Partial Addition and Ternary Product Based Γ-So-Semirings-2

Bhagyalakshmi Kothuru, V. Amarendra Babu

Abstract: *Here we are introducing thenotions i-system, idempotent, centre of a ternary Γ-SO semiring, Nilpotent are introduced and it is proved that some equivalent conditions. Further it is also proved that (i) if C be a ternary Γ - SO semiring, m is a "strongly regular element", then ∃ϑ, μ∈Γ also n∈C ∋m = mϑnμm,n = nμmϑn (ii) If "I be an Ideal of A strongly regular ternary Γ - SOsemiring R then I is strongly regular and any ideal J of I is an ideal of R" and many more properties were proved. Mathematical subject classification: 16Y60.*

Keywords: *Idempotent, i-system, strongly regular, m-system, n-system.*

I. INTRODUCTION

Sen in 1981 introduced the concepts of Γ - semigroup as generalization of semi group. In 1934 H.S Vandiver develops the theory of semi ring. In 1995 the notion of Γ - semiringswas introduced by M.MuralikrishnaRao. In this paper we are introducing some classical notions of ternary Γ - SO semiring.

II. MATHEMATICAL BACKGROUND:

Some of the main definitions and results are as follows.

Throughout this paper C is a CTΓSS means "complete ternary Γ- SO semiring" and "ternary Γ - SO semiring" is denoted by TΓ-S-SR.

Definition2.1: Let R is a TΓ-S-SR along with φ ≠S⊆R then S is known as

"*m-system*" if for any *s*, *t*, *u*∈ S implies that RΓRΓ*s*ΓRΓRΓ*t*ΓRΓRΓ*u*ΓRΓR⊆S.

Definition2.2: Let R be a TΓ-S-SR along withφ ≠S⊆R then S is known as "***n-system***" if any *l*∈S implies that RΓRΓ*l*ΓRΓRΓ*l*ΓRΓRΓ*l*ΓRΓR⊆S.

Notation2.3: For $d \in A$, CTΓSSC define (x] = {d∈C: d≤x} and for a subset X of a CTΓSSC. (x] =$\cup_{x \in X}(x]$.

Remark: **2.4:**"Let F be a PO-ternary semiring"and L⊆F, M⊆F, N⊆F, then

(i) L⊆(L] (ii) ((L]]=(L] (iii) (L]Γ(M]Γ(N]⊆(LΓMΓN] (iv) L⊆M⇒L⊆(M]
(v) L⊆M⇒(L]⊆(M] (vi) (L∩M]=(L]∩(M] (vii) (L∪M]=(L]∪(M]

Definition 2.5: F is any proper ideal of a TΓ-S-SRM is known as **"*prime*"**⇔for any ideals R, S, T of M, RΓSΓT ⊆ F⟹ R ⊆ F or S ⊆ F or T ⊆ F.

Definition2.6: F is any proper ideal of a TΓ-S-SRM is known as **"*semiprime*"**⇔ for any ideals R of M, RΓRΓR ⊆ F⟹ R ⊆ F.

And for more preliminaries the references [12] [13] [14] [15][16][17] and [18].

III. EXPERIMENTS AND RESULTS DESCRIPTION

Definition3.1: Let M be a TΓ-S-SRmoreoverφ ≠S⊆Mthen S is known as an **"*i-system*"**⇔ ∀ *r*, *t*, *u*∈ S, $< r > \cap < t > \cap < u > \cap S \neq \emptyset$

Example: 3.2: Let S= $\{0, m_1, m_2, m_3, m_4, m_5\}$, Γ ={$\vartheta$, μ} define Σ on S as

$$\sum_i x_i = \begin{cases} x_j \text{ if } x_i = 0 \text{ for all } i \neq j \text{, and for some } j \\ \text{not defined, elsewhere} \end{cases}$$

Here S is "ternary SO –monoid".

ϑ	0	m_1	m_2	m_3	m_4	m_5
0	0	0	0	0	0	0
m_1	0	0	0	0	0	0
m_2	0	0	0	0	0	0
m_3	0	0	0	0	0	0
m_4	0	0	0	0	0	0
m_5	0	0	0	0	0	0

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Bhagyalakshmi Kothuru, Research Scholar, Department of Mathematics, Acharya Nagarjuna University,KKR&KSR Institute of Technology &Sciences Vinjanampadu, Guntur,
E-mail:mblakshmi12@gmail.com
V. Amarendra Babu, Department of Mathematics, AcharyaNagarjuna University. Vinjanampadu, Guntur, India.
E-mail: amarendrab4@gmail.com

μ	0	m_1	m_2	m_3	m_4	m_5
0	0	0	0	0	0	0
m_1	0	0	0	0	0	m_1
m_2	0	0	0	0	0	m_2
m_3	0	0	0	0	0	m_3
m_4	0	0	0	0	0	m_4
m_5	0	m_1	m_2	m_3	m_4	m_5

Define the mapping S ×Γ× S×Γ× S⟶ S as follows:

Then S is a ternary Γ-SO-semiring. And $\{0, m_1\}$ is an "*i*-system"where as the subset $\{m_3, m_4, m_5\}$ is not an "*i*-system"while$<m_3>=\{0,m_3\}$, $<m_4>=\{0,m_4\}$,$<m_5>=\{0,m_5\}$and$<m_3>\cap<m_4>\cap<m_5>\cap Y=\emptyset$ Where Y is a subset of S.

