Spectrum Allocation by Sealed Bid Game Theory

Aritra De, Tirthankar Datta

Abstract: Wireless communication subscribers are increasing day by day specially in fifth generation (5G) wireless communication where multiple number of users (Multiple Input Multiple Output or MIMO) can be served in a specific time. The heavy data usage is also enhanced with the increasing the number of subscribers, this data transfer speed depends on the amount of spectrum allocation to the specific subscriber. Thus, spectrum allocation is a major criterion for wireless communication performance improvement. The spectrum allocation efficiency can be observed by Game Theory, which is a popular decision maker of modern era. Sealed Bid Game theory is one of the popular segment of the game theory. The spectrum allocation can be done by using Sealed Bid Game theory and spectrum equilibrium can be observed by using different sub division of Sealed Bid Game theory.

Keywords: 5G, MIMO, Game Theory, Sealed Bid Game Theory, Spectrum Allocation.

I. INTRODUCTION

Wireless communication spectrum allocation is major criteria for the performance improvement of the system [1]. Multiple number of subscribers can be served by modern mobile communication generation [2]. The multiple number of subscribers data speed and voice quality can be improved by using efficient spectrum allocation technique [3]. The spectrum can be done by simulation method or experimental method [4].

The spectrum also can be done by using different optimization technique [5]. The optimization technique is difficult to understand and mathematical calculation is time consuming [6].

Game Theory is less time consuming and it is used heavily in the modern era[7]. Game theory does not assume any knowledge of its players[8]. The only way to appreciate game theory is to see it in action, or better still to put in into action[9].

The user of the mobile subscriber can be static or dynamic [10].

In this work wireless generation is compared in the section II, Bayesian second price auction game theory with average value calculation is discussed in the section III and IV respectively, sealed bid first price auction is discussed with average value calculation is discussed in the section V and VI respectively, two player all pay auction is discuss with average value calculation is discussed in the section VII and VIII respectively. Conclusion of the work is discussed in the section IX.

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If \( C_2 > C_1 \) then \( Q_2 \) wins and pays the second highest bid \( C_1 \).

Now each and every player has a private valuation, consider the valuation of \( Q_1, Q_2 \) is \( U_1, U_2 \) respectively.

\( U_1, U_2 \) are independent, distributed and random variable as shown in figure (4).

The Nash Equilibrium of second price auction is
\[
C_1 = U_1 \quad \text{.................................} \quad (1A)  \\
C_2 = U_2 \quad \text{.................................} \quad (1B)  \\
\]

Now start with the assumption that
\[
C_2 = U_2 \quad \text{.................................} \quad (1C)  \\
\]

\[ A. \text{ Case 1} \]
Consider \( U_1 \geq U_2 \), the bidding of player 2 is \( C_2 = U_2 \) also if \( C_2 \geq U_2, Q_1 \) wins the auction and pays the second highest bid \( C_2 = U_2 \).

Net payoff = \( U_1 - U_2 \geq 0 \).

If he bids, \( C \leq C_2 = U_2 \) then player \( Q_1 \) loses the auction and his net payoff is 0.

Therefore, any bid \( C \geq U_2 \) is a best response that mean \( C = U_1 \) is a best response.

\[ B. \text{ Case 2} \]
If \( U_1 \leq U_2 \) player \( Q_2 \) is bidding \( C_2 = U_2 \). If \( Q_1 \) bids \( C \geq C_2 = U_2 \), then he wins the auction and pays second highest bid \( C_2 = U_2 \). Net payoff = \( U_1 - U_2 \leq 0 \).

If he bids \( C < C_2 = U_2 \) then he loses the auction and his payoff is 0. Therefore any bid \( C < C_2 = U_2 \) is a best response.

In particular \( C = U_1 \) is a best response.

If player \( Q_2 \) is bidding \( C_2 = U_2 \), then \( C_1 = U_1 \) is a best response for player \( Q_1 \).

Similarly it can be shown that if \( Q_2 \) is bidding \( C_1 = U_1 \) then \( C = U_2 \) is a best response for \( Q_2 \).

Hence the Nash equilibrium of second price auction is
\[
C_1 = U_1 \quad \text{.................................} \quad (1D)  \\
C_2 = U_2 \quad \text{.................................} \quad (1E)  \\
\]

So, each player bidding his true valuation is the Nash equilibrium for the second price auction.

II. WIRELESS GENERATION COMPARISON

<table>
<thead>
<tr>
<th>Generation Feature</th>
<th>1G</th>
<th>2G</th>
<th>3G</th>
<th>4G</th>
<th>5G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Band</td>
<td>800 MHz</td>
<td>900 MHz</td>
<td>2100 MHz</td>
<td>2600 MHz</td>
<td>3-90GHz</td>
</tr>
<tr>
<td>Speed</td>
<td>2 kbps</td>
<td>64 kbps</td>
<td>2Mbsp</td>
<td>1Gbps</td>
<td>Higher than 1Gbps</td>
</tr>
</tbody>
</table>

III. BAYESIAN SECOND PRICE AUCTION

The spectrum is allocated to two user \( Q_1 \) and \( Q_2 \). The spectrum is allocated to the user \( Q_1 \) is \( C_1 \) and maximum spectrum is allowed is \( U_1 \), the spectrum is allocated to the user \( Q_2 \) is \( C_2 \) and maximum spectrum is allowed is \( U_2 \).

