Elementary Ideals in Tsemi-Rings Andn(A)-Tsemi-Ring

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"ABSTRACT: In this paper" we will introduce l-elementary ideals, lt-elementary ideals, r-elementary ideals and elementary ideals in TSemi-rings and its properties. Also, we will discuss special type of Tsemi-rings i.e. N(I)-TSemi-ring.

Key words: Elementary ideal in T-Semi-rings, I-potent element, I-potent Γ-ideal, I-divisor,N(I)-Tsemi-ring.

I. INTRODUCTION

"Derric Henry Lehmer, An American Mathematician established ternary algebraic theory. Ideal theory in ternary semigroup was introduced in 1990 by Federic Sioson. The theory of ternary and semi heaps were developed by Santiago in 1990. Shabir and Khan learnt prime ideals and prime one side ideals in semi group in the year 2008. Shabir and Bashir initiated Prime ideals in ternary semigroups.

|Succeeding that W.G.Lister read about ternary semi rings.

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About T.K.Dutta and S.Kar initiated and considered some properties of ternary semirings which is generalization of ternary semi rings.

Dheena, P.Manvisan.S prepared a study on p-prime and small p-prime ideals in semi rings. S.Kar proved on Quasi ideals and bi-ideals in ternary semi rings. The same concept was established by Sarala, A.Anjaneyulu and D.Madhusudhanarao to ternary semi groups. In the year 2014 D.Madhusudhanarao and G.Srinivasarao investigated and read about classification of ternary semi rings and some particular elements in a ternary semirings, investigated structure of certain ideals in ternary semi rings."

II. BASIC RESULTS

For basic results, "we refer to the papers in the references".

III. I.ELEMENTARY IDEALS IN TSEMI-RINGS

Def "1.1 : An ideal I of a" Tsemi-ring T "is" said to be a ***l* (*lt*, r) elementary *ideal*** is

(i) if U_1, U_2, U_3 are "three ideals of T such that" $U_1U_2U_3 \subseteq I$ and "$U_2 \nsubseteq I$, $U_3 \nsubseteq I$", then "$U_1 \subseteq \sqrt{I}$". ($U_1U_2U_3 \subseteq I$ and $U_1 \nsubseteq I$, $U_3 \nsubseteq I$ then $U_2 \subseteq \sqrt{I}$, $U_1U_2U_3 \subseteq I$ and $U_1 \nsubseteq I$, $U_2 \nsubseteq I$ then $U_3 \subseteq \sqrt{I}$) . (ii) "$\sqrt{I}$ is a prime ideal of T".

Ex 1.2 : "Let T = { p_1, p_2, p_3, p_4 } be a" Tsemi-ring under the ternary operation .given by

+	p_1	p_2	p_3	p_4
p_1	p_1	p_2	p_3	p_4
p_2	p_2	p_3	p_4	p_1
p_3	p_3	p_4	p_1	p_2
p_4	p_4	p_1	p_2	p_3

·	p_1	p_2	p_3	p_4
p_1	p_1	p_1	p_1	p_1
p_2	p_1	p_1	p_1	p_2
p_3	p_1	p_1	q	p_1
p_4	p_1	p_2	p_2	p_1

Define the ternary operation [ ] as $[U_1U_2U_3] = U_1\ (U_2U_3) = (U_1U_2)U_3$. Then (T, +, [ ]) is a Tsemi-ring. Let "$U_1 = \{p_1, p_3\}$, $U_2 = \{p_2, p_2\}$", U_3 = {"p_1, p_2, p_3"} and U_4 = {"p_1, p_3, p_4"}. Then U_1, U_2, U_3, U_4 are all ideals of T. Now $U_1U_2U_3 \subseteq U_1$ and $U_1 \nsubseteq U_1$, $U_3 \nsubseteq U_1$ then $U_2 \subseteq \sqrt{U_1}$ and $\sqrt{U_1}$ "is a prime ideal of T". Therefore U_1 is a l-elementary ideal of T.

Ex 1.3: "In the Ex 1.2, $U_1U_2U_3 \subseteq U_1$ and $U_1 \nsubseteq U_1$, $U_3 \nsubseteq U_1$ then $U_2 \subseteq \sqrt{U_1}$ and $\sqrt{U_1}$ is a prime ideal of T. Therefore U_1 is a lt-elementary ideal of Tsemi-ring T."

Ex 1.4: "In the Ex 1.2.$U_1U_2U_3 \subseteq U_1$ and $U_1 \nsubseteq U_1$, $U_3 \nsubseteq U_1$ then $U_2 \subseteq \sqrt{U_1}$ and $\sqrt{U_1}$ is a prime ideal of T. Hence I is a r-elementary ideal of Tsemi-ring T."

