x \vee x^\sim) = \gamma_{T_1}(1)$

for all $x \in \mathcal{A}$. **(3.1)**

And also $\mu_{T_1}(x \vee x^-) \le \mu_{T_2}(x \vee x^-)$;

$\mu_{T_1}(x \vee x^\sim) \le \mu_{T_2}(x \vee x^\sim)$

Since T_1 is a subset of T_2.

From the equation of **(3.1)** and $\mu_{T_1}(1) = \mu_{T_2}(1)$

$\mu_{T_2}(1) \le \mu_{T_2}(x \vee x^-)$; $\mu_{T_2}(1) \le \mu_{T_2}(x \vee x^\sim)$ **(3.2)**

From (i) and (ii) of definition 3.1.,

We have $\mu_{T_2}(1) \ge \mu_{T_2}(x \vee x^-)$; $\mu_{T_2}(1) \ge \mu_{T_2}(x \vee x^\sim)$ **(3.3)**

From the equation of **(3.2)** and **(3.3),** we have

$\mu_{T_2}(1) = \mu_{T_2}(x \vee x^-)$; $\mu_{T_2}(1) = \mu_T(x \vee x^\sim)$for all $x \in \mathcal{A}$.

Similarly $\gamma_{T_1}(x \vee x^-) \ge \gamma_{T_2}(x \vee x^-)$;

$\gamma_{T_1}(x \vee x^\sim) \ge \gamma_{T_2}(x \vee x^\sim)$

Since T_1 is a subset of T_2.

From the equation of **(3.1)** and $\gamma_{T_1}(1) = \gamma_{T_2}(1)$

$\gamma_{T_2}(1) \ge \gamma_{T_2}(x \vee x^-)$; $\gamma_{T_2}(1) \ge \gamma_{T_2}(x \vee x^\sim)$ **(3.4)**

From (ii) and (ii) of definition 3.1.,

We have $\gamma_{T_2}(1) \le \gamma_{T_2}(x \vee x^-)$; $\gamma_{T_2}(1) \le \gamma_{T_2}(x \vee x^\sim)$ **(3.5)**

From the equation of **(3.4)** and **(3.5),** we have

$\gamma_{T_2}(1) = \gamma_{T_2}(x \vee x^-)$; $\gamma_{T_2}(1) = \gamma_{T_2}(x \vee x^\sim)$ for all $x \in \mathcal{A}$.

Hence T_2 an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$.

Proposition 3.7. Let $T = (\mu_T, \gamma_T)$be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$. Let T_1 and T_2 be two intuitionistic fuzzy Pseudo-Boolean implicative filters of $\mathcal{A}$. Then intersection of T_1 and T_2 is also an intuitionistic fuzzy Pseudo-Boolean implicative filter of $\mathcal{A}$.

Proof. Let T_1 and T_2 be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$.

Then , we have

$\mu_{T_1}(x \vee x^-) = \mu_{T_1}(1)$; $\gamma_{T_1}(x \vee x^-) = \gamma_{T_1}(1)$

$\mu_{T_1}(x \vee x^\sim) = \mu_{T_1}(1)$; $\gamma_{T_1}(x \vee x^\sim) = \gamma_{T_1}(1)$ and

$\mu_{T_2}(x \vee x^-) = \mu_{T_2}(1)$; $\gamma_{T_2}(x \vee x^-) = \gamma_{T_2}(1)$

$\mu_{T_2}(x \vee x^\sim) = \mu_{T_2}(1)$; $\gamma_{T_2}(x \vee x^\sim) = \gamma_{T_2}(1)$

for all $x \in \mathcal{A}$.

Let $\mu_{(T_1 \cap T_2)}(x \vee x^-) = \mu_{T_1}(x \vee x^-) \wedge \mu_{T_2}(x \vee x^-)$

$= \mu_{(T_1 \cap T_2)}(1)$ for all $x \in \mathcal{A}$.