Theorem: 3.3:Let C be a CTΓSSand $u\in C$implies $<u>_l=\{e\in C/\ e\leq\sum_m u+\sum r_j\alpha_j s_j\beta_j u,\ r_j, s_j\in C\ \alpha_j,\beta_j\in \Gamma, m\in N\}$ and Σ denotes the finite sum.

Proof: Let U= $\{e\in C/\ e\leq\sum_m u+\Sigma r_j\alpha_j s_j\beta_j u,\ r_j, s_j\in C\alpha_j,\beta_j\in \Gamma,\ m\in N\}$

Suppose e, f ∈ U. Then $e\leq\sum_m u+\Sigma r_j\alpha_j s_j\beta_j$ u and $f\leq\sum_m u+\Sigma r_i\alpha_i s_i\beta_i u$ for $r_i, s_i, r_j, s_j\in C$, $m\in N$

Now $e+f\leq\sum_m u+\Sigma r_j\alpha_j s_j\beta_j u+\sum_m u+\Sigma r_i\alpha_i s_i\beta_i u$

This implies $e+f\in U$ .Therefore U is an additive sub semigroup of C.

$r_1,r_2\in C$, $u\in U$ then $r_1\Gamma r_2\Gamma u\subseteq r_1\Gamma r_2\Gamma(\sum_m e+\Sigma r_j\alpha_j s_j\beta_j e)$

$=r_1\Gamma r_2\Gamma\sum_m e+r_1\Gamma r_2\Gamma\Sigma r_j\alpha_j s_j\beta_j e$

$=\sum_m(r_1\Gamma r_2\Gamma e)+\Sigma r_1\Gamma r_2\Gamma r_j\alpha_j s_j\beta_j e\in U$Therefore$r_1\Gamma r_2\Gamma u\subseteq U$

Let $r\in C$, $u\in U\ni r\leq u$. Because$u\in U$ implies$u\leq\sum_m e+\Sigma r_j\alpha_j s_j\beta_j e$

Now $r\leq u$, $u\leq\sum_m e+\Sigma r_j\alpha_j s_j\beta_j$ e. This implilesr$\leq e+\Sigma r_j\alpha_j s_j\beta_j e\Rightarrow r\in U$

ThereforeU is a "left ternary Γ-ideal of C".

We have (U] is a "left SO ternary Γ-ideal of C" containing *u*. Thus $<u>_l\subseteq$ (U].

On other hand $<u>_l$ is also a left SO ternary Γ-ideal containing *u*.

Hence U$\subseteq<u>_l$, thus (U] $\subseteq<u>_l$. Therefore $<u>_l=$U.

Theorem3.4"Let Cis a CTΓSSand $u\in C$. Then

$<u>_m=\{e\in C\ /e\leq\sum_n u+\sum_p g_p'\alpha_p u\beta_p g_p+\sum_q g_q''\alpha_q'' g_q\alpha_q' u\beta_q' h_q'\beta_q'' h_q''$ where $g_p', g_p, g_q'', g_q', h_q', h_q''\in C, \alpha_p,\beta_p,\alpha_q'',\alpha_q',\beta_q',\beta_q''\in\Gamma$, $n, p, q\in N\}$"

Proof:Similarto theorem3.3.

Theorem 3.5: "Let C be a CTΓSS and $u\in C$. Then $<u>_r=\{e\in C/\ e\leq\sum_n u+\sum_j u\,\alpha_j r_j\beta_j s_j$ where $r_j, s_j\in C$, $\alpha_j,\beta_j\in\Gamma$, $n,j\in N\}$".

Proof:Similar to theorem [3.3].

Theorem3.6: "Let C be a CTΓSS and $u\in C$. Then $<u>=\{e\in C\ /e\leq\sum_n u+\sum_p r_p\alpha_p s_p\beta_p u+\sum_k r_k'\alpha_k u\beta_k r_k+\sum_q r_q''\alpha_q'' r_q\alpha_q' u\beta_q' s_q'\beta_q'' s_q''+\sum_h u\alpha_h r_h\beta_h s_h$ where $r_p, s_p, r_k', r_k, r_q'', r_q', s_q', s_q'', r_h, s_h\in R$, $\alpha_j,\beta_j,\alpha_k,\beta_k,\alpha_l'',\alpha_l',\beta_l',\beta_l'',\alpha_m,\beta_m\in\Gamma$, $n,p,k,q\in N\}$"

Proof:Similar to theorem [3.3].

