A Decision-making Problem as an Applications of Intuitionistic Fuzzy Set

HemlataAggarwal, H.D. Arora, Vijay Kumar

Abstract: The fuzzy sets and Intuitionistic fuzzy sets are very useful concepts to elaborate the vagueness in real world problems. The objective of our study is to apply fuzzy set theory and Intuitionistic fuzzy set theory in decision making process. In this paper, we identify in which society a person has to purchase a house in order to fulfil his requirement to maximum extent. In our study we use intuitionistic fuzzy sets to find a relation between the societies and the parameters. And then we find a relation between a person and the parameters. We calculate Normalized Euclidean distance between two Intuitionistic fuzzy sets to make a decision of purchasing house in a society.

Keywords: Fuzzy sets, Intuitionistic fuzzy sets, distance between two intuitionistic fuzzy sets.

I. INTRODUCTION

Term fuzzy set was firstly defined by L.A. Zadeh in 1965[1]. He defined the fuzzy sets for the ambiguity in the real life. He overcomes the problem of confusion about inclusion and exclusion of any element to a set. He defined membership value for each element of a set in between zero and one and the non-membership value is one minus the membership value. The term intuitionistic fuzzy set is the extension of fuzzy sets was defined by K.Atanassov in 1986[2]. He defines membership value, non-membership value as well as the hesitation index. He says that the sum of membership value and non-membership value is lies between zero and one and the hesitation index is one minus the sum of membership value and non-membership value of an element of asset. The hesitation index for a fuzzy set is zero. The fuzzy sets and intuitionistic fuzzy sets are very useful tools in real life application areas like decision making problems, medical problems, engineering problems, control systems and various fields [3]. In this work we use the concept of distance between two intuitionistic fuzzy sets [4-6] in decision making process. The decision has been taken by measuring the smallest Euclidean distance between a person and a society. Many real-world decision-making problems such as academic career of the students, high school determination problem, medical problems, student performance determination of a course, career determinations etc. have been carried out by various researchers by using intuitionistic fuzzy sets [7-10].

II. PRELIMINARIES

In this section, we present the basic definitions.

2.1 DEFINITION

Let X be a non-empty set. A fuzzy set A drawn from X is defined as

\[ A = \{(x, \mu_A(x)) : x \in X\}, \]

where \( \mu_A(x) : X \rightarrow [0,1] \) called the membership function of the fuzzy set A and \( \mu_A(x) \) is called the membership value of the element x [1].

2.2 DEFINITION

[2,3] Let X be a non-empty set. An intuitionistic fuzzy set A drawn from X is defined as

\[ A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}, \]

where \( \mu_A(x) : X \rightarrow [0,1] \) called the membership function of the intuitionistic fuzzy set A and \( \mu_A(x) \) is called the membership value of the element x. \( \nu_A(x) : X \rightarrow [0,1] \) called the non-membership function of the intuitionistic fuzzy set A and \( \nu_A(x) \) is called the non-membership value of the element x.

Furthermore, we have \( \pi_A(x) = 1 - (\mu_A(x) + \nu_A(x)) \) called the hesitation index of x in X.

\( \pi_A(x) \) express the lack of knowledge about inclusion and exclusion of x in X.

2.3 DEFINITION

Let X be a non-empty set and \( P, Q, R \) be intuitionistic fuzzy sets of X. A function \( d : X \times X \rightarrow [0,1] \) is said to be distance measure between two intuitionistic fuzzy sets [6] if it satisfies the following axioms:

1. \( 0 \leq d(P, Q) \leq 1 \)
2. \( d(P, Q) = 0 \text{ if and only if } P = Q \)
3. \( d(P, Q) = d(Q, P) \)
4. If \( P \subseteq Q \text{ and } Q \subseteq R \text{ then } \)
   \[ d(P, R) \geq d(P, Q) + d(Q, R) \]

