On the Seidel Energy of Certain Mesh Derived Networks

Sophia Shalini G. B., Anwar Saleh, Dhananjayamurthy B.V

Abstract: *The energy of graph G is defined as the sum of the absolute values of eigenvalues of the adjacency matrix A(G). The manual calculation of energy of graphs consumes several man hours. In this paper, we use MATLAB to generate the Seidel matrix and hence calculate the Seidel energy of some mesh derived networks.*

Keywords: Seidel matrix, Seidel energy, Grid, Cylinder, Torus, Extended grid. **AMS classification.** 05C15,05C50

I. INTRODUCTION

Let G=(V,E) be an undirected simple graph with n vertices $\{v_1, v_2, \ldots, v_n\}$. The adjacency matrix $A(G) = [a_{ij}]$ is defined as

$$a_{ij} = \begin{cases} 1, if \ \ v_i v_j \in E(G) \\ 0, otherwise \end{cases}$$

Let $\lambda_1, \lambda_2, \ldots \lambda_n$ be the eigenvalues of A(G) assumed in the non-increasing order. Since the matrix A(G) is a real symmetric matrix, the eigenvalues of the graph G are real and their sum is equal to zero. The concept of energy of graphs was introduced by Ivan Gutman in the year 1978 [5]. The energy of a graph G is defined as the sum of the absolute values of eigenvalues of A(G). That is,

$$E(G) = \sum_{i=1}^{n} |\lambda_i| \qquad (1)$$

For further facts and the results on the energy of graphs we refer the following papers [3, 6, 7]. The various bounds for the energy of a graph and the characteristics of it have been established in [1, 4]. The Seidel energy of a graph G was defined by Willem.H. Haemers [10] in 2012. Latterly the Laplacian energy of a gridbased networks was calculated by Bharathi Rajan *et al.* [2]. Followed by this, Rajesh Kanna M.R. *et al.* have calculated the maximum degree energy of certain mesh derived networks [8]. Also, Pradeep Kumar *et al.* calculated the Harary energy of some of the networks using MATLAB [9]. From the review of the above-mentioned papers, we calculate the Seidel energy of some of the graph networks such as Grid, Cylinder, Torus and Extended grid.

II. SEIDEL ENERGY

Consider a simple graph G of order n and size m. The Seidel matrix of G is the $n \times n$ matrix denoted by $S(G) = [s_{ij}]$, where

$$s_{ij} = \begin{cases} -1, if \ v_i v_j \in E \\ 1, \ \ if \ \ v_i v_j \notin E \\ 0, \ \ otherwise \end{cases}$$

Let $\mu_1, \mu_2 \ldots \mu_n$ be the eigenvalues of the Seidel matrix S(G). The characteristic polynomial of S(G) denoted by $f_n(G, \mu) = det(\mu I - S(G))$. The Seidel energy of a graph G is defined as
$$SE(G) = \sum_{i=1}^{n} |\mu_i|.$$

III. SEIDEL ENERGY OF SOME STANDARD GRAPH NETWORKS

A. Grid Graph $\mathcal{G}$(m,n)

The grid network $\mathcal{G}$(m,n) is defined as the Cartesian product $P_m \times P_n$ of undirected paths P_m and P_n. The eigenvalues of the graph do not depend on the labeling of its vertices. Nevertheless, we follow a specific way of labeling so that the Seidel matrix has a pattern which is similar for any order. Here we number the vertices in vertical order as shown in Fig.1.

