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# A Cost Effective Mathematical Model for Strategic Workforce Planning

**K. B. Priya Iyer, Fernandes Jeyshree Felix**

***Abstract: Strategic workforce planning has become increasing essential for modern organisations. It is the key approach used by organisation to improve and maintain the capabilities of its workforce. However many experts distinguish between Training and Development, being that training tends to be more closely focused and adapted demands short-term performance concerns, while development tends to be adapted more towards expanding an individual skills for future responsibilities. Enhancement of skills through training programmes leads to different performance results. The success of any training organisation depends on the number of customers who have enrolled for the training programmes. This paper presents novel mathematical models and algorithms to accurately represent and efficiently solve workforce planning problems. The paper emphasizes on the optimum allocation of suitable trainer operational hours and minimise the cost of conducting the programme. A cost function derived is based on the assumption and the minimum cost is obtained satisfying the constraints. Lagrangian multiplier method is employed to find the minimum cost. In this paper, the efficiency of the organisation is analysed by using the relative service efficiency method and cost model is derived.***

***Keyword: A cost function derived is based on the assumption and the minimum cost is obtained satisfying the constraints.***

## I. INTRODUCTION

In the present digitalization world, the success of any industry depends on three factors such as production, workforce, and capacity. The workforce planning is an important process which helps the organisation to determine the optimal workforce composition and decides the need for recruitment. Every organisation prepares a workforce plan or framework for planning staff resources based on organisation goals and mission. An effective plan gives the right number of workers with right skills at right time and right place. This is a not a trivial task for any organisation.

Workforce planning is a key task that ensures the level of employees skill set which leads to productivity in the organisation. As the cost involved in training process is high, the resources allocated to the training must be used efficiently. The capacity of the training facility and number of trainers are the limiting factors of workforce planning task. Organisations trains its workers at right time to meet the market demand with all necessary skill set and it manages the idleness of highly skilled workers. Therefore, workforce planning and facility utilization is one of the most important aspects that organizations.

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* Correspondence Author
**K B Priya Iyer,** Associate Professor, Dept of Computer Science, M.O.P.Vaishnav College for Women (Autonmous), India.
**Dr FernandesJayshree Felix,** Associate Professor & Head Dept. of Mathematics, M.O.P. Vaishnav College for Women (Autonomous), India



The use of mathematical models for planning workforce for manpower planning has increased to a large extent in recent times. This in-turn helps for better manpower planning quantitatively. In respect of organizational management, numerous previous studies have applied Markov chain models in describing the actual manpower needs of an organization or predict the future manpower needs.

This paper provides a Lagrangian multiplier based mathematical model to determine the best possible combination of required capacity, workforce, and lot size for an organisation. The method of Lagrange multipliers is a technique for finding the local maxima and minima of a function subject to equality constraints or conditions. The Lagrange multipliers method is a very efficient for the nonlinear optimization problems and is capable of dealing with both equality constrained and inequality constrained nonlinear optimization problems.

This paper is organised as follows. The various studies on workforce planning is reviewed in Section 2. The conditions, variable parameters, and the proposed mathematical model are presented in Section 3. Section 4 validates and discusses the numerical illustrations of the model. Section 5 concludes the work.

## II. LITERATURE REVIEW

[7] proposes an efficient strategy that focuses on cost and delivery. The paper captures the link between operations strategy with its theory. Meeting with middle managers help to analyse the perceptions of operations strategies. The paper suggests that infrastructural categories are difficult to reproduce and its full benefit is derived only if it is sinks with business strategy. [8] studied the financial costs and cost efficiency of training. The workshop based training and LDHF training approaches are analysed. The paper examines the preferences and perceptions of training. They conducted structured interviews with trainees and found the cost per unit of provider.

[9] allocated the training hours and minimised total quality cost. The cost considered are prevention cost, appraisal cost and failure cost. The model developed includes organisational and individual learning-forgetting approach. [10] analyses the various models available in literature and understood the usage of Kirkpatrick model.

[11] proposes a mathematical model based on linear programming to determine optimal number of trainees. The optimal number of trainees required in various training courses is calculated using the model. The model is applied to real life situation and results max the utilization of the training resources.

