

# Edge Vertex Prime Labeling of Union of Graphs

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**Abstract:** A graph  $G(p, q)$  is said to be an edge vertex prime labeling if its vertices and edges are labeled with distinct positive numbers not exceeding  $p + q$  such that for any edge  $e = xy$ ,  $f(x)$ ,  $f(y)$  and  $f(xy)$  are pairwise relatively prime. We prove that some class of union of graphs such as  $p + q$  is even for  $G \cup K_{1,n}$ ,  $G \cup P_n$  and  $C_m \cup K_{1,n}$ ,  $C_m \cup P_n$ ,  $C_n \cup C_n$  when  $n \equiv 0, 2 \pmod{3}$ ,  $K_{2,m} \cup C_n$  and one point union of wheel and cycle related graphs are edge vertex prime.

**Keywords:** edge vertex prime labeling, relatively prime, star, path, cycle, one point union of graphs.

**Mathematics Subject Classification:** 05C78

## I. INTRODUCTION

Finite, simple and undirected graphs should alone be considered. A graph  $G$  is an ordered pair  $G = (V, E)$ , where  $V(G)$  refer a finite set of elements called vertices, while  $E(G)$  is a finite set of unordered pairs of vertices called edges. The cardinality of the sets of vertices  $V(G)$  and edges  $E(G)$  is denoted by  $|V(G)|$  and  $|E(G)|$  respectively. For all standard notation and terminology in graph theory, Balakrishnan and Ranganathan [1] are followed. A graph of order  $n$  is *prime* if one can bijectively label its vertices with positive numbers  $1, 2, 3, \dots, n$ , so that any two adjacent vertices are relatively prime. Tout, Dobboucy, Howalla [10], first introduced a first kind of graph labeling called prime labeling and later developed by Roger Entriger. There are several types of labeling for a dynamic survey of various graph labeling problems with extensive bibliography we refer to Gallian [2]. Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  be two simple graphs. The graph  $G = (V(G), E(G))$ , where  $V = V_1 \cup V_2$  and  $E = E_1 \cup E_2$ , is called the *union of  $G_1$  and  $G_2$*  is denoted by  $G_1 \cup G_2$ . For  $n \geq 2$ , an  $n$ -*path* or simply *path* is denoted  $P_n$ , is a connected graph consisting of two vertices, with degree 1 and  $n - 2$  vertices of degree 2. For  $n \geq 3$ , an  $n$ -*Cycle* or Simply *cycle*, denoted  $C_n$ , is a connected graph

consisting of  $n$  vertices, each of degree 2. Note that both  $P_n$  and  $C_n$  have  $n$  vertices while  $P_n$  has  $n - 1$  edges and  $C_n$  has  $n$  edges. An  $n$ -*star* or simply *star*, denoted  $S_n$ , is a graph consisting of one vertex of degree  $n$ , called the *centre* and  $n$  vertices of degree 1. Note that  $S_n$  consists of  $n + 1$  vertices and  $n$  edges. The graph  $W_n^m$  obtained from  $m$  copies of  $W_n$  by identifying their center. Prime labeling is a variant of an edge vertex prime labeling. An edge vertex prime labeling starts with the definition of a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, |V(G) \cup E(G)|\}$  is an *edge vertex prime labeling* if for any edge  $uv \in E(G)$ , we have

$$\gcd(f(u), f(v)) = \gcd(f(u), f(uv)) = \gcd(f(v), f(uv)) = 1$$

. A graph  $G$  which admits an edge vertex prime labeling is called an *edge vertex prime graph*. Jagadesh and Baskar Babujee [3] originated the concept of an edge vertex prime labeling the existence of the same paths, cycles and star  $K_{1,n}$  are proved by them. In [4], they also proved that an edge vertex prime graph for some class of graphs such as generalized star, generalized cycle star,  $p + q$  is even for  $G \cup K_{1,n}$ ,  $G \cup P_n$ ,  $G \cup C_n$ . An edge vertex prime graph of wheel graph, fan graph, friendship graph is Parmer [5] investigated. An edge vertex prime graph can be [6] determined that  $K_{2,n}$ , for every  $n$  and  $K_{3,n}$  for  $n = \{3, 4, \dots, 29\}$ .

In [7], we proved that triangular and rectangular book, butterfly graph with shell, Drums  $D_n$ , Jahangir  $J_{n,3}$  and  $J_{n,4}$  are an edge vertex prime graphs. Also in [8], double star  $B_{m,n}$ , subdivision of  $B_{m,n}$  and  $K_{1,n}$ , comb graph, spider, H-graph of path  $P_n$  and coconut tree are an edge vertex prime graph are determined by us. Some class of graphs such as  $p + q$  is odd for  $G \cup W_n$ ,  $G \cup f_n$ ,  $G \cup F_n$ ,  $p + q$  is even for  $G \cup P_n$ ,  $C_1 \cup K_{1,m} \cup P_n$ , Umbrella graph  $U(m, n)$ , crown graph, union of cycles for  $C_n \cup C_n \cup C_n$ ,  $n \equiv 0 \pmod{3}$ ,  $C_n \cup C_n \cup C_n \cup \dots \cup C_n$ ,  $n \equiv 0 \pmod{5}$  are an edge vertex prime graph.

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In section 2. An edge vertex prime is an investigation of union of some graphs  $p + q$  is even for  $GUK_{1,n}$ ,  $GUP_n$ , and  $C_m UK_{1,n}$ ,  $C_m UP_n$ ,  $C_n UC_n$  when  $n \equiv 0, 2(mod 3)$ ,  $K_{2,m} UC_n$  when  $m$  is even,  $n \equiv 0(mod 3)$  and  $m$  is odd  $n \equiv 0, 1(mod 3)$ .

In section 3, finalise that one point union of graphs such as  $W_n^m$ ,  $n$  is even and  $n = 3, 5, 7, 9$  and cycle  $C_n^m$ ,  $n = 3, 4, 5, 6, 7, 9, 11$  are an edge vertex prime.

## II. UNION OF GRAPHS

We have proved some union of graphs are edge vertex prime in the section below.

**Theorem 2.1** If  $G(p, q)$  has an edge vertex prime graph with  $p + q$  is even, then there exists a graph from the class  $GUK_{1,n}$ ,  $n \geq 1$  that admits an edge vertex prime graph.