Fig 1. Gridgraph(3,3)
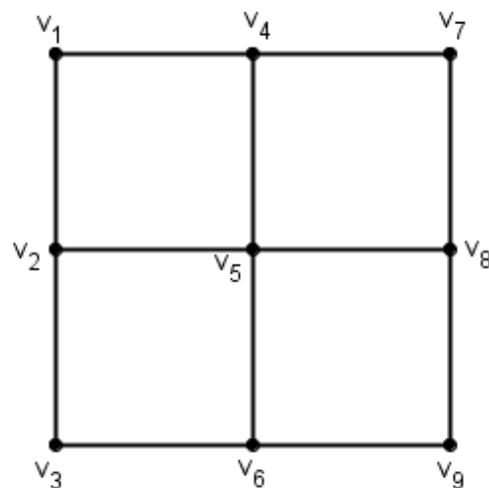
The Seidel matrix of 3x3 grid is given by

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$$S = \begin{bmatrix} B & C & D \\ C & B & C \\ D & C & B \end{bmatrix}, where B = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}, D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

In general, the Seidel matrix of $\mathcal{G}$(m,n) can be written as

$$S = \begin{bmatrix} B & C & D & . & . & ... & D \\ C & B & C & D & . & ... & D \\ D & C & B & C & D & ... & D \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & C \\ D & D & . & . & . & C & B \end{bmatrix}, \text{ where}$$

$$B = \begin{bmatrix} 0 & -1 & 1 & . & . & ... & 1 \\ -1 & 0 & -1 & 1 & . & ... & 1 \\ 1 & -1 & 0 & -1 & 1 & ... & 1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & -1 \\ 1 & 1 & . & . & . & -1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 1 & . & . & . & ... & 1 \\ 1 & -1 & 1 & . & . & ... & 1 \\ 1 & 1 & -1 & . & . & ... & 1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & 1 \\ 1 & 1 & . & . & . & 1 & -1 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 1 & . & . & . & ... & 1 \\ 1 & 1 & 1 & . & . & ... & 1 \\ 1 & 1 & 1 & . & . & ... & 1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & 1 \\ 1 & 1 & . & . & . & 1 & 1 \end{bmatrix}$$

The MATLAB program given below generates the Seidel energy of a grid $\mathcal{G}$(m,n)

```
function energygrid(m,n)
n=3;

m=3;

v=m*n;

A=ones(v);

for a=1:n:v
```

for b=a:a+(n-2)

A(b,b+1)=-1;

A(b+1,b)=-1;

end

end

for a=1:v-n

A(a,a+n)=-1;

A(a+n,a)=-1;

end

for a=1:v

A(a,a)=0;

end

seidelspectrum=eig(A)

seidelenergy=sum(abs(seidelspectrum))

B. Cylinder $\mathcal{C}$(m,n)

The cylinder graph $\mathcal{C}$(m,n) is defined as the Cartesian product $P_m \times C_n$ of undirected path P_m and an undirected cycle C_n.
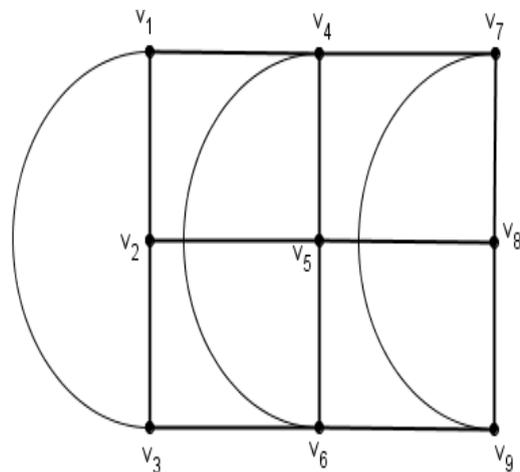

Fig 2. Cylinder(3,3)

The Seidel matrix of $\mathcal{C}$(m,n) can be written as

$$S = \begin{bmatrix} B & C & D & . & . & ... & D \\ C & B & C & D & . & ... & D \\ D & C & B & C & D & ... & D \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & C \\ D & D & . & . & . & C & B \end{bmatrix}, \text{ where}$$

$$B = \begin{bmatrix} 0 & -1 & -1 & . & . & ... & -1 \\ -1 & 0 & -1 & -1 & . & ... & -1 \\ -1 & -1 & 0 & -1 & -1 & ... & -1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & -1 \\ -1 & -1 & . & . & . & -1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 1 & . & . & . & ... & 1 \\ 1 & -1 & 1 & . & . & ... & 1 \\ 1 & 1 & -1 & . & . & ... & 1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & 1 \\ 1 & 1 & . & . & . & 1 & -1 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 1 & . & . & . & ... & 1 \\ 1 & 1 & 1 & . & . & ... & 1 \\ 1 & 1 & 1 & . & . & ... & 1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & 1 \\ 1 & 1 & . & . & . & 1 & 1 \end{bmatrix}$$