The factors like capacity, workforce, and lot size are combined by [12] to determine the ideal ratios of the same for a multi-product, multi-stage, and multi-model production system. The system provides a detailed workforce distribution plan that optimizes the capacity, lot size. The model is based on linear programming to reduce the manufacturing cost. The numerical illustration proves the model efficiency.

Strategic staff planning affects a firm performance[13]. An mathematical optimization model is proposed for solving staff planning. The model takes as input the company's strategies, policies and objectives and optimizing the cost and the staff. The model is tested real time in an multinational firm. [14] explains the use of decision support system (DSS) for efficiently optimising and managing workforce planning. The system solves the various decision levels such as tactical, operational and strategic. [15] helps to grasp the perception of operations strategy in decisions. Two alternative sets of operations strategies like resources and flow are found.

A conceptual model for data driven decisions is presented[16]. A mathematical model and analytical results for two category organisation is found. Mena and variance of the employee time of joining is calculated[17]. [18] aims to study the proportions of staff recruited, promoted and withdrawn and forecast the future requirement. In the paper, academic staff structure of university of Uyo, Nigeria using Markov chain models were studied. The model developed works within the scope that no control on recruitment.

A novel framework for optimising hiring and firing is proposed[19]. The model sets several parameters such as labour rights, technical and managerial constraints. Optimal mode and duration of employment for different workers based on their skills is identified using a dynamic programming algorithm. A lexicographical method to solve the multi-objective programming model for workforce distribution to optimise production planning is proposed[20]. A single-piece flow-based cellular manufacturing industry is considered for simulation. A linear programming model with three objective functions was developed and solved to obtain the optimal solution. For labour intensive industries a new model for work force allocation is developed [21]. The paper focuses on workforce allocation, and a support system is developed from the concepts of holonic manufacturing systems and PROSA reference architecture. The results suggest an improved manufacturing throughput performance.

## III. MODEL DESCRIPTION:

A training organisation has trainers offering training programmes to varied grades of customers of various/same organisation. The arrival of the customers follows a Poisson distribution and the service process follows an exponential distribution. This paper emphasizes on the optimum allocation of suitable trainer operational hours and minimise the cost of conducting the programme. A cost function derived is based on the assumption and the minimum cost is obtained satisfying the constraints. Lagrangian multiplier method is employed to find the minimum cost.

### A. Assumption:

1. Customers arrive for the training programme in a Poisson manner and they are assigned to the training programs in the order of their registration.
2. The training process provided by the different trainers follows an exponential distribution.

### B. Notations:

**n:** number of trainees who are willing to join the training program.
**j:** number of trainers providing training (j = 1,2,…,k).
**λ:** average rate at which the customers arrive per unit time.
**$\mu_j$:** average service rate of the $j^{th}$ trainer per unit time.
**$\Phi_j$:** probability that the $j^{th}$trainer provides training to the customers.
**$x_j$:** actual time(in hours) taken by the $j^{th}$ trainer to offer the training programme.
**$€_j(x_j)$:** mean time spent by the $j^{th}$ trainer with training hour $x_j$.

$$€j(xj) = \frac{1}{\mu j - \frac{\lambda \Phi j}{xj}}$$

$$€j(xj) = \frac{xj}{xj\mu j - \lambda \Phi j}$$

**X:** $X=(x_1, x_2,\ldots,x_k)$,$x_j$the vector

**Tx:** expected time for a customer spent in the organisation with $x_j$ trainer operation hours.

$$Tx = \sum_{j=1}^{k} \Phi j\, €j(xj)$$

**T:** expected time spent by a customer in the organisation

$$T = \sum_{j=1}^{k} \Phi j \,/\, \mu j$$

**R(X):** Relative service efficiency of the trainer,

$R(X) = T/T_X \quad 0<R(X)<1$

**$C_j$:** The training charges of each trainer per unit time

**S:** expected relative service efficiency of the organisation

**C:** Total expenditure of all the training programmes in the organisation

### C. Analysis of the model:

Cost function model can be formulated as a linear programming problem as

$$\min_{x} \quad C = \sum_{j=1}^{k} C_j x_j$$

Subject to

$$R(X) = T/T_X = S,\ x_j \geq 0\ \forall\ j$$

(i.e.)

