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Multiply Divisor Cordial Labeling

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Abstract - A Graph G^* having multiply divisor cordial labeling with node set V^* is a bijective. t on V^* to $\{1,2,\ldots,V^*\}$ such that an edge ab is allocate the label 1 if 2 divides (t (a)·t (b)) and 0 otherwise, then the number of edges having label 0 and the number of edges having label 1 differ by maximum 1. A graph having multiply divisor cordial labeling is said to be multiply divisor cordial graph. In this paper, we prove that cycle, cycle having 1 chord, cycle having 2 chords, cycle having triangle, path, jellyfish, coconut tree, star and bistar graph are multiply divisor cordial graphs.

Subject classification number: 05C78.
Keywords: Subtract divisor cordial, jellyfish, coconut tree.

I. INTRODUCTION

All graphs included here are strict, finite, connected and undirected. We use primary notations and terminologies of graph theory as in [2]. Labeling of a graph is a correspondence that carries the graph components to the set of numbers, usually to the set of whole numbers excluding zero. If the set of inputs is set of nodes the labeling is said to be node labeling. If the set of inputs is the set of edges, then we are talking edge labeling. If the labels are allocate to both nodes and edges then the labeling is said to be total labeling. For a dynamic survey of variegated labeling of graphs, we make reference to Gallian [1].

Definition 1: Taking $G^* = (V^*, E^*)$ be strict graph and $t : V^* \rightarrow \{1,2,\ldots,V^*\}$ be bijective. For each edge ab, allocate the label 1 if 2 | (t (a)·t (b)) and the label 0 otherwise. The function t is said to be multiply divisor cordial labeling if $|e^*_t(0) - e^*_t(1)| \leq 1$. A graph which acknowledges multiply divisor cordial labeling is said to be multiply divisor cordial graph.

II. METHDOLOGY

Multiply Divisor Cordial Labeling that implied a new labeling pattern to various graphs. As given in definition 1, first given labeling pattern has been identified. Some graphs have been taken for the implementation of given labeling. By hit and trial method some graphs have been identified then labeling was given as per the conditions of multiply divisor cordial labeling and hypothesis was tested. Using node and edge labeling cycles with different number of chords were taken and tried and then applied to another graphs. After all labeling pattern respective functions have been identified under the certain conditions. And by this way given conclusive and applied research is drafted.

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III. MAIN RESULTS

Theorem 1: Cycle C_m is multiply divisor cordial graph.

Proof: Taking $a_1,a_2,\ldots,a_n$ be the nodes of cycle C_m. Here cycle C_m has m nodes and m edges.

To define labeling function $t: V^* \rightarrow \{1,2,\ldots,m\}$, we appraise following cases.

Case 1: Whereas m is even.

Then C_m will not be multiply divisor cordial graph. For attainment of edge set-up for multiply divisor cordial graph it is requisite to accord label 1 to $\frac{m}{2}$ edges and label 0 to $\frac{m}{2}$ edges from total m edges. Labeling of edge will heighten for minimum $\frac{m}{2}+1$ edges having label 1 and maximum $\frac{m}{2}$ -1 edges having label 0 from total m edges. Therefore $|e_t(0) - e_t(1)| = 2$. So, the edge set-up is going in opposition to multiply divisor cordial graph.

Therefore, cycle C_m is not multiply divisor cordial whereas m is even.

Case 2: whereas m is odd.

$t(a_i) = 2i - 1; \quad 1 \leq i \leq \left\lceil \frac{m}{2} \right\rceil$.

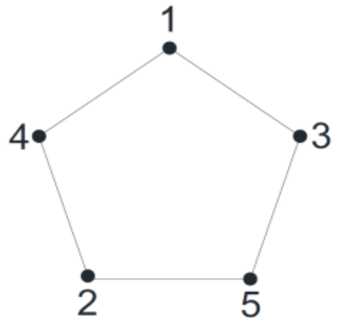
Accord the remaining labels to the remaining nodes $a_{\left\lceil \frac{m}{2} \right\rceil + 1}$ **to** a_m in any order.