The MATLAB program given below generates the Seidel energy of a cylinder $\mathcal{C}(m,n)$

```
function energycylinder(m,n)
n=3;

m=2;

v=m*n;

A=ones(v);

for a=1:n:v

for b=a:a+(n-2)

A(b,b+1)=-1;

A(b+1,b)=-1;

end
```

```
A(a,a+(n-1))=-1;

A(a+(n-1),a)=-1;

end

for a=1:v-n

A(a,a+n)=-1;

A(a+n,a)=-1;

end

for a=1:v

A(a,a)=0;

end

seidelspectrum=eig(A)

seidelenergy=sum(abs(seidelspectrum))
```

C. Torus $\mathcal{T}$(m,n)

The torus network $\mathcal{T}$(m,n) is defined as the Cartesian product $C_m \times C_n$ of undirected cycles C_m and C_n.

The Seidel matrix of $\mathcal{T}$(m,n) can be written as

$$S = \begin{bmatrix} B & C & D & . & . & ... & C \\ C & B & C & D & . & ... & C \\ D & C & B & C & D & ... & C \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & C \\ C & C & . & . & . & C & B \end{bmatrix}, \text{ where}$$

$$B = \begin{bmatrix} 0 & -1 & -1 & . & . & ... & -1 \\ -1 & 0 & -1 & -1 & . & ... & -1 \\ -1 & -1 & 0 & -1 & -1 & ... & -1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & -1 \\ -1 & -1 & . & . & . & -1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 1 & . & . & . & ... & 1 \\ 1 & -1 & 1 & . & . & ... & 1 \\ 1 & 1 & -1 & . & . & ... & 1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & 1 \\ 1 & 1 & . & . & . & 1 & -1 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 1 & . & . & . & ... & 1 \\ 1 & 1 & 1 & . & . & ... & 1 \\ 1 & 1 & 1 & . & . & ... & 1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & 1 \\ 1 & 1 & . & . & . & 1 & 1 \end{bmatrix}$$

The MATLAB program given below generates the Seidel energy of a torus $\mathcal{T}(m,n)$.

```
function energytorus(m,n)

n=4;

m=3;

v=m*n;

A=ones(v);

for a=1:n:v

for b=a:a+(n-2)

A(b,b+1)=-1;

A(b+1,b)=-1;

end

A(a,a+(n-1))=-1;

A(a+(n-1),a)=-1;

end

for a=1:v-n

A(a,a+n)=-1;

A(a+n,a)=-1;

end

for a=1:n
 A(a,a+(v-n))=-1;

A(a+(v-n),a)=-1;

end

for a=1:v

A(a,a)=0;

end

seidelspectrum=eig(A)

seidelenergy=sum(abs(seidelspectrum))
```

D. Extended Grid EX(m,n)

The extended grid $\mathcal{EX}(m,n)$ is constructed by forming every 4-cycle in a network to a complete graph. The order of $\mathcal{EX}(m,n)$ is mn and the size of $\mathcal{EX}(m,n)$ is 4mn-3m-3n+2. The following diagram (Fig.3.) illustrates the numbering chosen.
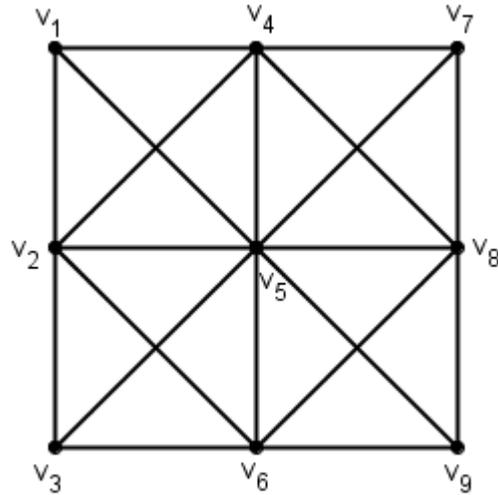