$$R(X) = \frac{\sum_{j=1}^{k} \frac{\Phi_j}{\mu_j}}{\sum_{j=1}^{k} \frac{\Phi_j x_j}{\Phi_j x_j - \Phi_j \lambda}} = S$$

λ, S, $C_j$, $\mu_j$&$\Phi_j$ are given parameters and X=($x_1$, $x_2$ … $x_n$) are decision variables of the cost function model.

$$F = \sum_{j=1}^{k} C_j x_j + \lambda' \left(S - \frac{T}{T_x}\right)$$, where $\lambda^*$ is the Lagrangian multiplier

Let $X^* = (x_1^*, x_2^* \ldots x_n^*)$ and $\lambda'^*$ be the optimal solutions for the model

$$\frac{\partial F}{\partial x_j} = C_j + \lambda' \left(\frac{T(-\Phi_j^2 \lambda)}{T_x^2 (\mu_j x_j - \Phi_j \lambda)^2}\right) = 0$$

Then $\lambda' \left(\frac{T(\Phi_j^2 \lambda)}{T_x^2 (\mu_j x_j - \Phi_j \lambda)^2}\right) = C_j$

$$(\mu_j x_j - \lambda \Phi_j)^2 = \frac{T\lambda\lambda' \Phi j2}{T_x^2 C_j}$$

$$\mu j x j - \lambda \Phi j = \frac{\sqrt{T\lambda\lambda'}\Phi_j}{T_x \sqrt{C_j}}$$

Therefore, the optimal solution for $x_j^*$

$$x_j^* = \frac{\lambda \Phi_j}{\mu_j} + \frac{1}{\mu_j} \frac{\sqrt{T\lambda\lambda'^*}\Phi_j}{T_x^* \sqrt{C_j}}$$

Also, $\frac{\partial F}{\partial \lambda'} = 0 \Rightarrow S - \frac{T}{T_x} = 0$

$$T_x^* = \frac{T}{S}$$

$$x_j^* = \frac{\Phi_j}{\mu_j}\left[\lambda + \frac{\sqrt{T\lambda\lambda'^*}}{\frac{T}{S}\sqrt{C_j}}\right]$$

$$= \frac{\Phi_j}{\mu_j}\left[\lambda + S \frac{\sqrt{\lambda\lambda'^*}}{\sqrt{TC_j}}\right]$$

Since, $x_j^*$ is dependent on the Lagrangian multiplier $\lambda'^*$ the value of $\lambda'^*$ is found as follows.

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$$\frac{T}{S} = \sum_{j=1}^{k} \frac{\Phi_j}{\mu_j} \left[ \frac{\left[\lambda + S\frac{\sqrt{\lambda\lambda'^*}}{\sqrt{TC_j}}\right]}{\Phi_j S\frac{\sqrt{\lambda'^*\lambda}}{\sqrt{TC_j}}} \right]$$

$$= \sqrt{\frac{\lambda T}{\lambda'^*}} \sum_{j=1}^{k} \frac{\Phi_j}{\mu_j} \left( \frac{1}{s} \sqrt{C_j} \right) + T$$

$$\lambda'^* = \lambda \left[ \sum_{j=1}^{k} \frac{\frac{\Phi_j}{\mu_j}\sqrt{C_j}}{(1-S)\sqrt{T}} \right]^2$$

**D. Result :**

The Langrangian multiplier λ' is the increment of unit relative service efficiency S.

(i.e.)$\frac{\partial F}{\partial S} = \lambda'$

We consider F as a function of S:

Let F(S) = $\sum_{j=1}^{k} C_j x_j^*(S) + \lambda'^*(S)\left[S - \frac{T}{T_x^*(S)}\right]$

We have

$R^*(S) = \frac{T}{T_x^*(S)}$

$\frac{\partial F(S)}{\partial S} = \sum_{j=1}^{k} C_j x_j^*(S) + \lambda'^*(S)\left[S - \frac{T}{T_x^*(S)}\right] + \lambda'^*(S)(1 - R^*(S))$

$\frac{\partial F(S)}{\partial S} = \sum_{j=1}^{k} C_j x_j^*(S) - \lambda'^*(S) R^*(S) + \lambda'^*(S)$

Using the values of $x_j^*(S)$& $\lambda'^*(S)$, on simplification. The desired result $\frac{\partial F(S)}{\partial S} = \lambda(S)$ is obtained.