Then we have $e^*_t(0) = \left\lfloor \frac{m}{2} \right\rfloor$, $e^*_t(1) = \left\lceil \frac{m}{2} \right\rceil$

Therefore $|e^*_t(0) - e^*_t(1)| \leq 1$.

Hence, cycle C_m is multiply divisor cordial graph.

Exemplar 1: Multiply divisor cordial labeling of cycle C_5 to be viewed in Drawing 1.



Drawing 1

Theorem 2: Cycle C_m having 1 chord is multiply divisor cordial graph, except m = 4.

Proof: Taking $a_1, a_2, \ldots, a_n$ be consecutive nodes of cycle C_m and $e^* = a_2a_m$ be a chord of cycle C_m. The nodes a_1, a_2, a_n constructs a triangle with chord e^*.

To define labeling function $t: V^* \rightarrow \{1,2,\ldots,m\}$ we appraise following cases.

Case 1: whereas m = 4.

Then C_4 is not multiply divisor cordial graph.

For attainment of the edge set-up for multiply divisor cordial graph it is requisite to accord label 1 to 3 edges and label 0 to 2 edges from total 5 edges. The edge label will heighten minimum 4 edges having label 1 and maximum 1 edge having label 0 from total 5 edges. Therefore $|e^*_t(0) - e^*_t(1)| = 3$. So, the edge set-up is going in opposition to multiply divisor cordial graph. Therefore, cycle C_4 having 1 chord is not multiply divisor cordial graph.

Case 2: $\mathbf{m \equiv 0, 1 (mod\ 2)}$ **except** $\mathbf{m = 4}$.

$\mathbf{f(a_m) = 1}$.

$f(a_i) = \mathbf{2i + 1}$; $\mathbf{1 \le i \le \left\lceil \frac{m}{2} \right\rceil}$.

Accord the remaining labels to the remaining nodes $\mathbf{a_{\left\lceil \frac{m}{2} \right\rceil + 1}}$ **to** $\mathbf{a_{m-1}}$ in any order.

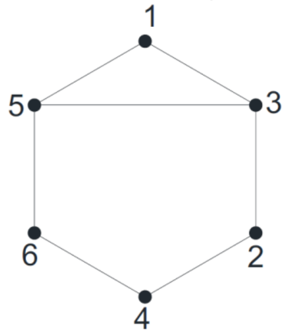
From the above labeling pattern we have

Cases of m	Edge set-up
m is odd	$e^*_t(0) = e^*_t(1) = \left\lceil \frac{m}{2} \right\rceil$
m is even	$e^*_t(0) = \frac{m}{2}$, $e^*_t(1) = \frac{m}{2} + 1$

Therefore $| e^*_t(0) - e^*_t(1) | \le 1$.

Hence, cycle C_m having one chord is multiply divisor cordial graph.

Exemplar 2: A multiply divisor cordial labeling of cycle C_6 having 1 chord to be viewed in Drawing 2.



Drawing 2

Theorem 3: Cycle C_m having 2 chords is multiply divisor cordial graph, except for m = 6.

Proof: Taking G^* be the cycle with 2 chords, where chords construct 2 triangles and 1 cycle C_{m-2}. Here number of nodes p = m and number of edges q = m + 2. Taking $a_1, a_2, . . . , a_m$ be successive nodes of G^*. Taking $e^*_1 = a_m a_2$ and $e^*_2 = a_m a_3$ be the chords of cycle C_m.

To define labeling function t: $V^* \rightarrow \{1,2,\ldots,m\}$ we appraise following cases.

Case 1: whereas m = 6.

Then C_6 is not multiply divisor cordial graph. For attainment of the edge set-up for multiply divisor cordial graph it is requisite to accord label 1 to 4 edges and label 0 to 4 edges from total 8 edges. The edge label will heighten minimum 5 edges having label 1 and maximum 3 edges having label 0 from total 8 edges. Therefore $|e^*_t(0) - e^*_t(1)| = 2$. So, the edge set-up is going in opposition to for multiply divisor cordial graph. Therefore, cycle C_6 with 2 chords is not multiply divisor cordial graph.