Proof. Let  $G(p, q)$  be an edge vertex prime graph when  $p + q$  is even, with bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, p + q\}$  with property that given any edge  $uv \in E(G)$ , the numbers  $f(u), f(v)$  and  $f(uv)$  are pairwise relatively prime. Consider the graph  $K_{1,n}$  with vertex set  $\{u, v_i: 1 \leq i \leq n\}$  and edge set  $\{uv_i: 1 \leq i \leq n\}$ . We define a new graph  $G_1 = GUK_{1,n}$  with vertex set  $V_1 = V(G) \cup \{u, v_i: 1 \leq i \leq n\}$  and edge set  $E_1 = EU\{uv_i: 1 \leq i \leq n\}$ . Define a bijective function  $g: V_1 \cup E_1 \rightarrow \{1, 2, 3, \dots, p + q, p + q + 1, \dots, p + q + 2n + 1\}$

by  $g(v) = f(v)$ , for all  $v \in V(G)$  and  $g(uv) = f(uv)$  for all  $uv \in E(G)$ ,  $g(u) = p$ , where  $p$  is choose the largest prime number in the set  $\{p + q + 1, p + q + 2, \dots, p + q + 2n + 1\}$  and label the edge set  $\{uv_i: 1 \leq i \leq n\}$  by remaining even labels and label the vertex set  $\{v_i: 1 \leq i \leq n\}$  by the remaining odd labels. To analyse that  $G_1$  is an edge vertex prime graph. Earlier,  $G$  is an edge vertex prime graph, it is possible to prove that for any edge  $uv \in E_1$ , which is not in  $G$ , the numbers  $g(u)$ ,  $g(v)$  and  $g(uv)$  are pairwise relatively prime. It is easily proved that, for any edge  $uv \in E_1$ ,  $gcd(g(u), g(v)) = 1$ ,  $gcd(g(u), g(uv)) = 1$ ,  $gcd(g(v), g(uv)) = 1$ . Hence  $G_1 = GUK_{1,n}$ ,  $n \geq 1$  is an edge vertex prime graph.

**Theorem 2.2** If  $G(p, q)$  has an edge vertex prime graph with  $p + q$  is even, then there exists a graph from the class  $GUP_n$  that admits an edge vertex prime graph.

Proof. Let  $G(p, q)$  be an edge vertex prime labeling graph when  $p + q$  is even, with bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, p + q\}$  with property that given any edge  $uv \in E(G)$ , the numbers  $f(u), f(v)$  and  $f(uv)$  are pairwise relatively prime. Consider the graph  $P_n$  with vertex set  $\{u_i: 1 \leq i \leq n\}$  and edge set  $\{u_i u_{i+1}: 1 \leq i \leq n - 1\}$ . We define a new graph  $G_1 = GUP_n$  with vertex set  $V_1 = V \cup \{u_i: 1 \leq i \leq n\}$  and  $E_1 = EU\{u_i u_{i+1}: 1 \leq i \leq n - 1\}$ . Define a bijective function

$$g: V_1 \cup E_1 \rightarrow \{1, 2, 3, \dots, p + q, p + q + 1, \dots, p + q + 2n - 1\}$$

by  $g(v) = f(v)$  for all  $v \in V(G)$  and  $g(uv) = f(uv)$  for all  $uv \in E(G)$ ,  $g(u_i) = p + q - 1 + 2i$  for  $1 \leq i \leq n$ ,  $g(u_i u_{i+1}) = p + q + 2i$  for  $1 \leq i \leq n - 1$ . We have to prove that  $G_1$  is an edge vertex prime labeling.

Earlier,  $G$  is an edge vertex prime labeling, it is enough to prove that for any edge  $uv \in E_1$ , which is not in  $G$ , the numbers  $g(u)$ ,  $g(v)$  and  $g(uv)$  are pairwise relatively prime. Label the vertices and edges of path  $P_n$  is consecutive positive numbers. It is easily verified that, for any edge  $\in E_1$ ,  $gcd(g(u), g(v)) = 1$ ,  $gcd(g(u), g(uv)) = 1$ ,  $gcd(g(v), g(uv)) = 1$ .

So  $G_1 = GUP_n$  is an edge vertex prime graph.

**Theorem 2.3** The disconnected graph  $C_m UK_{1,n}$ ,  $m \geq 3$  is an edge vertex prime graph.

Proof. Consider the disconnected graph  $G = C_m UK_{1,n}$ .

Let  $V(C_m) = \{v_i: 1 \leq i \leq m\}$  and  $V(K_{1,n}) = \{u, u_i: 1 \leq i \leq n\}$ , where  $u$  is the centre of  $K_{1,n}$ ,  $E(C_m) = \{v_1 v_m, v_i v_{i+1}: 1 \leq i \leq m - 1\}$ ,  $E(K_{1,n}) = \{uu_i: 1 \leq i \leq n\}$ , Also,

$|V(G)| = m + n + 1$  and  $|E(G)| = m + n$ . Define a bijective function

$f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 2m + 2n + 1\}$ , by

$f(u) = 1$ ,  $f(u_i) = 2i + 1$  for  $1 \leq i \leq n$ ,

$f(uu_i) = 2i$  for  $1 \leq i \leq n$ ,

$f(v_1) = 2m + 2n + 1$ ,  $f(v_i) = 2n + 2i - 1$  for

$2 \leq i \leq m$ ,  $f(v_i v_{i+1}) = 2n + 2i$  for

$1 \leq i \leq m - 1$ ,

$f(v_1 v_m) = 2m + 2n$ .

Next, we show that the property of an edge vertex prime graph.

For any  $1 \leq i \leq n$ ,

$$\gcd(f(u), f(u_i)) = \gcd(1, 2i + 1) = 1,$$

$$\gcd(f(u), f(uu_i)) = \gcd(1, 2i) = 1,$$

$$\gcd(f(u_i), f(uu_i)) = \gcd(2i + 1, 2i) = 1, \text{ since}$$

they are consecutive positive numbers. For any  $2 \leq i \leq n$ ,

$$\gcd(f(v_i), f(v_{i+1})) = \gcd(2n + 2i - 1, 2n + 2i + 1) = 1,$$

$$\gcd(f(v_i), f(v_i v_{i+1})) = \gcd(2n + 2i - 1, 2n + 2i) = 1,$$

$$\gcd(f(v_{i+1}), f(v_i v_{i+1})) = \gcd(2n + 2i + 1, 2n + 2i) = 1,$$

$$\gcd(f(v_1), f(v_2)) = \gcd(2m + 2n + 1, 2n + 3) = 1,$$

$$\gcd(f(v_1), f(v_1 v_2)) = \gcd(2m + 2n + 1, 2n + 2) = 1,$$

$$\gcd(f(v_2), f(v_1 v_2)) = \gcd(2n + 3, 2n + 2) = 1,$$

$$\begin{aligned} \gcd(f(v_1), f(v_m)) &= \gcd(2m + 2n + 1, 2m + 2n - 1) = 1, \\ \gcd(f(v_1), f(v_1 v_m)) &= \gcd(2m + 2n + 1, 2m + 2n - 1) = 1, \\ \gcd(f(v_m), f(v_1 v_m)) &= \gcd(2m + 2n - 1, 2m + 2n) = 1. \end{aligned}$$

Therefore, for any edge  $uv \in E(G)$ ,  $\gcd(f(u), f(v)) = 1$ ,  $\gcd(f(u), f(uv)) = 1$ ,  $\gcd(f(v), f(uv)) = 1$ . Hence  $G = C_m \cup K_{1,n}$ ,  $m \geq 3$  admits an edge vertex prime graph.