The Relative efficiency can be found as follows

R=$\frac{T}{Tx}$, Tx = $\sum_{j=1}^{k} \frac{\Phi j x j}{x j \mu j - \lambda \Phi j}$

## IV. NUMERICAL ILLUSTRATION

The following table finds the value of T=0.788 and Tx=0.934for various cost and efficiency is found at 75%.

**Table 1**

| $C_j$ | $\Phi_j$ | $\mu_j$ | λ | T | Tx | S |
|---|---|---|---|---|---|---|
| 50 | 0.1 | 1.2 | 6 | 0.788827838 | 0.934612865 | 0.75 |
| 80 | 0.2 | 2.1 | | | | |
| 60 | 0.1 | 1.3 | | | | |
| 120 | 0.3 | 0.9 | | | | |
| 60 | 0.3 | 1.5 | | | | |

From the below table 2, the cost of all x1*,x2*, x3*,x4* and x5* five trainers are calculated. The optimal solution is arrived at TC=199.3873 where x1*=0.8571, x2*=0.8227, x3*=0.7195, x4*=3.5810, x5*=2.5415 based on given constraints. It is concluded the Total Cost is 199.3873.

**Table 2: Optimised Solution**

| TC | S | $\lambda'^{*}$ | $x_1^*$ | $x_2^*$ | $x_3^*$ | $x_4^*$ | $x_5^*$ |
|---|---|---|---|---|---|---|---|
| 146.7263 | 0.1 | 40.2421 | 0.5266 | 0.5949 | 0.4828 | 2.1304 | 1.3106 |
| 152.8528 | 0.2 | 45.2724 | 0.5580 | 0.6214 | 0.5066 | 2.2766 | 1.4347 |
| 159.5619 | 0.3 | 51.7399 | 0.5905 | 0.6515 | 0.5339 | 2.4435 | 1.5763 |
| 167.2031 | 0.4 | 60.3632 | 0.6304 | 0.6868 | 0.5657 | 2.6388 | 1.7420 |
| 176.0229 | 0.5 | 72.4359 | 0.6785 | 0.7295 | 0.6042 | 2.8747 | 1.9422 |
| 271.8417 | 0.6 | 90.5449 | 1.0868 | 1.0909 | 0.9306 | 4.8747 | 3.6393 |
| **199.3873** | **0.7** | **120.7265** | **0.8227** | **0.8571** | **0.7195** | **3.5810** | **2.5415** |
| 218.7731 | 0.8 | 181.0898 | 0.9517 | 0.9713 | 0.8226 | 4.2130 | 3.0778 |
| 590.175 | 0.9 | 362.1796 | 1.2186 | 1.2076 | 1.0360 | 5.5208 | 4.1875 |

## V. CONCLUSION

The number of hours the trainer is busy depends on the relative service efficiency and on the average arrival rate of the trainees, it is also inversely proportional to the cost. Therefore as λ' increases, R(X) also increases. Thus trainees prefer trainers with good efficiency and so a large number of trainees enrol for the training programme. The cost function model developed can be used in any training organisation to improve the service efficiency of the trainers. Also, the optimum hours to be allocated to trainers for training can be obtained through this model. The total cost obtained will be minimum and the total trainer work hours is proportional to the expected relative service efficiency. This coincides with the natural phenomena and can be applied for any training organisation.

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## AUTHORS PROFILE

**Dr K B Priya Iyer,** is currently Associate Professor, Dept of Computer Science at M.O.P.Vaishnav College for Women(Autonmous), India. She has published more than 26 papers in national and international journals. Her areas of interest includes Location based services, Privacy and Spatial Queries, Spatial Data Mining, Mobile Computing.

**Dr FernandesJayshree Felix,** iscurrently the Head and Associate Professor, Dept. of Mathematics at M.O.P. Vaishnav College for Women(Autonomous), India. She has published morethan 6 papers in national and international journals. Her areas of interest includes Stochastic Modelling, Queuing System, Graph theory and Linear Programming.

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