Case 2: $m \equiv 0, 1 (mod\ 2)$ except $m = 6$.

$\mathbf{t(a_m) = 1}$.

$t(a_i) = \mathbf{2i + 1}$; $\mathbf{1 \le i \le \left\lceil \frac{m}{2} \right\rceil}$.

Accord the remaining labels to the remaining nodes $\mathbf{a_{\left\lceil \frac{m}{2} \right\rceil + 1}}$ **to** $\mathbf{a_{m-1}}$ in any order.

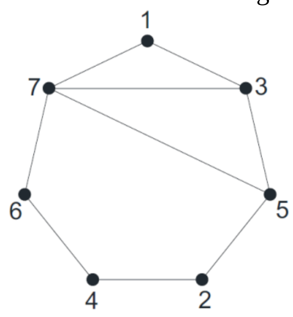
From the above labeling pattern we have

Cases of m	Edge set-up
m is odd	$e^*_t(0) = \left\lceil \frac{m}{2} \right\rceil + \mathbf{1}$, $e^*_t(1) = \left\lceil \frac{m}{2} \right\rceil$
m is even	$e^*_t(0) = e^*_t(1) = \frac{m}{2} + 1$

Therefore $|e^*_t(0) - e^*_t(1)| \le 1$.

Hence, cycle C_m with twin chords is multiply divisor cordial graph.

Exemplar 3: Multiply divisor cordial labeling of cycle C_7 having twin chords to be viewed in Drawing 3.



Drawing 3

Theorem 4: Cycle C_m having triangle is multiply divisor cordial graph, except m = 6.

Proof: Taking G^* be cycle with triangle $C_m(1, 1, m - 5)$. Taking $a_1, a_2, . . . , a_m$ be successive nodes of G. Taking a_1, a_3 and a_5 be the nodes of triangle constructed by edges $e^*_1 = a_1a_3$, $e^*_2 = a_3a_5$ and $e^*_3 = a_1a_5$.

To define labeling function t : $V^* \rightarrow \{1,2,\ldots,m\}$ we appraise following cases.

Case 1: whereas m = 6.

Then C_6 is not multiply divisor cordial graph. For attainment of the edge set-up for multiply divisor cordial graph it is requisite to accord label 1 to 5 edges and label 0 to 4 edges from total 9 edges. The edge label will heighten minimum 6 edges having label 1 and maximum 3 edges having label 0 from total 9 edges. Therefore, $|e^*_t(0) - e^*_t(1)| = 3$. So, the edge set-up is going in opposition to for multiply divisor cordial graph. Therefore, cycle C_6 having triangle is not multiply divisor cordial graph.

Case 2: $\boldsymbol{m \equiv 0, 1 (mod\ 2)}$ **except** $\mathbf{m = 6}$.

$\mathbf{t(a_m) = m - 1, t(a_{m-1}) = m}$.

$t(a_i) = 2i - 1$; $\mathbf{1 \le i \le \left\lceil \frac{m}{2} \right\rceil - 1}$.

Accord the remaining labels to the remaining nodes $\mathbf{a_{\left\lceil \frac{m}{2} \right\rceil}}$ **to** $\mathbf{a_{m-2}}$ in any order.

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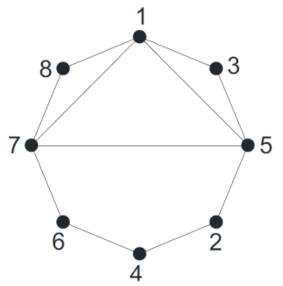
From the above labeling pattern we have

Cases of m	Edge set-up
m is odd	$e^*_t(0) = e^*_t(1) = \left\lceil \frac{m}{2} \right\rceil + 1$
m is even	$e^*_t(0) = \frac{m}{2} + 1$, $e^*_t(1) = \frac{m}{2} + 2$

Therefore $|e^*_t(0) - e^*_t(1)| \leq 1$.

Hence, cycle C_m with triangle is multiply divisor cordial graph.