**Theorem 2.4** The disconnected graph  $C_m \cup P_n$ ,  $m \geq 3$  is an edge vertex prime graph.

Proof. Let  $u_1, u_2, \dots, u_m$  be the vertices of cycle  $C_m$  and  $v_1, v_2, \dots, v_n$  be the vertices of path  $P_n$ . Consider  $G = C_m \cup P_n$  be a graph. Then  $V(G) = \{u_i, v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$  and  $E(G) = \{u_1 u_m, u_i u_{i+1} : 1 \leq i \leq m - 1\} \cup \{v_j v_{j+1} : 1 \leq j \leq n - 1\}$ .

Here,  $|V(G)| = m + n$  and  $|E(G)| = m + n - 1$ . Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 2m + 2n - 1\}$  by  $f(u_i) = 2i - 1$  for  $1 \leq i \leq m$ ,  $f(u_i u_{i+1}) = 2i$  for

$$\begin{aligned} 1 \leq i \leq m - 1, & & f(u_1 u_m) &= 2m, \\ f(v_j) &= 2m + 2j - 1 & \text{for } & 1 \leq j \leq n, \\ f(v_j v_{j+1}) &= 2m + 2j & \text{for } & 1 \leq j \leq n - 1. \end{aligned}$$

Next, we prove the property of an edge vertex prime graph.

For any edge  $u_i u_{i+1} \in E(G)$ ,

$$\gcd(f(u_i), f(u_{i+1})) = \gcd(2i - 1, 2i + 1) = 1,$$

$$\gcd(f(u_i), f(u_i u_{i+1})) = \gcd(2i - 1, 2i) = 1,$$

$$\gcd(f(u_{i+1}), f(u_i u_{i+1})) = \gcd(2i + 1, 2i) = 1.$$

For any  $u_1 u_m \in E(G)$ ,

$$\gcd(f(u_1), f(u_m)) = \gcd(1, 2m - 1) = 1,$$

$$\gcd(f(u_1), f(u_1 u_m)) = \gcd(1, 2m) = 1,$$

$$\gcd(f(u_m), f(u_1 u_m)) = \gcd(2m - 1, 2m) = 1.$$

Similarly, the other edges are pairwise relatively prime.

Therefore, for any edge  $uv \in E(G)$ ,

$$\gcd(f(u), f(v)) = 1, \gcd(f(u), f(uv)) = 1,$$

$$\gcd(f(v), f(uv)) = 1. \text{ Hence } C_m \cup P_n, m \geq 3 \text{ has}$$

an edge vertex prime graph.

**Theorem 2.5** The disconnected graph  $C_n \cup C_n$ ,  $n \geq 3$  admits an edge vertex prime graph, where  $n \equiv 0, 2 \pmod{3}$ .

Proof. Let  $G = C_n \cup C_n$  be a graph. Then  $V(G) = \{v_i : 1 \leq i \leq 2n\}$  and

$$E(G) = \{v_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_1 v_n\} \cup \{v_i v_{i+1} : n + 1 \leq i \leq 2n - 1\} \cup \{v_{n+1} v_{2n}\}.$$

Also,  $|V(G)| = 2n$  and  $|E(G)| = 2n$ . Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 4n\}$  by  $f(v_i) = 2i - 1$  for  $1 \leq i \leq 2n$ ,  $f(v_i v_{i+1}) = 2i$  for  $1 \leq i \leq n - 1$ ,  $f(v_1 v_n) = 2n$ ,  $f(v_{n+1} v_{2n}) = 4n$ ,  $f(v_i v_{i+1}) = 2i$  for  $n + 1 \leq i \leq 2n - 1$ . Clearly, for any edge  $uv \in E(G)$ ,  $\gcd(f(u), f(v)) = 1$ ,  $\gcd(f(u), f(uv)) = 1$ ,  $\gcd(f(v), f(uv)) = 1$ .

Hence  $G = C_n \cup C_n$ ,  $n \geq 3$  is an edge vertex prime graph, where  $n \equiv 0, 2 \pmod{3}$ .

**Theorem 2.6** The graph obtained by the duplication of vertex  $v_2$  in path  $P_n$  or cycle  $C_n$  is an edge vertex prime graph.

Proof. Let  $G'$  be the graph obtained by duplicating a vertex  $v_2$  of degree 2 in  $P_n$ . Let  $v'_2$  be the duplication of  $v_2$  in  $G'$ . Then  $V(G') = \{v'_2, v_i : 1 \leq i \leq n\}$  and  $E(G') = \{v_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{v_1 v'_2\} \cup \{v_3 v'_2\}$ .

Here,  $|V(G')| = n + 1$  and  $|E(G')| = n + 1$ . Define a bijective labeling  $f: V(G') \cup E(G') \rightarrow \{1, 2, \dots, 2n + 2\}$  by  $f(v_i) = 2i - 1$  for  $1 \leq i \leq n$ ,  $f(v_i v_{i+1}) = 2i$  for  $1 \leq i \leq n - 1$ ,  $f(v'_2) = 2n + 1$ ,  $f(v_1 v'_2) = 2n$ ,  $f(v_3 v'_2) = 2n + 2$ .

Next, we show that the property of an edge vertex prime graph.