Def 1.5 : "An ideal I of a Tsemi-ring T is said to be a ***elementary ideal*** provided I is a l-elementary ideal, a lt-elementary ideal and a r-elementary ideal."

Ex 1.6 : In Ex 1.2. the subset U_4 is a "elementary ideal of Tsemi-ring T."

"Th 1.7 : Let I be anideal of a Tsemi-ring T. Then U_1, U_2 and U_3 are threeideals of T such that$U_1U_2U_3$**$\subseteq$Iand$U_2 \nsubseteq$I,W$\nsubseteq$I$\Rightarrow U_1 \subseteq \sqrt{I}$ifandonlyif*$U_1$*, U_2, $U_3 \in$ T,<*U_1*><U_2><U_3>$\subseteq$I and U_2, $U_3 \notin$I$\Rightarrow$*U_1*$\in \sqrt{I}$."**

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"Th 1.8 : Let I be an ideal of a Tsemi-ring T. Then U_1, U_2, U_3 are threeideals of T such that$U_1U_2U_3$**$\subseteq$I and $U_1 \not\subseteq$I, $U_3 \not\subseteq$ I$\Rightarrow U_2 \subseteq \sqrt{I}$ if and only if U_1, U_2, $U_3 \in$ T, $\langle U_1\rangle\langle U_2\rangle\langle U_3\rangle \subseteq$I and U_1,$U_3 \notin$I$\Rightarrow U_2 \in \sqrt{I}$.**

Th 1.9 : Let I be an ideal of a Tsemi-ring T. Then U_1, U_2, U_3 **are threeideals of T such that** $U_1U_2U_3$**$\subseteq$ I and $U_1 \not\subseteq$I, $U_2 \not\subseteq$ I$\Rightarrow U_3 \subseteq \sqrt{I}$ if and only ifU_1, U_2, $U_3 \in$ T, $\langle U_1\rangle\langle U_2\rangle\langle U_3\rangle \subseteq$I and U_1, $U_2 \notin$I$\Rightarrow U_3 \in \sqrt{I}$."**

Th1.10 : "Let T be a commutative Tsemi-rings and I be an ideal of T. Then the following conditions are equivalent.

1.I isa l-elementary(lt, r) ideal.

2. U_1, U_2, U_3 are three ideals of T such that $U_1U_2U_3 \subseteq$ **I and $U_2 \not\subseteq$** I, **$U_3 \not\subseteq$** I$\Rightarrow$ **$U_1 \subseteq \sqrt{I}$. (U_1, U_2, U_3 are three ideals of T such that** $U_1U_2U_3 \subseteq$ **I and $U_1 \not\subseteq$I,$U_3 \not\subseteq$**I$\Rightarrow$**$U_2 \subseteq \sqrt{I}$. ; U_1, U_2, U_3 are three ideals of T such that** $U_1U_2U_3 \subseteq$ **I and $U_1 \not\subseteq$I,"$U_3 \not\subseteq$"**I$\Rightarrow$**$U_3 \subseteq \sqrt{I}$").**

3. "U_1,U_2, U_3" $\in$T, <"U_1><U_2 "><U_3>$\subseteq$I and "$U_2 \notin$I, $U_3 \notin$I implies $U_1 \in \sqrt{I}$". (U_1,"U_2, U_3 "$\in$T, <"U_1><U_2 "><U_3>$\subseteq$I and "$U_1 \notin$I, $U_3 \notin$I" implies "$U_2 \in \sqrt{I}$", U_1, U_2," U_3 "$\in$T, $U_1U_2U_3 \subseteq$ **I and $U_1 \notin$A, $U_2 \notin$I implies $U_3 \in \sqrt{I}$.).**

Th 1.11 : "Let T be a commutative Tsemi-ring and I be anideal of T. Then the following conditions are equivalent.

1) I is a elementary ideal. 2) U_1, U_2, U_3 are threeideals of T, $U_1U_2U_3$**$\subseteq$I and $U_2 \not\subseteq$I, $U_3 \not\subseteq$ I$\Rightarrow U_1 \subseteq \sqrt{I}$.**

3) U_1, U_2, $U_3 \in$ T, $U_1U_2U_3$**$\in$I, U_2, $U_3 \notin$I$\Rightarrow U_1 \in \sqrt{I}$."**

NOTE 1.12 : "In an arbitraryTsemi-ring a l-elementary ideal is not necessarily a r-elementary ideal."

Ex 1.13 : Let T = {p_1, p_2, p_3}. Define binary operations addition and multiplication**.**"in T as shown in the following table."