Exemplar 4: Multiply divisor cordial labeling of cycle C_8 having triangle to be viewed in Drawing 4.



Drawing 4

Theorem 5: Path P_m is multiply divisor cordial graph.

Proof: Taking $a_1, a_2, \ldots, a_m$ be nodes of the path P_m.

Define labeling t: V^* $(K^*_{1,m}) \rightarrow \{1, 2, \ldots, m\}$ as follows.

$t(a_i) = \mathbf{2\mathit{i} - 1}$; $\quad \mathbf{1 \leq i \leq \left\lceil \frac{m}{2} \right\rceil}$.

Accord the remaining labels to the remaining nodes $\mathbf{a_{\left\lceil \frac{m}{2} \right\rceil + 1}}$ **to** $\mathbf{au_m}$ in any order.

From the above labeling pattern we have

Cases of m	Edge set-up
m is odd	$e^*_t(0) = e^*_t(1) = \left\lfloor \frac{m}{2} \right\rfloor$
m is even	$e^*_t(0) = \frac{m}{2} - 1$, $e^*_t(1) = \frac{m}{2}$

Therefore $|e^*_t(0) - e^*_t(1)| \leq 1$.

Hence, path P_m is multiply divisor cordial graph.

Exemplar 5: Multiply divisor cordial labeling of path P_9 to be viewed in Drawing 5.

Drawing 5

Theorem 6: Star graph $K_{1,m}$ is multiply divisor cordial graph.

Proof: Taking V^* $(K_{1,m}) = \{a, b_i: 1 \leq i \leq m\}$ and E^* $(K_{1,m}) = \{(ab_i) : 1 \leq i \leq m\}$.

Here $|V^*$ $(K_{1,m})| = m + 1$, $|E^*$ $(K_{1,m})| = m$.

Define labeling t: V^* $(K_{1,m}) \rightarrow \{1, 2, \ldots, m + 1\}$ as follows.

$t(a_0) = 1$,

$t(a_i) = i$; $1 \leq i \leq m$.

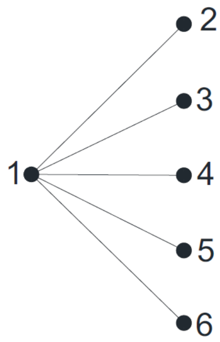
From the above labeling pattern we have,

Cases of m	Edge set-up
m is odd	$e^*_t(0) = \left\lfloor \frac{m}{2} \right\rfloor$, $e^*_t(1) = \left\lceil \frac{m}{2} \right\rceil$
m is even	$e^*_t(0) = e^*_t(1) = \frac{m}{2}$

Therefore $|e^*_t(0) - e^*_t(1)| \leq 1$.

Hence, star graph $K_{1,m}$ is multiply divisor cordial graph.

Exemplar 6: A multiply divisor cordial labeling of star $K_{1,5}$ to be viewed in Drawing 6.



Drawing 6

Theorem 7: Jellyfish $J_{m,m}$ is multiply divisor cordial graph.

Proof: Taking V^* $(J_{m,m}) = \{a, b, p, q, a_i, b_i : 1 \leq i \leq m\}$ and E^* $(J_{m,m}) = \{ap, aq, bp, bq, pq, aa_i, bb_i : 1 \leq i \leq m\}$.

Define labeling t: V^* $(J_{m,m}) \rightarrow \{1, 2, \ldots, 2m + 4\}$ as follows.

$t(a) = 1, t(a_m) = 2m + 4, t(b) = 2$.

$t(p) = 3, t(q) = 5$.

$t(a_i) = 2i + 3; 1 \leq i \leq m$.

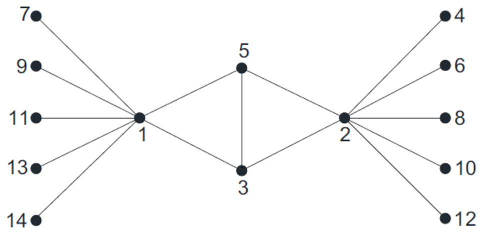
$t(b_i) = 2i + 2; 1 \leq i \leq m$.