For any  $1 \leq i \leq n - 1$ ,

$$\begin{aligned} \gcd(f(v_i), f(v_{i+1})) &= \gcd(2i - 1, 2i + 1) = 1, \\ \gcd(f(v_i), f(v_i v_{i+1})) &= \gcd(2i - 1, 2i) = 1, \\ \gcd(f(v_{i+1}), f(v_i v_{i+1})) &= \gcd(2i + 1, 2i) = 1, \\ \gcd(f(v_1), f(v'_2)) &= \gcd(1, 2n + 1) = 1, \\ \gcd(f(v_1), f(v_1 v'_2)) &= \gcd(1, 2n) = 1, \\ \gcd(f(v'_2), f(v_1 v'_2)) &= \gcd(2n + 1, 2n) = 1, \\ \gcd(f(v_3), f(v'_2)) &= \gcd(5, 2n + 1) = 1, \\ \gcd(f(v_3), f(v_3 v'_2)) &= \gcd(5, 2n + 2) = 1, \\ \gcd(f(v'_2), f(v_3 v'_2)) &= \gcd(2n + 1, 2n + 2) = 1. \end{aligned}$$

Therefore, for any edge  $uv \in E(G)$ ,  $\gcd(f(u), f(v)) = 1$ ,  $\gcd(f(u), f(uv)) = 1$ ,  $\gcd(f(v), f(uv)) = 1$ . Hence the graph  $G'$  is duplicating a vertex  $v_2$  in  $P_n$  has an edge vertex prime labeling.

Let  $G''$  be the graph obtained by duplication of  $v_2$  of degree 2 in  $C_n$ . Let  $v''_2$  be the duplication of  $v_2$  in  $G''$ . Then  $V(G'') = \{v''_2, v_i; 1 \leq i \leq n\}$ , and  $E(G'') = \{v_i v_{i+1}; 1 \leq i \leq n - 1\} \cup \{v_1 v_n\} \cup \{v_1 v''_2\} \cup \{v_3 v''_2\}$ .

Here,  $V(G'') = n + 1$  and  $E(G'') = n + 2$ . Define a bijective labeling  $f: V(G'') \cup E(G'') \rightarrow \{1, 2, \dots, 2n + 3\}$  by  $f(v_i) = 2i - 1$  for  $1 \leq i \leq n$ ,  $f(v_i v_{i+1}) = 2i$  for  $1 \leq i \leq n - 1$ ,  $f(v_1 v_n) = 2n$ ,  $f(v''_2) = 2n + 2$ ,  $f(v_1 v''_2) = 2n + 1$ ,  $f(v_3 v''_2) = 2n + 3$ . Clearly, for any edge  $uv \in E(G)$ ,  $\gcd(f(u), f(v)) = 1$ ,  $\gcd(f(u), f(uv)) = 1$ ,  $\gcd(f(v), f(uv)) = 1$ . Hence the graph  $G''$  is duplication of  $v_2$  in  $C_n$  has an edge vertex prime graph.

**Theorem 2.7** The disconnected graph  $K_{2,m} \cup C_n$ , ( $n \geq 3, n \equiv 0 \pmod{3}, m$  is even) is an edge vertex prime graph.

Proof. Consider the disconnected graph  $G = K_{2,m} \cup C_n$ , ( $n \geq 3, n \equiv 0 \pmod{3}, m$  is even). Let  $V(K_{2,m}) = \{u_1, u_2\} \cup \{v_i; 1 \leq i \leq m\}$ ,  $V(C_n) = \{w_i; 1 \leq i \leq n\}$  and

$$\begin{aligned} E(K_{2,m}) &= \{u_1 v_i, u_2 v_i; 1 \leq i \leq m\}, \\ E(C_n) &= \{w_1 w_n, w_i w_{i+1}; 1 \leq i \leq n - 1\}. \end{aligned}$$

Also,  $|V(G)| = m + n + 2$  and  $|E(G)| = 2m + n$ .

Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 3m + 2n + 2\}$  as follows.

First, consider  $K_{2,m}$ , we use (Parmer [6] proved that the same technique  $K_{2,m}$  is an edge vertex prime graph for all  $m$  in theorem 2.1). Next, consider  $C_n$ ,  $f(w_i) = 3m + 2i + 1$  for  $1 \leq i \leq n$ ,  $f(w_i w_{i+1}) = 3m + 2i + 2$  for  $1 \leq i \leq n - 1$ .

Clearly, for any edge  $uv \in E(G)$ , the numbers  $f(u), f(v)$  and  $f(uv)$  are pairwise relatively prime. Hence  $G = K_{2,m} \cup C_n$ , ( $n \geq 3, n \equiv 0 \pmod{3}, m$  is even) admits an edge vertex prime graph.

**Theorem 2.8** The disconnected graph  $K_{2,m} \cup C_n$ , ( $n \geq 3, n \equiv 0, 1 \pmod{3}, m$  is odd) is an edge vertex prime graph.

Proof. Similar to the even case, above theorem 2.7, only changes in first cycle, we stated the lowest label by an edge.  $f(w_i) = 3m + 2i$  for  $2 \leq i \leq n$ ,  $f(w_1) = 3m + 2n + 2$ ,  $f(w_i w_{i+1}) = 3m + 2i + 1$  for  $1 \leq i \leq n - 1$ ,  $f(w_1 w_n) = 3m + 2i + 1$ . It is easily verified that, for any edge  $uv \in E(G)$ , the numbers  $f(u), f(v)$  and  $f(uv)$  are pairwise relatively prime.

Hence  $G = K_{2,m} \cup C_n$ , ( $n \geq 3, n \equiv 0, 1 \pmod{3}, m$  is even) admits an edge vertex prime graph.

### III. ONE POINT UNION OF GRAPHS

In this section, we investigate one point union of some graphs are an edge vertex prime.

**Theorem 3.1** One point union of  $m$  copies  $W_n$ , that is,  $W_n^m$  ( $n$  is even, except  $n = 10n - 6, 10n - 2, m \geq 1$  and  $n \geq 1$ ) is an edge vertex prime graph.

Proof. Let  $G = W_n^m$  be a graph. Then  $V(G) = \{v, v_{ij}; 1 \leq i \leq m, 1 \leq j \leq n\}$  and  $E(G) = \{vv_{ij}; 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{v_{ij} v_{i+1}; 1 \leq i \leq m, 1 \leq j \leq n - 1\} \cup \{v_{i1} v_{in}; 1 \leq i \leq m\}$ . Also,  $|V(G)| = mn + 1$  and  $|E(G)| = 2mn$ . Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 3mn + 1\}$  by  $f(v) = 1$ ,

$$f(v_{ij}) = \begin{cases} 3n(i-1) + 3j; & j = 1,3,5, \dots, n-1 \\ 3n(i-1) + 3j - 1; & j = 2,4,6, \dots, n \end{cases} \quad f(v_{i1}v_{ij}) = 3(10n-6)(i-1) + 3j + 1, \quad j = 1,3,5, \dots, n-1$$

$$f(vv_{ij}) = \begin{cases} 3n(i-1) + 3j - 1; & j = 1,3,5, \dots, n-1 \\ 3n(i-1) + 3j; & j = 2,4,6, \dots, n \end{cases}$$

$$f(v_{ij}v_{ij+1}) = 3n(i-1) + 3j + 1, \quad j = 1,3,5, \dots, n-1$$

$$f(v_{i1}v_{ij}) = 3n(i-1) + 3j + 1, \quad j = n.$$

It is easily verified that, for any edge  $uv \in E(G)$ ,  $\gcd(f(u), f(v)) = 1$ ,  $\gcd(f(u), f(uv)) = 1$ ,  $\gcd(f(v), f(uv)) = 1$ . Hence  $G = W_n^m$  ( $n$  is even, except  $n = 10n - 6, 10n - 2, m \geq 1$  and  $n \geq 1$ ) admits an edge vertex prime graph.