Define a

+	p_1	p_2	p_3
p_1	p_1	p_2	p_3
p_2	p_2	p_3	p_1
p_3	p_3	p_1	p_2

·	p_1	p_2	p_3
p_1	p_1	p_1	p_1
p_2	p_1	p_1	p_1
p_3	p_1	p_2	p_3

mapping from "T × T × T → T by$[p_1p_2p_3] = p_1.p_2.p_3$" for all p_1, p_2, $p_3 \in$ T. "It is easy to see that T is a" Tsemi-ring. Now consider the ideal, $<p_1> = T^eT^e p_1 T^eT^e$ = {p}. "Let abc$\subseteq <p_1>$, a, b$\notin <p_1> \Rightarrow$b$\in \sqrt{<p_1>} \Rightarrow b^n \subseteq <p_1>$ for some odd natural number$n \in$ N. Since qrr$\subseteq <p_1>$,"r$\notin <p_1> \Rightarrow p_2 \in \sqrt{<p_1>}$. Therefore $<p_1>$ is l-elementary. If $p_2 \notin <p_1>$ then $p_3^n \nsubseteq <p_1>$ for any$n \in$ N $\Rightarrow p_2 \notin \sqrt{<p_2>}$. Therefore $<p_1>$ is not r-elementary and lt-"elementary."

"Th 1.14 : Every ideal I in a Tsemi-ring T is l(lt, r) elementary if and only if every ideal I satisfies condition (i) of definition 1.1(1.3, 1.5)."

We now introduce the terms, a l-elementary T semi ring, a lt-elementary Tsemi-ring, and a r-elementary Tsemi-ring and anelementary Tsemi-ring.

Def 1.15 : A "T semi-ring" T is said to be ***l(lt, r)elementary*** provided every "ideal of T is a" l(lt, r) elementary "ideal" of T.

Def1.16:ATsemi-ring"Tissaidtobe***elementary***providedeveryideal of T is a elementary ideal of T."

"Th 1.17 : Let T be a T semi-ring with identity and let M be the unique maximal ideal of T. If √I = M for some ideal of T, then I is a elementary ideal."

NOTE 1.18: If a Tsemiring T "has no identity, then the Th 1.17, is not true," even if the Tsemi-ring T has a unique maximal ideal. In Ex 1.13, $\sqrt{< p>}$ = M where M = {p, q} "is the unique maximal ideal. But < p > is not a elementary ideal."

"Th 1.19: If T is a Tsemi-ring with identity, then for any natural number *n*, M^n is elementary ideal of T where M is the unique maximal ideal of T.

NOTE 1.20: If T has no identity then Th1.17, is not true. In **E**x 1.13,

$M = \{p, q\}$ is the unique maximal ideal, but $M^2 = \{p\}$ is not elementary."

"Th 1.21: In quasi commutative Tsemi-ring T, an ideal I of T is l(lt, r) elementary if and only if r(l, lt) elementary."

Cor 1.22: If "I is an ideal of a quasi commutative Tsemi-ring T, then the following are equivalent.

1) I is elementary 2) I is l-elementary 3) I is lt-elementary 4) I is r-elementary."

IV. N(I) –TSEMI-RING

"We now introduce the terms" "I-potent element and I-potent ideal for a T ideal of a" T semi-ring.

Def 2.1 : "Let I be an ideal in a Tsemi-ring T. An element $x \in T$ is said to be I-***potent*** $\exists$ an odd natural number n such that $x^n \in I$."

Def 2.2 :"Let I be an ideal in a Tsemi-ring T. An ideal J of T is said to be I-***potent ideal*** provided $\exists$ an odd natural number n such that $J^n \subseteq I$."

NOTE 2.3 : "If I is an ideal of a Tsemi-ring T, then every element of I is a I-potent element of T and I itself an I-potent ideal of T.

Def 2.4 : Let I be an ideal of a Tsemi-ring T. An I-potent element U is said to be a ***nontrivial I-potent*** element of T if $U \notin I$.

NOTATION 2.5 : $N_o(I)$ = The set of all I-potent elements in T.

$N_1(I)$ = The largest ideal contained in $N_o(I)$.

$N_2(I)$ = The union of all I-potent ideals.

Th 2.6 : If I is an ideal of a Tsemi-ring T, then $I \subseteq N_2(I) \subseteq N_1(I) \subseteq N_0(I)$

Th 2.7 : If I is an ideal in a Tsemi-ring T, then the following are true.

1. **$N_0(I) = A_2$. 2.$N_1(I)$ is a semiprime ideal of T containing I. 3.$N_2(I) = A_4$.**

NOTE 2.8 : It is natural to ask whether $N_1(I) = A_3$. This is not true.