Then we have $e^*_t(0) = m + 2$ and $e^*_t(1) = m + 3$.

Therefore $|e^*_t(0) - e^*_t(1)| \leq 1$.

Hence, jellyfish $J_{m,m}$ is multiply divisor cordial graph.

Exemplar 7: Multiply divisor cordial labeling of jellyfish $J_{5,5}$ to be viewed in Drawing 7.



Drawing 7

Theorem 8: Bistar $B_{m,m}$ is multiply divisor cordial graph.

Proof: Taking a_0, b_0 be apex nodes of $B_{m,m}$. Taking $a_1, a_2, \ldots, a_m$ be the pendant nodes adjacent to the node a_0 and $b_1, b_2, \ldots, b_m$ be the pendant nodes adjacent to the node b_0.
We define labeling function t: $V^* \rightarrow \{1, 2, \ldots, 2m + 1\}$ as follows

$t(a_0) = 1,$

$t(a_i) = 2i + 1; 1 \leq i \leq m.$

$t(b_0) = 2,$

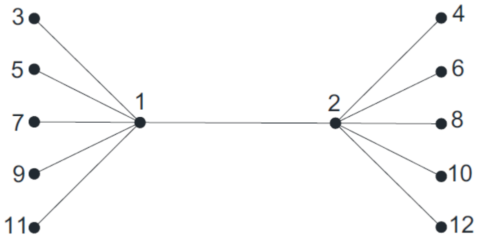
$t(b_i) = 2i + 2;\ 1 \leq i \leq m.$

Then we have $e^*_t(0) = m$ and $e^*_t(1) = m + 1$.

Therefore $|e^*_t(0) - e^*_t(1)| \leq 1$.

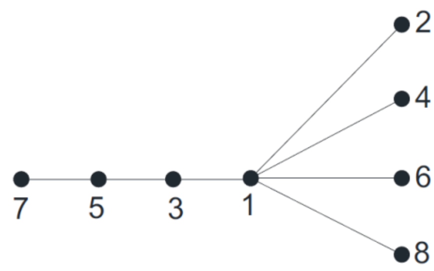
Hence, bistar $B_{m,m}$ is multiply divisor cordial graph.

Exemplar 8: Multiply divisor cordial labeling of bistar $B_{5,5}$ to be viewed in Drawing 8.



Drawing 8

Theorem 9: Coconut tree $CT_{m,m}$ is multiply divisor cordial graph.
Proof: Taking $a_1, a_2, \ldots, a_m$ be nodes of path P_m and $b_1, b_2, \ldots, b_m$ be the pendant nodes being adjacent with a_1 in the coconut tree. Taking e_i denote the edge a_ia_{i+1} of P_m for $1 \leq i \leq m - 1$ and a_1b_i for $1 \leq i \leq m$. The coconut tree has $|V| = 2m$ and $|E| = 2m - 1$.
We define labeling function t: V $\rightarrow \{1,2,3, \ldots, 2m\}$, as follows.
$t(a_i) = 2i - 1\,; 1 \leq i \leq m.$
$t(b_i) = 2i;\ 1 \leq i \leq m.$
Then we have $e^*_t(1) = m$ and $e^*_t(0) = m - 1$.
Therefore $|e^*_t(1) - e^*_t(0)| \leq 1$.
Hence, Coconut tree is multiply divisor cordial labeling.
Exemplar 9: A multiply divisor cordial labeling of coconut tree $CT_{4,4}$ to be viewed in Drawing 9.



Drawing 9

IV. CONCLUSION

We instigate here advanced notion of multiply divisor cordial labeling. The multiply divisor cordial labeling is a version of divisor cordial labeling. It is engrossing to inquire graph or families of graphs which are multiply divisor cordial as all the graphs do not acknowledge multiply divisor cordial labeling. It will elevate aspect to the research work in the area tethering two branches - labeling of graphs and number theory. Here, we inquired nine new families of graphs which grant multiply divisor cordial labeling.

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