**Theorem 3.2** One point union of  $W_{10n-6}^m$ ,  $m \geq 1$  and  $n \geq 1$  is an edge vertex prime graph.

Proof. Let  $G = W_{10n-6}^m$  be a graph. Then

$$V(G) = \{v, v_{ij} : 1 \leq i \leq m, 1 \leq j \leq 10n - 6\}$$

and

$$E(G) = \{vv_{ij} : 1 \leq i \leq m, 1 \leq j \leq 10n - 6\} \cup \{v_{ij}v_{ij+1} : 1 \leq i \leq m, 1 \leq j \leq 10n - 7\} \cup \{v_{i1}v_{i(10n-6)} : 1 \leq i \leq m\}$$

$$|V(G)| = m(10n - 6) + 1 \quad \text{Also, and}$$

$$|E(G)| = 2m(10n - 6).$$

Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 3m(10n - 6) + 1\}$

by  $f(v) = 1$ ,

$$f(v_{ij}) = \begin{cases} 3(10n - 6)(i - 1) + 3j; & j = 1,3,5, \dots, 10n - 7 \\ 3(10n - 6)(i - 1) + 3j - 1; & j = 2,4,6, \dots, 10n - 6 \end{cases}$$

$$f(vv_{ij}) = \begin{cases} 3(10n - 6)(i - 1) + 3j - 1; & j = 1,3,5, \dots, 10n - 7 \\ 3(10n - 6)(i - 1) + 3j; & j = 2,4,6, \dots, 10n - 6 \end{cases}$$

$$f(v_{ij}v_{ij+1}) = 3(10n - 6)(i - 1) + 3j + 1, \quad j = 1,2,3, \dots, 10n - 7$$

Consider the following cases.

Case 1.  $m \not\equiv 2 \pmod{5}$

$$f(v_{ij}v_{ij+1}) = 3(10n - 6)(i - 1) + 3j + 1, \quad j = 1,2,3, \dots, 10n - 7$$

Case 2.  $m \equiv 2 \pmod{5}$

$$f(v_{ij}v_{ij+1}) = \begin{cases} 3(10n - 6)(i - 1) + 3j + 1; & j = 1,2,3, \dots, 10n - 8 \\ 3(10n - 6)(i - 1) + 3(j + 1) + 1; & j = 10n - 7 \end{cases}$$

$$f(v_{i1}v_{ij}) = 3(10n - 6)(i - 1) + 3(j - 1) + 1, \quad j = 10n - 6$$

Clearly, for any edge  $uv \in E(G)$ ,  $\gcd(f(u), f(v)) = 1$ ,  $\gcd(f(u), f(uv)) = 1$ ,  $\gcd(f(v), f(uv)) = 1$ . Hence  $G = W_{10n-6}^m$ ,  $m \geq 1$  and  $n \geq 1$  admits an edge vertex prime.

**Theorem 3.3** One point union of  $W_{10n-2}^m$ ,  $m \geq 1$  and  $n \geq 1$  is an edge vertex prime graph.

Proof. Let  $G = W_{10n-2}^m$  be a graph. Then

$$V(G) = \{v, v_{ij} : 1 \leq i \leq m, 1 \leq j \leq 10n - 2\}$$

$$E(G) = \{vv_{ij} : 1 \leq i \leq m, 1 \leq j \leq 10n - 2\} \cup \{v_{ij}v_{ij+1} : 1 \leq i \leq m, 1 \leq j \leq 10n - 3\} \cup \{v_{i1}v_{i(10n-2)} : 1 \leq i \leq m\}$$

$$|V(G)| = m(10n - 2) + 1$$

$$\text{and } |E(G)| = 2m(10n - 2).$$

Define a bijective function

$$f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 3m(10n - 2) + 1\}$$

by  $f(v) = 1$ ,

$$f(v_{ij}) = \begin{cases} 3(10n - 2)(i - 1) + 3j; & j = 1,3,5, \dots, 10n - 3 \\ 3(10n - 2)(i - 1) + 3j - 1; & j = 2,4,6, \dots, 10n - 4 \end{cases}$$

$$f(vv_{ij}) = \begin{cases} 3(10n - 2)(i - 1) + 3j - 1; & j = 1,3,5, \dots, 10n - 3 \\ 3(10n - 2)(i - 1) + 3j; & j = 2,4,6, \dots, 10n - 2 \end{cases}$$

$$f(v_{ij}v_{ij+1}) = 3(10n - 2)(i - 1) + 3j + 1, \quad j = 1,2,3, \dots, 10n - 2$$

Consider the following cases.

Case 1.  $m \not\equiv 4 \pmod{5}$

$$f(v_{ij}v_{ij+1}) = 3(10n - 2)(i - 1) + 3j + 1, \quad j = 1, 2, 3, \dots, 10n - 3$$

$$f(vv_{ij}) = \begin{cases} 9(i - 1) + 3j - 1; & j = 1, 3 \\ 9(i - 1) + 3j & j = 2 \end{cases}$$

$$f(v_{i1}v_{ij}) = 3(10n - 2)(i - 1) + 3j + 1, \quad j = 10n - 2 \quad f(v_{ij}v_{ij+1}) = \begin{cases} 9(i - 1) + 3j + 1; & j = 1, 2, \\ 9(i - 1) + 3(j + 1); & j = 2 \end{cases}$$

Case 2.  $m \equiv 4 \pmod{5}$

$$f(v_{ij}v_{ij+1}) = \begin{cases} 3(10n - 2)(i - 1) + 3j + 1; & j = 1, 2, 3, \dots, 10n - 4 \\ 3(10n - 2)(i - 1) + 3(j + 1) - 1; & j = 2, 4, 6, \dots, 10n - 3 \end{cases}$$

$$f(v_{i1}v_{ij}) = 3(10n - 2)(i - 1) + 3(j - 1) + 1, \quad j = 10n - 2$$

$$f(v_{i1}v_{ij}) = 9(i - 1) + 3j + 1, \quad j = 3.$$

It is easily verified, for any edge  $uv \in E(G)$ , the numbers  $f(u)$ ,  $f(v)$  and  $f(uv)$  are pairwise relatively prime. Hence  $G = W_3^m$  admits an edge vertex prime graph.