Fig 3. Extended grid $\mathcal{EX}(3,3)$

The general Seidel matrix of $\mathcal{EX}(n,n)$

can be written as

$$S = \begin{bmatrix} B & C & D & . & . & ... & D \\ C & B & C & D & . & ... & D \\ D & C & B & C & D & ... & D \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & C \\ D & D & . & . & . & C & B \end{bmatrix}, \text{ where }$$

$$B = \begin{bmatrix} 0 & -1 & 1 & . & . & ... & 1 \\ -1 & 0 & -1 & 1 & . & ... & 1 \\ 1 & -1 & 0 & -1 & 1 & ... & 1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & -1 \\ 1 & 1 & . & . & . & -1 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} -1 & 1 & . & . & . & ... & 1 \\ -1 & -1 & -1 & . & . & ... & 1 \\ 1 & -1 & -1 & . & . & ... & 1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & -1 \\ 1 & 1 & . & . & . & -1 & -1 \end{bmatrix},$$

$$D = \begin{bmatrix} 1 & 1 & . & . & . & ... & 1 \\ 1 & 1 & 1 & . & . & ... & 1 \\ 1 & 1 & 1 & . & . & ... & 1 \\ . & . & . & . & . & ... & . \\ . & . & . & . & . & ... & 1 \\ 1 & 1 & . & . & . & 1 & 1 \end{bmatrix}$$

The MATLAB program give below generates the Seidel energy of an extended grid $\mathcal{EX}(n,n)$

```
function energyextendedgrid(n,n)

n=3;

v=n*n;

A=ones(v);

for a=1:n:v

for b=a:a+(n-2)

A(b,b+1)=-1;

A(b+1,b)=-1;

end

end

for a=2:n:v-(n-1)

for b=a:a+(n-2)

A(b,b+(n-1))=-1;

A(b+(n-1),b)=-1;

end

end

for a=1:v-n

A(a,a+n)=-1;

A(a+n,a)=-1;

end

for a=1:n:v-(n+1)

for b=a:a+(n-2)
 A(b,b+(n+1))=-1;

A(b+(n+1),b)=-1;

end
end
for a=1:v
```

```
A(a,a)=0;
end
seidelspectrum=eig(A)
seidelenergy=sum(abs(seidelspectrum))
```

IV. RESULT AND DISCUSSION

The energy of a graph G is the sum of the absolute values of the eigenvalues of the adjacency matrix A(G). Graphs whose energy satisfies $E(G)>2(n-1)$ are called hyper-energetic graphs. Graphs whose energy satisfies $E(G)<n$ are called hypo-energetic graphs. The aim of this paper is to determine the Seidel energy of some of the famous mesh derived networks.

1). Grid: Calculating the Seidel energy of grid using the MATLAB code generated above, we find that all the graphs are hyper-energetic.

2). Cylinder: Calculating the Seidel energy of cylinder using the MATLAB code generated above, we find that all the graphs are hyper-energetic and for n>3, $E(\mathcal{G}(m,n))<E(\mathcal{C}(m,n))$.

3). Torus: Calculating the Seidel energy of Torus using the MATLAB code generated above, we find that all the graphs are hyper-energetic.

4). Extended Grid: Calculating the Seidel energy of Extended grid using the MATLAB code generated above, we find that all the graphs are hyper-energetic. We also find that among all the families of graphs mentioned above, the energy of extended grid is maximum.

V. CONCLUSION

In this paper, we have developed a procedure to calculate the eigenvalues of the Seidel matrix and hence the Seidel energy of some of the mesh derived networks.

This is of great significance in studying the characteristics of these networks in a different dimension. Finding the energy of the interconnection networks are under study. In the further research, we find the Seidel matrix for any graph.

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