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* Correspondence Author

Hemlata Aggarwal, Department of Mathematics, Amity University, Noida, UP, India. Email: agz.hem@gmail.com
Dr. H. D Arora, Department of Mathematics, Amity University, Noida, UP, India.
Dr. Vijay Kumar, Department of Mathematics, Manav Rachna International University, Faridabad, Haryana, India.
2.4 DEFINITION

Let \( A = \{(x_i, \mu_A(x_i), \nu_A(x_i), \pi_A(x_i)) : x_i \in X \} \) and \( B = \{(x_i, \mu_B(x_i), \nu_B(x_i), \pi_B(x_i)) : x_i \in X \} \) be two intuitionistic fuzzy sets in \( X \). Szmidt and Kacprzyk [4,5] proposed the following four distance measures between \( A \) and \( B \):

The Hamming distance;

\[
d_H(A, B) = \frac{1}{2} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)
\]

The Euclidean distance;

\[
d_E(A, B) = \left[ \frac{1}{2} \sum_{i=1}^{n} (\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2 \right]^{1/2}
\]

The Normalized Hamming distance;

\[
d_{n-H}(A, B) = \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|)
\]

The Normalized Euclidean distance;

\[
d_{n-E}(A, B) = \left[ \frac{1}{2n} \sum_{i=1}^{n} (\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2 + (\pi_A(x_i) - \pi_B(x_i))^2 \right]^{1/2}
\]

III. APPLICATION

In this section, we present an application of intuitionistic fuzzy set. Many researchers present the applications of intuitionistic fuzzy sets by Normalized Euclidean distance method [11,12]. We also present an application of intuitionistic fuzzy sets. The problem we consider is as follows:

Suppose a set of persons want to purchase a house in a society. Let \( \{S_1, S_2, S_3\} \) be the set of societies and \( \{A_1, A_2\} \) be the set of persons who want to purchase a house in the said societies. The parameters for purchasing a house are wooden work, cheap, green surroundings, parking facility and water supply. Let \( \{E_1, E_2, E_3, E_4, E_5\} \) be the set of parameters where \( E_1 \) stands for wooden work, \( E_2 \) stands for cheap, \( E_3 \) stands for green surroundings, \( E_4 \) stands for parking facility and \( E_5 \) stands for water supply.

The linguistic terms can be converted into a number between zero and one with the help of following table.

<table>
<thead>
<tr>
<th>Linguistic Terms</th>
<th>Membership value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Completely satisfy</td>
<td>1</td>
</tr>
<tr>
<td>Very strongly satisfy</td>
<td>0.9</td>
</tr>
<tr>
<td>Strongly satisfy</td>
<td>0.7</td>
</tr>
<tr>
<td>Satisfy</td>
<td>0.5</td>
</tr>
<tr>
<td>Strongly dissatisfy</td>
<td>0.3</td>
</tr>
<tr>
<td>Very strongly dissatisfy</td>
<td>0.1</td>
</tr>
<tr>
<td>Completely dissatisfy</td>
<td>0</td>
</tr>
</tbody>
</table>

Any value in between the given values represents the opinion on particular parameter is somewhat, in between the given linguistic terms.

Let \( X = \{E_1, E_2, E_3, E_4, E_5\} \) be the set of parameters. We define the intuitionistic fuzzy sets \( S_1 \) over \( X \) as follows:

\[
S_1 = \{(E_1, 0.6, 0.3, 0.1), (E_2, 0.5, 0.2, 0.3), (E_3, 0.5, 0.1, 0.4), (E_4, 0.1, 0.1, 0.8), (E_5, 0.5, 0.4, 0.1)\}
\]

\((E_1, 0.6, 0.3, 0.1) \in S_1 \) represents that the society \( S_1 \) satisfy the parameter \( E_1 \) (wooden work) with membership value 0.6, non-membership value 0.3 and hesitation index 0.1.