If \( C_1 \geq C_2 \) then \( Q_1 \) will win else if \( C_2 > C_1 \) then \( Q_2 \) wins so it mean player with highest bid wins the auction.

If \( C_1 \geq C_2 \) then \( Q_1 \) will win and pays the second highest bid \( C_2 \).

IV. EXPECTED REVENUE OF SECOND PRICE AUCTION

\[
C_1 = U_1 \quad \text{.................................} \quad (2A)  \\
C_2 = U_2 \quad \text{.................................} \quad (2B)  \\
\]

Noted that \( U_1 \geq U_2 \) then \( C_1 \leq C_2 \), so \( Q_1 \) wins the auction and pays second highest bid \( C_2 = U_2 \).

If \( U_1 < U_2 \) then \( C_1 > C_2 \), so \( Q_2 \) wins the auction and pays second highest bid \( U_2 \).

So, it can be concluded that the revenue to the auctioneer in the Bayesian second price auction is minimum \( U_1, U_2 \).

\( U_1, U_2 \) are independent and Probability Density Function (PDF) \( F_{U_1}(U_1) \) and \( F_{U_2}(U_2) \) of Maximum spectrum allowed is distributed uniformly in the interval \( [0, 1] \) as shown in the figure 5.

\[ A. \text{ Case 1} \]
If \( U_1 \leq U_2 \) then \( U_1 \in [U, U+dU] \)
\( U_2 \in [U+dU, 1] \)
\[
P_r = P_r(U_1 \in [U, U + dU]) \times P_r(U_2 \in [U + dU, 1])  \\
= dU \times (1-U+dU)  \\
= dU(1-U)  \\
\]
Revenue to the auctioneer=minimum \( \{U_1, U_2\} \)
Since minimum lies in \([U, U+dU]\), revenue=U. Expected revenue\(=P_U \times U \)
Expected revenue 
\[
\int_0^1 2(1-U)du
\]
\[
=1/3
\]
Hence expected revenue \(=1/3\) also the revenue is independent.

\section*{V. SEALED BID FIRST PRICE AUCTION}

The spectrum is allocated to two user \(Q_1\) and \(Q_2\). The spectrum is allocated to the user \(Q_1\) is \(C_1\) and maximum spectrum is allowed is \(U_1\), the spectrum is allocated to the user \(Q_2\) is \(C_2\) and maximum spectrum is allowed is \(U_2\).

The Probability Density Function (PDF) \(F_U\) of Maximum spectrum allowed is distributed uniformly in the interval \([0, 1]\) as shown in the figure 4.

The Game Theory bidding strategy is as following:

\(C_1=\frac{1}{2}U_1\) \hspace{1cm} (1)
\(C_2=\frac{1}{2}U_2\) \hspace{1cm} (2)

\(\pi(C)\) Denotes the payoff to the user \(Q_1\) as a function of \(C\) where \(C\) is maximum spectrum allocated to the mobile tower.

User \(Q_1\) wins the auction game i.e. \(C\geq C_2\), then the payoff is \(=\text{Valuation of the user 1-Bid paid on winning the auction} =U_1-C\). \hspace{1cm} (3)

Average payoff to player 1 is given by,
\[\pi(C)=P_U(\text{win})\times(U_1-C)+P_L(\text{loss})\times 0\]
So,
\[\pi(C)=P_U(\text{win})\times(U_1-C)\] \hspace{1cm} (4)

The winning condition for the player 1 is
\[C\geq C_2=\frac{1}{2}U_2\] \hspace{1cm} (5)
\[C\geq \frac{1}{2}U_2\] \hspace{1cm} (6)
\[U_2\leq 2C\] \hspace{1cm} (7)

Since \(U_2\) is distributed uniformly in \([0,1]\), \(U_2\) must have in \([0,2C]\).

\[\text{Probability } U_2 \text{ lies in } [0,2C] = \int_0^{2C} F_{U_2}(U_2) dU_2 \]
\[= \int_0^{2C} du \]
\[=U_2|_0^{2C} \]
\[=2C\] \hspace{1cm} (11)

Hence, \(P_U(\text{win})\) for player 1 is \(2C\), therefore
\[\pi(C)=P_U(\text{win})\times(U_1-C)=2C\times(U_1-C)\] \hspace{1cm} (13)

\section*{VI. EXPECTED REVENUE OF THE FIRST PRICE AUCTION}

Nash equilibrium is given by
\[C_1=\frac{1}{2}U_1\] \hspace{1cm} (17)
\[C_2=\frac{1}{2}U_2\] \hspace{1cm} (18)

Now the player wins who called for maximum bid. Hence
\[\text{Revenue}=\text{maximum } \{C_1, C_2\} \hspace{1cm} (19)
\[=\text{maximum } \{\frac{1}{2}U_1, \frac{1}{2}U_2\}\] \hspace{1cm} (20)
\[=\text{maximum } \{U_1, U_2\} \hspace{1cm} (21)
\[U_1, U_2\] are uniform distributed in \([0, 1]\) and probability for equation (21) lies in the infinitesimal interval \([U, U+dU]\) as shown in the figure 4.