**Theorem 3.5** One point union of  $m$  copies  $W_5$  is an edge vertex prime graph.

Proof. Let  $G = W_5^m$  be a graph. Then  $V(G) = \{v, v_{ij} : 1 \leq i \leq m, 1 \leq j \leq 5\}$  and  $E(G) = \{vv_{ij} : 1 \leq i \leq m, 1 \leq j \leq 5\}$

Clearly, for any edge  $uv \in E(G)$ ,  $\gcd(f(u), f(v)) = 1$ ,  $\gcd(f(u), f(uv)) = 1$ ,  $\gcd(f(v), f(uv)) = 1$ . Hence  $G = W_{10n-2}^m$ ,  $m \geq 1$  and  $n \geq 1$  admits an edge vertex prime graph.

$$\cup \{v_{ij}v_{ij+1} : 1 \leq i \leq m, 1 \leq j \leq 4\} \cup \{v_{i1}v_{i5} : 1 \leq i \leq m\}$$

. Also,  $|V(G)| = 5m + 1$  and  $|E(G)| = 10m$ .

**Theorem 3.4** One point union of  $m$  copies  $W_3$  is an edge vertex prime graph.

Proof. Let  $G = W_3^m$  be a graph. Then  $V(G) = \{v, v_{ij} : 1 \leq i \leq m, 1 \leq j \leq 3\}$  and  $E(G) = \{vv_{ij} : 1 \leq i \leq m, 1 \leq j \leq 3\} \cup \{v_{ij}v_{ij+1} : 1 \leq i \leq m, 1 \leq j \leq 2\} \cup \{v_{i1}v_{i3} : 1 \leq i \leq m\}$

Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 15m + 1\}$  by  $f(v) = 1$ .

. Also,  $|V(G)| = 3m + 1$  and  $|E(G)| = 6m$ . Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 9m + 1\}$  by  $f(v) = 1$ . Consider  $i^{th}$  copy of the following cases.

Case 1. Even number of copies, that is,  $i = 2, 4, 6, \dots$

$$f(v_{ij}) = \begin{cases} 9(i - 1) + 3j - 1; & j = 1, 3 \\ 9(i - 1) + 3j & j = 2 \end{cases}$$

$$f(v_{ij}) = \begin{cases} 15(i - 1) + 3j - 1; & j = 1, 3, 5 \\ 15(i - 1) + 3j & j = 2, 4 \end{cases}$$

$$f(vv_{ij}) = \begin{cases} 9(i - 1) + 3j; & j = 1, 3 \\ 9(i - 1) + 3j - 1; & j = 2 \end{cases}$$

$$f(vv_{ij}) = \begin{cases} 15(i - 1) + 3j; & j = 1, 3, 5 \\ 15(i - 1) + 3j - 1; & j = 2, 4 \end{cases}$$

$$f(v_{ij}v_{ij+1}) = 9(i - 1) + 3j + 1, \quad j = 1, 2$$

$$f(v_{ij}v_{ij+1}) = 15(i - 1) + 3j + 1, \quad j = 1, 2, 3, 4.$$

$$f(v_{i1}v_{ij}) = 9(i - 1) + 3j + 1, \quad j = 3.$$

$$f(v_{i1}v_{ij}) = 15(i - 1) + 3j + 1, \quad j = 5.$$

Case 2. Odd number of copies, that is,  $i = 1, 3, 5, \dots$

$$f(v_{ij}) = \begin{cases} 9(i - 1 + 3j); & j = 1 \\ 9(i - 1) + 3j - 1; & j = 2 \\ 9(i - 1) + 3j - 2; & j = 3 \end{cases}$$

Case 2. Odd number of copies, that is,  $i = 1, 3, 5, \dots$

$$f(v_{ij}) = \begin{cases} 15(i - 1 + 3j); & j = 1, 3 \\ 15(i - 1) + 3j - 1; & j = 2, 4 \\ 15(i - 1) + 3j - 2; & j = 5 \end{cases}$$

$$f(vv_{ij}) = \begin{cases} 15(i - 1) + 3j - 1; & j = 1, 3, 5 \\ 15(i - 1) + 3j & j = 2, 4 \end{cases}$$

$$f(v_{ij}v_{ij+1}) = \begin{cases} 15(i - 1) + 3j + 1; & j = 1, 2, 3 \\ 15(i - 1) + 3j + 3; & j = 4 \end{cases}$$

$$f(v_{i1}v_{ij}) = 15(i - 1) + 3j + 1, \quad j = 5.$$

Therefore, for any edge  $uv \in E(G)$ ,  $\gcd(f(u), f(v)) = 1$ ,  $\gcd(f(u), f(uv)) = 1$ ,  $\gcd(f(v), f(uv)) = 1$ . Hence  $G = W_5^m$  admits an edge vertex prime graph.

**Theorem 3.6** One point union of  $m$  copies  $W_7$  is an edge vertex prime graph.

Proof. Let  $G = W_7^m$  be a graph. Then  $V(G) = \{v, v_{ij} : 1 \leq i \leq m, 1 \leq j \leq 7\}$  and  $E(G) = \{vv_{ij} : 1 \leq i \leq m, 1 \leq j \leq 7\} \cup \{v_{ij}v_{ij+1} : 1 \leq i \leq m, 1 \leq j \leq 6\} \cup \{v_{i1}v_{i7} : 1 \leq i \leq m\}$

. Also,  $|V(G)| = 7m + 1$  and  $|E(G)| = 14m$ . Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 21m + 1\}$  by  $f(v) = 1$ . Consider  $i^{th}$  copy of the following cases.