In similar way we can define the intuitionistic fuzzy sets \( S_2, S_3, S_4, S_5 \) over \( X \) as follows:

<table>
<thead>
<tr>
<th>( E_1 )</th>
<th>( E_2 )</th>
<th>( E_3 )</th>
<th>( E_4 )</th>
<th>( E_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6, 0.3</td>
<td>0.5, 0.2</td>
<td>0.5, 0.1</td>
<td>0.1, 0.8</td>
<td>0.5, 0.4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( E_2 )</th>
<th>( E_3 )</th>
<th>( E_4 )</th>
<th>( E_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8, 0.1</td>
<td>0.5, 0.1</td>
<td>0.5, 0.2</td>
<td>0.1, 0.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( E_3 )</th>
<th>( E_4 )</th>
<th>( E_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3, 0.1</td>
<td>0.7, 0.2</td>
<td>0.3, 0.4</td>
</tr>
</tbody>
</table>

Also we define the intuitionistic fuzzy sets \( A_1 \) over \( X \) as follows:

\[
A_1 = \{(E_1, 0.5, 0.2, 0.3), (E_2, 0.6, 0.2, 0.2), (E_3, 0.7, 0.1, 0.2), (E_4, 0.1, 0.5, 0.4), (E_5, 0.5, 0.4, 0.1)\}
\]

\((E_1, 0.5, 0.2, 0.3) \in A_1 \) represents that the person \( A_1 \) satisfy the parameter \( E_1 \) (wooden work) with membership value 0.5, non-membership value 0.2 and hesitation index 0.3.
In similar way we can define the intuitionistic fuzzy sets $A_2, A_3, A_4, A_5, A_6$ over $X$ as follows:

$$
\begin{array}{|c|c|c|c|c|}
\hline
 & E_1 & E_2 & E_3 & E_4 \\
\hline
A_1 & (0.5, 0.6) & (0.7, 0.1) & (0.5) \\
0.2 & (0.5) & 0.1 & 0.5 & 0.4 \\
0.3 & (0.2) & (0.2) & 0.4 & 0.1 \\
\hline
A_2 & (0.6, 0.6) & (0.5) & (0.4, 0.2) \\
0.2 & (0.3) & 0.3 & 0.2 \\
0.2 & (0.1) & (0.2) & 0.4 \\
\hline
\end{array}
$$

Now we calculate the Normalized Euclidean distance between $S_i$ and $A_j$, where $i = 1, 2, 3$ and $adj = 1, 2$.

The table given below calculates the Normalized Euclidean distance between $\{S_1, S_2, S_3\}$ and $\{A_1, A_2\}$ by considering the five parameters $\{E_1, E_2, E_3, E_4, E_5\}$.

$$
\begin{array}{|c|c|c|c|}
\hline
 & S_1 & S_2 & S_3 \\
\hline
A_1 & 0.2191 & 0.2966 & 0.2932 \\
A_2 & 0.2098 & 0.1789 & 0.2966 \\
\hline
\end{array}
$$

From the above table we analyze that the distance between the person $A_1$ and the society $S_1$ is lesser than the distance between the person $A_1$ and the society $S_2$ and with society $S_3$ also. It shows that if a person $A_1$ wants to purchase a house in society then he can choose a house in society $S_1$. Similarly, if a person $A_2$ wants to purchase a house in society then he can choose a house in society $S_2$.

IV. RESULT

In this paper, we discussed an application of intuitionistic fuzzy sets in decision making problem. We took an example to explain the process. We took a set of persons who want to purchase a house in a society and a set of shortlisted societies with a set of parameters. From the above calculations we analyzed that the society $S_1$ is more suitable for the person $A_1$ with his parameters and the society $S_2$ is more suitable for the person $A_2$ with his parameters.

<table>
<thead>
<tr>
<th>Person</th>
<th>Suitable Society</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$S_1$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$S_2$</td>
</tr>
</tbody>
</table>

V. CONCLUSION

In this paper we have studied the application of intuitionistic fuzzy sets in decision making. In this paper two persons were randomly selected who wants to purchase a house in shortlisted three societies with some parameters. We calculated the Normalized Euclidean distance between a person and a society. Lesser the distance will provide the better choice for a person to decide in which society he can purchase a house with defined parameters. This method can be applied for various decision-making problems. The above method gives the sensible results.