\subsection*{A. Case 1}

\(U_1\) is the maximum \(U_1\) lies in \([U, U+dU]\) and \(U_2\) lies in \([0, U]\).

\[P_U=\begin{cases} P_1 & (U_1 \in [U, U+dU]) \times P_2 & (U_2 \in [0, U]) \end{cases} \hspace{1cm} (22)
\]
\[=dU \times U \]
\[=dU \times U \hspace{1cm} \] \hspace{1cm} (23)
\[=dU \times U \hspace{1cm} \] \hspace{1cm} (24)

\subsection*{B. Case 2}

\(U_2\) is the maximum \(U_2\) lies in \([U, U+dU]\) and \(U_1\) lies in \([0, U]\).

\[P_U=\begin{cases} P_1 & (U_1 \in [0, U]) \times P_2 & (U_2 \in [U, U+dU]) \end{cases} \hspace{1cm} (22)
\]
\[=dU \times U \]
\[=dU \times U \hspace{1cm} \] \hspace{1cm} (23)
\[=dU \times U \hspace{1cm} \] \hspace{1cm} (24)

\subsection*{C. Equations}

Probability that maximum \(\{U_1, U_2\}\) is lies in \([U, U+dU]\)
\[=dU \times U \hspace{1cm} \] \hspace{1cm} (25)
\[=dU \times U \hspace{1cm} \] \hspace{1cm} (26)

So, Average revenue corresponding to maximum \(\{U_1, U_2\}\) is lies in \([U, U+dU]\)
\[=dU \times U \hspace{1cm} \] \hspace{1cm} (27)
\[=dU \times U \hspace{1cm} \] \hspace{1cm} (28)

So, total average revenue to the auctioneer is
\[=\int_0^1 U^2 dU \hspace{1cm} \] \hspace{1cm} (29)
\[=\frac{3}{3} \hspace{1cm} \] \hspace{1cm} (30)
\[=\frac{1}{3} \hspace{1cm} \] \hspace{1cm} (31)

The expected revenue of the auctioneer is \(=1/3\).
VII. TWO PLAYER ALL PAY PRICE AUCTION

The spectrum is allocated to two user $Q_1$ and $Q_2$. The spectrum is allocated to the user $Q_1$ is $C_1$ and maximum spectrum is allowed is $U_1$. The spectrum is allocated to the user $Q_2$ is $C_2$ and maximum spectrum is allowed is $U_2$.

User with highest spectrum allocation wins the game. Both the player pay their bid irrespectively of their outcome.

Now, assume that $U_1$ and $U_2$ denotes the valuations of $Q_1$ and $Q_2$.

The Probability Density Function (PDF) $F_U(U_1)$ and $F_U(U_2)$ of Maximum spectrum allocated is distributed uniformly in the interval $[0, 1]$ as shown in the figure 4.

The Nash Equilibrium is as following

\[
C_1 = \frac{1}{2} U_1^2 \\
C_2 = \frac{1}{2} U_2^2
\]

Now, assume that player $Q_2$ is bidding $C_2 = \frac{1}{2} U_2^2$, also assume $Q_1$ bids $C$.

\[
\pi(C) = \text{expected payoff to player } 1 \text{ as a function } C.
\]

\[
\pi(C) = P_1(\text{win}) \times (U_1 - C) + P_1(\text{loss}) \times (-C).
\]

If $C \geq C_2 = \frac{1}{2} U_2^2$, then $U_1 \leq \sqrt{2C}$. So, from equation (34) it can be derived that $\frac{\partial \pi}{\partial C} = \frac{1}{2} U_1^2 - 1 = 0$. Similarly, it can be shown that $C_1 = \frac{1}{2} U_1^2$ is a best response bid for user $Q_2$.

\[
C_1 = \frac{1}{2} U_1^2 \\
C_2 = \frac{1}{2} U_2^2
\]

So, equation (46) and (47) are Nash Equilibrium.

VIII. EXPECTED REVENUE OF TWO PLAYER ALL PAY PRICE AUCTION

Revenue $= C_1 + C_2$.

\[
= \frac{1}{2} U_1^2 + \frac{1}{2} U_2^2
\]

Expected Revenue $= 1 \int_0^1 \left( \frac{1}{2} U_1^2 + \frac{1}{2} U_2^2 \right) dU_1 = \frac{1}{2} \int_0^1 U_1^2 dU_1 + \frac{1}{2} \int_0^1 U_2^2 dU_2$.

\[
= \frac{1}{2} \int_0^1 U_1^2 dU_1 + \frac{1}{2} \int_0^1 U_2^2 dU_2.
\]

Equation (55) is the revenue.

IX. CONCLUSION

This work is completed by using different bidding game theoretical approach. The Nash equilibrium is calculated in each sub part of sealed bid game theory, which is nothing but best approach of spectrum allocation.

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