Case 1. Even number of copies, that is,  $i = 2, 4, 6, \dots$

$$f(v_{ij}) = \begin{cases} 21(i - 1) + 3j - 1; & j = 1, 3, 5, 7 \\ 21(i - 1) + 3j & j = 2, 4, 6 \end{cases}$$

$$f(vv_{ij}) = \begin{cases} 21(i - 1) + 3j; & j = 1, 3, 5, 7 \\ 21(i - 1) + 3j - 1; & j = 2, 4, 6 \end{cases}$$

Subcase 1a.  $m \not\equiv 4 \pmod{10}$

$$f(v_{ij}v_{ij+1}) = 21(i - 1) + 3j + 1, \quad j = 1, 2, 3, 4, 5, 6$$

$$f(v_{i1}v_{ij}) = 21(i - 1) + 3(j + 1) + 1, \quad j = 7.$$

Subcase 1b.  $m \equiv 4 \pmod{10}$

$$f(v_{ij}v_{ij+1}) = 21(i - 1) + 3j + 1, \quad j = 1, 2, 3, 4, 5$$

$$f(v_{ij}v_{ij+1}) = 21(i - 1) + 3(j + 1) + 1, \quad j = 6$$

$$f(v_{i1}v_{ij}) = 21(i - 1) + 3j - 2, \quad j = 7.$$

Case 2. Odd number of copies, that is,  $i = 1, 3, 5, \dots$

$$f(v_{ij}) = \begin{cases} 21(i - 1) + 3j; & j = 1, 3, 5 \\ 21(i - 1) + 3j - 1; & j = 2, 4, 6 \\ 21(i - 1) + 3j - 2; & j = 7 \end{cases}$$

$$f(vv_{ij}) = \begin{cases} 21(i - 1) + 3j - 1; & j = 1, 3, 5, 7 \\ 21(i - 1) + 3j & j = 2, 4, 6 \end{cases}$$

$$f(v_{ij}v_{ij+1}) = 21(i - 1) + 3j + 1, \quad j = 1, 2, 3, 4, 5$$

$$f(v_{i1}v_{ij}) = 21(i - 1) + 3j + 3, \quad j = 6.$$

$$f(v_{i1}v_{i7}) = 21(i - 1) + 3j + 1, \quad j=7$$

Clearly, for any edge  $uv \in E(G)$ , the numbers  $f(u), f(v)$  and  $f(uv)$  are pairwise relatively prime. Hence  $G = W_7^m$  admits an edge vertex prime graph.

**Theorem 3.7** One point union of  $m$  copies  $W_9$  is an edge vertex prime graph.

Proof. Let  $G = W_9^m$  be a graph. Then  $V(G) = \{v, v_{ij} : 1 \leq i \leq m, 1 \leq j \leq 9\}$  and

$$E(G) = \{vv_{ij} : 1 \leq i \leq m, 1 \leq j \leq 9\} \cup \{v_{ij}v_{ij+1} : 1 \leq i \leq m, 1 \leq j \leq 8\} \cup \{v_{i1}v_{i9} : 1 \leq i \leq m\}$$

. Also,  $|V(G)| = 9m + 1$  and  $|E(G)| = 18m$ .

Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 27m + 1\}$  by  $f(v) = 1$ . Consider  $i^{th}$  copy of the following cases.

Case 1. Even number of copies, that is,  $i = 2, 4, 6, \dots$

$$f(v_{ij}) = \begin{cases} 27(i - 1) + 3j - 1; & j = 1, 3, 5, 7, 9 \\ 27(i - 1) + 3j & j = 2, 4, 6, 8 \end{cases}$$

$$f(vv_{ij}) = \begin{cases} 27(i - 1) + 3j; & j = 1, 3, 5, 7, 9 \\ 27(i - 1) + 3j - 1; & j = 2, 4, 6, 8 \end{cases}$$

$$f(v_{ij}v_{ij+1}) = 27(i - 1) + 3j + 1, \quad j = 1, 2, 3, 4, 5, 6, 7, 8$$

$$f(v_{i1}v_{ij}) = 27(i - 1) + 3j + 1, \quad j = 9.$$

Case 2. Odd number of copies, that is,  $i = 1, 3, 5, \dots$

$$f(v_{ij}) = \begin{cases} 27(i - 1) + 3j; & j = 1, 3, 5, 7 \\ 27(i - 1) + 3j - 1; & j = 2, 4, 6, 8 \\ 27(i - 1) + 3j - 2; & j = 9 \end{cases}$$

$$f(vv_{ij}) = \begin{cases} 27(i - 1) + 3j - 1; & j = 1, 3, 5, 7 \\ 27(i - 1) + 3j & j = 2, 4, 6, 8 \end{cases}$$

$$f(v_{ij}v_{ij+1}) = 27(i - 1) + 3j + 1, \quad j = 1, 2, 3, 4, 5, 6, 7$$

Subcase 2a.  $m \not\equiv 7 \pmod{10}$

$$f(vv_{ij}) = 27(i - 1) + 3j - 1, \quad j = 9,$$

$$f(v_{ij}v_{ij+1}) = 27(i - 1) + 3j + 1, \quad j = 8,$$

$$f(v_{i1}v_{ij}) = 27(i - 1) + 3j - 2, \quad j = 9.$$

Subcase 2b.  $m \equiv 7 \pmod{10}$

$$f(vv_{ij+1}) = 27(i - 1) + 3j + 1, \quad j = 9$$

$$f(v_{ij}v_{ij+1}) = 27(i - 1) + 3(j + 1), \quad j = 8$$

$$f(v_{i1}v_{ij}) = 27(i - 1) + 3j - 1, j = 9.$$

Clearly, for any edge  $uv \in E(G)$ , the numbers  $f(u), f(v)$  and  $f(uv)$  are pairwise relatively prime. Hence  $G = W_9^m$  admits an edge vertex prime graph.

**Theorem 3.8** One point union of  $m$  copies of  $C_n^m$ ,  $n = 3, 5, 7, 9, 11$  is an edge vertex prime graph.

Proof. Let  $G = C_n^m$ , ( $n = 3, 5, 7, 9, 11$ ) be a graph. Then  $V(G) = \{v, v_{ij}: 1 \leq i \leq m, 1 \leq j \leq n - 1\}$  and  $E(G) = \{vv_{i1}, vv_{i(n-1)}: 1 \leq i \leq m\} \cup \{v_{ij}v_{ij+1}: 1 \leq i \leq m, 1 \leq j \leq n - 2\}$ . Also,  $|V(G)| = m(n - 1) + 1$  and  $|E(G)| = mn$ .

Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 2mn - m + 1\}$  by  $f(v) = 1$ . Consider  $i^{th}$  copy of the following cases.