Similarly, we prove $\gamma_{(T_1 \cap T_2)}(x \vee x^-) = \gamma_{(T_1 \cap T_2)}(1)$ for all $x \in \mathcal{A}$.

Hence, intersection of T_1 and T_2 is also an intuitionistic fuzzy Pseudo-Boolean implicative filter of $\mathcal{A}$.

Proposition 3.8. Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$, then which satisfies the following inequalities

(i) $\mu_T(1) = \mu_T(x)$ (or) $\mu_T(1) = \mu_T(x^-)$;

$\gamma_T(1) = \gamma_T(x)$ (or) $\gamma_T(1) = \gamma_T(x^-)$

(ii) $\mu_T(1) = \mu_T(x)$ (or) $\mu_T(1) = \mu_T(x^\sim)$;

$\gamma_T(1) = \gamma_T(x)$ (or) $\gamma_T(1) = \gamma_T(x^\sim)$ for all $x \in \mathcal{A}$.

Proof. Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$.

So $\mu_T(1) = \mu_T(x \vee x^-)$

$\geq \min\{\mu_T(x^-), \mu_T(x)\}$ [From (i) of definition 2.4]

$= \mu_T(x)$ (or) $\mu_T(x^-)$

$\mu_T(1) = \mu_T(x)$ (or) $\mu_T(1) = \mu_T(x^-)$ and also

$\gamma_T(1) = \gamma_T(x \vee x^-)$

$\leq \max\{\gamma_T(x^-), \gamma_T(x)\}$

$= \gamma_T(x)$ (or) $\gamma_T(x^-)$ [From (ii) of definition 2.4]

$\gamma_T(1) = \gamma_T(x)$ (or) $\gamma_T(1) = \gamma_T(x^-)$

Similarly, we prove

$\mu_T(1) = \mu_T(x)$ (or) $\mu_T(1) = \mu_T(x^\sim)$; $\gamma_T(1) = \gamma_T(x)$ (or) $\gamma_T(1) = \gamma_T(x^\sim)$ for all $x \in \mathcal{A}$.

Proposition 3.9. Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$, then which satisfies the following

(i) $\mu_T(1) = \mu_T(x)$ (or) $\mu_T(1) = \mu_T(x^-)$ if and only if $\mu_T(x \to x^-) = \mu_T(1)$ (or) $\mu_T(x^- \to x) = \mu_T(1)$

(ii) $\gamma_T(1) = \gamma_T(x)$ (or) $\gamma_T(1) = \gamma_T(x^-)$ if and only if $\gamma_T(x \to x^-) = \gamma_T(1)$ (or) $\gamma_T(x^- \to x) = \gamma_T(1)$

(iii) $\mu_T(1) = \mu_T(x)$ (or) $\mu_T(1) = \mu_T(x^\sim)$ if and only if $\mu_T(x \to x^\sim) = \mu_T(1)$ (or) $\mu_T(x^\sim \to x) = \mu_T(1)$

(iv) $\gamma_T(1) = \gamma_T(x)$ (or) $\gamma_T(1) = \gamma_T(x^\sim)$ if and only if $\gamma_T(x \to x^\sim) = \gamma_T(1)$ (or) $\gamma_T(x^\sim \to x) = \gamma_T(1)$

for all $x \in \mathcal{A}$.

Proof. Let $T = (\mu_T, \gamma_T)$ be an intuitionistic fuzzy Pseudo-Boolean implicative filter of LPWA $\mathcal{A}$.