Ex 2.9 : In the free Tsemi-ring T over the alphabet p,*q*,*r*. For the ideal I= Tb^3T, $N_0(I)=\{b\} \cup T^e p^3 T^e$ and $N_1(I)=\{p^3\} \cup Tp^3T = T^e p^3 T^e$. But Tp^3T is a prime ideal, let U,V,W are three ideals of S such that $UVW \subseteq Ta^3T$, implies all words containing $p^3 \in I$ or all words containing $p^3 \in V$ or all words containing $p^3 \in W \Rightarrow I \subseteq Tp^3T$" or $V \subseteq Tp^3T$ or $W \subseteq Tp^3T$. Therefore Tp^3T isa prime ideal. We have $A_3= Tp^3T$, so $N_1(I) \neq A_3$. Therefore we can remark that the inclusions in $A_3 \subseteq N_1(I) \subseteq N_0(I)=A_2$"may be proper in an arbitraryTsemi-ring."

"Th 2.10 : If I is a semipseudo symmetric ideal in a Tsemi-ring T, then $N_0(I) = N_1(I) = N_2(I)$.

Th 2.11 : For any semipseudo symmetric ideal I in a Tsemi-ring T, a nontrivial A-potent element *U*($U \notin I$) cannot be semisimple.

Th 2.12 : If I is an ideal in a Tsemi-ring T, such that $N_0(I)$=I, then I is a completely semiprime ideal and I is a pseudo symmetric ideal.

Th 2.13 : If I is a semi-pseudo symmetric ideal of a semi simple Tsemi-ring the I = $N_0(I)$."

Def 2.14 : "Let I be an ideal in a T semi-ring T. An element $x \in T$ is said to be a ***l*** **(*lt*, r)I-*divisor*** provided" $\exists$ two elements $y,z \in T \backslash I$ "such that" $xyz \in I (yxz \in I, yzx \in I)$.

Def 2.15 : "Let I be an ideal in a T semi-ring T. An element $U \in T$ is said to be an I-***divisor*** if *U* is both a l-I-divisor and a r-I-divisor element."

"We now introduce a l-I-divisor T ideal, lt-I-divisor T ideal, r-I-divisor T ideal and" an "I-divisor T ideal corresponding to a T ideal I in a" T semi-ring.

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Def 2.”16 : Let I be an ideal in a” Tsemi-ring “T. An” ideal J in “T is said to be a” ***l* (*lt,* r)I-*divisor* “*ideal*** provided every element of J is a” l (lt, r) “I-divisor element”.

Def 2.17 : Let I “be an ideal in a” Tsemi-ring “T. An ideal” J “in T is said to be an I-***divisor ideal***provided if it is both a” l-I-“divisor ideal and a r-I-divisor ideal of a T”semi-ring T.

“NOTATION 2.18 : $R_l(I)$ = The union of all l-I-divisor ideals in T”.

$R_r(I)$ = The union of all r-I-divisor ideals in T.

$R_m(I)$= The union of all lt- or medial I-divisor ideals in T.

$R(I) = R_l(I) \cap R_m(I) \cap R_r(I)$. We call R(I), the divisor radical of T.”

Th 2.19: “If Iis any ideal of a Tsemi-ring T, then $N_1(I) \subseteq R(I)$.

Th 2.20 : If I is an ideal in a Tsemi-ring T, then R(I) is the union of all I-divisor ideals in T.

COROLLARY 2.21 : If A is a pseudo symmetric ideal in a Tsemi-ring T, then R(I) is the set of all I-divisor elements in T.”

“We now introduce the notion of” N(I)-Tsemi-ring.

“Def2.22: Let I be an ideal in a T semi-ringT. T is said to be a ***N(I)-Tsemi-ring*** provided every I-divisor is I-potent.

NOTATION 2.23 : Let T be a Tsemi-ring with zero. If I={0}, then we write R for R(I) and N for $N_0(I)$ and N- Tsemi-ring for N(I)-Tsemi-ring.

Th 2.24 : If T is an N(I)-Tsemi-ring, then $R(I) = N_1(I)$.

Th 2.25 : Let I be a semipseudo symmetric ideal in a Tsemi-ring T. Then T is an N(I)-Tsemi-ring if and only if $R(I) = N_0(I)$.”

Cor 2.26 : “Let I be a pseudo symmetric ideal in a Tsemi-ring T. Then T is an N(I)-Tsemi-ring if and only if $R(I) = N_0(I)$.

Cor 2.27 : Let T be a semi-ringwith 0 and < 0 > is a pseudo symmetric ideal. Then R = N if and only if T is an N-Tsemi-ring.

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Th 2.28: If M is a maximal ideal in a Tsemi-ring T containing a pseudo symmetric ideal I, then M contains all A-potent elements in T or T\M is singleton which is I-potent.

Cor2.29 : If M is a nontrivial maximal ideal in a Tsemi-ring T containing *a* pseudo symmetric ideal I. Then $N_0(I) \subseteq M$.

Cor 2.30 : If M is a maximal ideal in a semisimple Tsemi-ring T containing *a* semipseudo symmetric ideal I. Then $N_0(I) \subseteq M$."

V. Conclusion :

"We are expressing thankful to every reviewer and give your valuable suggestions."

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