Case 1. Odd number of copies, that is,  $i = 1, 3, 5, \dots$   
 $f(v_{ij}) = 2n(i - 1) + 2(j + 1) - i, j = 1, 2, 3, \dots, n - 1$   
 $f(v_{ij}v_{ij+1}) = 2n(i - 1) + 2(j + 2) - (i + 1), j = 1, 2, 3, \dots, n - 2$   
 $f(vv_{i1}) = (2n - 1)i - (2n - 3),$   
 $f(vv_{i(n-1)}) = (2n - 1)i + 1.$

Case 2. Even number of copies, that is  $i = 2, 4, 6, \dots$   
 $f(v_{ij}) = 2n(i - 1) + 2(j + 1) - (i + 1), j = 1, 2, 3, \dots, n - 1.$   
 $f(v_{ij}v_{ij+1}) = 2n(i - 1) + 2(j + 2) - (i + 2), j = 1, 2, 3, \dots, n - 2.$

Consider the following subcases.

Subcase 2a. Consider  $n = 3, 5, 9$ , if we take  $n = 7$ , then  $m \not\equiv 2 \pmod{6}$  and if we take  $n = 11$ , then  $m \not\equiv 4 \pmod{10}$ .

$$f(vv_{i1}) = (2n - 1)i + 1,$$

$$f(vv_{i(n-1)}) = (2n - 1)i.$$

Subcase 2b. If we take  $n = 7$ , then  $m \equiv 2 \pmod{6}$  and if we take  $n = 11$ , then  $m \equiv 4 \pmod{10}$ .

$$f(vv_{i1}) = (2n - 1)i,$$

$$f(vv_{i(n-1)}) = (2n - 1)i + 1.$$

Clearly, for any edge  $uv \in E(G)$ ,  $\gcd(f(u), f(v)) = 1$ ,  $\gcd(f(u), f(uv)) = 1$ ,  $\gcd(f(v), f(uv)) = 1$ . Hence  $G = C_n^m$  admits an edge vertex prime graph.

**Theorem 3.9** One point union of  $m$  copies of  $C_4$  is an edge vertex prime graph.

Proof. Let  $G = C_4^m$  be a graph. Then  $V(G) = \{v, v_{ij}: 1 \leq i \leq m, 1 \leq j \leq 3\}$  and  $E(G) = \{vv_{i1}, vv_{i3}: 1 \leq i \leq m\} \cup \{v_{ij}v_{ij+1}: 1 \leq i \leq m, 1 \leq j \leq 2\}$ . Also,  $|V(G)| = 3m + 1$  and  $|E(G)| = 4m$ .

Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 7m + 1\}$  by  $f(v) = 1$

Consider  $i^{th}$  copy of the following cases.

Case 1. Odd number of copies, that is,  $i = 1, 3, 5, \dots$

$$f(v_{ij}) = 8(i - 1) + 2(j + 1) - i, j = 1, 2, 3$$

$$f(v_{ij}v_{ij+1}) = 8(i - 1) + 2(j + 2) - (i + 1), j = 1, 2$$

$$f(vv_{i1}) = 7i - 5, f(vv_{i3}) = 7(i + 1) + 1.$$

Case 2. Even number of copies, that is,  $i = 2, 4, 6, \dots$

$$f(v_{ij}) = 8(i - 1) + 2(j + 1) - (i + 1), j = 1, 2, 3$$

$$f(v_{ij}v_{ij+1}) = 8(i - 1) + 2(j + 2) - (i + 2), j = 1, 2$$

$$f(vv_{i1}) = 7(i - 1) + 1, f(vv_{i3}) = 7i.$$

Therefore, for any edge  $uv \in E(G)$ , the numbers  $f(u), f(v)$  and  $f(uv)$  are pairwise relatively prime. Hence  $G = C_4^m$  admits an edge vertex prime graph.

**Theorem 3.10** One point union of  $m$  copies of  $C_6$  is an edge vertex prime graph.

Proof. Let  $G = C_6^m$  be a graph. Then  $V(G) = \{v, v_{ij}: 1 \leq i \leq m, 1 \leq j \leq 5\}$  and

$$E(G) = \{vv_{i1}, vv_{i5}: 1 \leq i \leq m\} \cup \{v_{ij}v_{ij+1}: 1 \leq i \leq m, 1 \leq j \leq 4\}$$

Also,  $|V(G)| = 5m + 1$  and  $|E(G)| = 6m$ .

Define a bijective function  $f: V(G) \cup E(G) \rightarrow \{1, 2, \dots, 11m + 1\}$  by  $f(v) = 1$ . Consider  $i^{th}$  copy of the following cases.

Case 1. Odd number of copies, that is,  $i = 1, 3, 5, \dots$

$$f(v_{ij}) = 12(i - 1) + 2(j + 1) - i, j = 1, 2, 3, 4, 5$$

$$f(v_{ij}v_{ij+1}) = 12(i - 1) + 2(j + 2) - (i + 1), j = 1, 2, 3, 4$$

$$f(vv_{i1}) = 11i - 9, f(vv_{i5}) = 11i + 1.$$

Case 2. Even number of copies, that is,  $i = 2, 4, 6, \dots$

$$f(v_{ij}) = 12(i - 1) + 2(j + 1) - (i + 1), j = 1, 2, 3, 4, 5$$

$$f(v_{ij}v_{ij+1}) = 12(i - 1) + 2(j + 2) - (i + 2), j = 1, 2, 3, 4$$

$$f(vv_{i1}) = 11i + 1, f(vv_{i5}) = 11i.$$

Subcase 2a.  $m \not\equiv 4 \pmod{10}$

$$f(vv_{i1}) = 11i + 1, f(vv_{i5}) = 11i.$$

Subcase 2b.  $m \equiv 4 \pmod{10}$

$$f(vv_{i1}) = 11i, f(vv_{i5}) = 11i + 1.$$

Therefore, for any edge  $uv \in E(G)$ ,  $\gcd(f(u), f(v)) = 1$ ,  $\gcd(f(u), f(uv)) = 1$ ,  $\gcd(f(v), f(uv)) = 1$ . Hence  $G = C_6^m$  admits an edge vertex prime graph.



#### IV. CONCLUSION

We proved that if  $G(p, q)$  has an edge vertex prime graph with  $p + q$  is even, then there exists a graph from the class  $GUP_n$  that admits an edge vertex prime graph. One point union of  $m$  copies  $W_5$  is an edge vertex prime graph. One point union of  $m$  copies  $W_7$  is an edge vertex prime graph. One point union of  $m$  copies  $W_9$  is an edge vertex prime graph. One point union of  $m$  copies of  $C_n^m$ ,  $n = 3, 5, 7, 9, 11$  is an edge vertex . One point union of  $m$  copies of  $C_6$  is an edge vertex prime graph.

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