(i) Let $\mu_T(1) = \mu_T(x)$ (or) $\mu_T(1) = \mu_T(x^-)$

To Prove : $\mu_T(x \to x^-) = \mu_T(1)$ (or)

$\mu_T(x^- \to x) = \mu_T(1)$

From (i) of definition 3.1, we have

$\mu_T(x \to x^-) \vee \mu_T(x^- \to x) = \mu_T(1)$

From (i) the proposition 3.8, we have

$\mu_T(x \to x^-) = \mu_T(1)$ (or) $\mu_T(x^- \to x) = \mu_T(1)$

Conversely,

let $\mu_T(x \to x^-) = \mu_T(1)$ (or) $\mu_T(x^- \to x) = \mu_T(1)$ and $\mu_T(x \vee x^-) = \mu_T(1)$

Now $(x^- \vee x) \to x = (x^- \to x) \wedge (x \to x)$

[From (iv) of proposition 2.3]

$= (x^- \to x) \wedge 1 = (x^- \to x)$

[From (i) of proposition 2.3]

It follows that $\mu_T((x^- \vee x) \to x) = \mu_T(x^- \to x) = \mu_T(1)$

Since, $\mu_T(x) \geq \min\{\mu_T((x^- \vee x) \to x), \mu_T(x^- \vee x)\}$

[From (ii) of definition 2.4]

So, $\mu_T(x) = \mu_T(1)$

Now $(x \vee x^-) \to x^- = (x \to x^-) \wedge (x^- \to x^-)$

[From (iv) of proposition 2.3]

$= (x \to x^-) \wedge 1 = (x \to x^-)$

[From (i) of proposition 2.3]

It follows that $\mu_T((x \vee x^-) \to x^-) = \mu_T(x \to x^-) = \mu_T(1)$

Since, $\mu_T(x^-) \geq \min\{\mu_T((x \vee x^-) \to x), \mu_T(x \vee x^-)\}$

[From (ii) of definition 2.4]

Thus, $\mu_T(x^-) = \mu_T(1)$.

(ii) Assume that $\gamma_T(1) = \gamma_T(x)$ (or) $\gamma_T(1) = \gamma_T(x^-)$

Prove that $\gamma_T(x \to x^-) = \gamma_T(1)$ (or) $\gamma_T(x^- \to x) = \gamma_T(1)$

From (i) of definition 3.1,

We have $\gamma_T(x \to x^-) \vee \gamma_T(x^- \to x) = \gamma_T(1)$

From (i) the proposition 3.8, we have

$\gamma_T(x \to x^-) = \gamma_T(1)$ (or) $\gamma_T(x^- \to x) = \gamma_T(1)$

Conversely,

Let $\gamma_T(x \to x^-) = \gamma_T(1)$ (or) $\gamma_T(x^- \to x) = \gamma_T(1)$ and $\gamma_T(x \vee x^-) = \gamma_T(1)$ and

$(x^- \vee x) \to x = (x^- \to x) \wedge (x \to x)$

[From (iv) of proposition 2.3]

$= (x^- \to x) \wedge 1 = (x^- \to x)$

[From (i) of proposition 2.3]

It follows that $\gamma_T((x^- \vee x) \to x) = \gamma_T(x^- \to x) = \gamma_T(1)$

Since $\gamma_T(x) \leq \max\{\gamma_T((x^- \vee x) \to x), \gamma_T(x^- \vee x)\}$

[From (ii) of definition 2.4]

Thus, $\gamma_T(x) = \gamma_T(1)$

and $(x \vee x^-) \to x^- = (x \to x^-) \wedge (x^- \to x^-)$

[From (iv) of proposition 2.3]

$= (x \to x^-) \wedge 1 = (x \to x^-)$

[From (i) of proposition 2.3]

It follows that $\gamma_T((x \vee x^-) \to x^-) = \gamma_T(x \to x^-) = \gamma_T(1)$

Since $\gamma_T(x^-) \leq \max\{\gamma_T((x \vee x^-) \to x), \gamma_T(x \vee x^-)\}$

[From (ii) of definition 2.4]

So, $\gamma_T(x^-) = \gamma_T(1)$

Similarly, we prove that (iii) and (iv).

IV. CONCLUSION

In this paper, we have introduced the notion of an intuitionistic fuzzy pseudo-Boolean implicative filter of LPWA and discussed some properties with illustrations.

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