Theorem3.7: "Let C be a CTΓSSandS be a proper ideal of C. The following are (i) S is prime (ii) $\ddot{e}_1,\ddot{e}_2,\ddot{e}_3\in C$ such that $<\ddot{e}_1>\Gamma<\ddot{e}_2>\Gamma<\ddot{e}_3>\subseteq S\Rightarrow\ddot{e}_1\in S$ or $\ddot{e}_2\in S$ or $\ddot{e}_3\in S$ are equivalent."

(i)⇒(ii) If S is a "prime ideal" of C.Let $\ddot{e}_1,\ddot{e}_2$, $\ddot{e}_3\in C\ni<\ddot{e}_1>\Gamma<\ddot{e}_2>\Gamma<\ddot{e}_3>\subseteq S$.

As$<\ddot{e}_1>$, $<\ddot{e}_2>$, $<\ddot{e}_3>$ are ideals of C, S is prime implies that $<\ddot{e}_1>\subseteq S$ or $<\ddot{e}_2>\subseteq S$ or $<\ddot{e}_3>\subseteq S$.We have $\ddot{e}_1\in<\ddot{e}_1>$, $\ddot{e}_2\in<\ddot{e}_2>$, $\ddot{e}_3\in<\ddot{e}_3>$. Therefore $\ddot{e}_1\in S$ or $\ddot{e}_2\in S$ or $\ddot{e}_3\in S$.

(ii)⇒ (i) from condition (i) $\ddot{e}_1,\ddot{e}_2$, $\ddot{e}_3\in C$,$<\ddot{e}_1>\Gamma<\ddot{e}_2>\Gamma<\ddot{e}_3>\subseteq S\Rightarrow\ddot{e}_1\in S$ or $\ddot{e}_2\in S$ or $\ddot{e}_3\in S$..

HereE_1, E_2,E_3arethree ideals of C∋ $E_1\Gamma E_2\Gamma E_3\subseteq S$.

Presume$E_1\nsubseteq S$, $E_2\nsubseteq S$ there exists $\ddot{e}_1\in E_1\ni\ddot{e}_1\notin S$& there exists $\ddot{e}_2\in E_2\ni\ddot{e}_2\notin S$.

Let $\ddot{e}_3\in E_3$ then $<\ddot{e}_1>\Gamma<\ddot{e}_2>\Gamma<\ddot{e}_3>\subseteq S$.

Since by our supposition $\ddot{e}_1\in S$ or $\ddot{e}_2\in S$ or $\ddot{e}_3\in S$.But $\ddot{e}_1\notin S$ and $\ddot{e}_2\notin S$ there $\ddot{e}_3\in S\Rightarrow E_3\subseteq S$.

Then S is a "prime ideal" of C.

Theorem3.8:"Let R be a CTΓSSand Z be a "proper ideal" of R. Then the conditions

(i) S is prime

(ii) Let a_1, b_1, c_1 ∈R∋RΓRΓa_1ΓRΓRΓb_1ΓRΓRΓc_1ΓRΓR⊆Z⇒ a_1 ∈ Z or b_1 ∈ Z or c_1 ∈Z.

(iii)If Z_1, Z_2, Z_3 are three right ideals of R such that Z_1Γ Z_2Γ Z_3 ⊆Z then Z_1 ⊆Z or Z_2 ⊆Z or Z_3 ⊆Z

(iv)If T_1, T_2, T_3 are three middle ideals of R such that T_1Γ T_2Γ T_3 ⊆Z then T_1 ⊆Z or T_2 ⊆Z or T_3 ⊆Z.

(v) If Y_1, Y_2, Y_3 are three left ideals of R such that Y_1Γ Y_2Γ Y_3 ⊆Z then Y_1 ⊆Z or Y_2 ⊆Z or Y_3 ⊆Z are equivalent".

Proof: **(i) ⇒ (ii)** If Zbe a "prime ideal" of R.

Let a_1, b_1, c_1 ∈ C such that RΓRΓa_1ΓRΓRΓb_1ΓRΓRΓc_1ΓRΓR⊆Z

Now < a_1 > Γ< b_1 > Γ< c_1 >= (RΓRΓa_1ΓRΓR)Γ(RΓRΓb_1ΓRΓR)Γ(RΓRΓc_1ΓRΓR)

= RΓRΓ a_1 ΓRΓRΓ b_1 ΓRΓRΓ c_1 ΓRΓR ⊆ Z ⇒ a_1 ∈ Z or b_1 ∈ Z or c_1 ∈Z.(by above theorem)

(ii) ⇒ (iii) Suppose that condition (ii) holds. Let Z_1, Z_2, Z_3 are three right ideals of R such that $Z_1 \Gamma Z_2 \Gamma Z_3$ ⊆ Z. Suppose if possible Z_1 ⊈Z, Z_2 ⊈Z,Z_3 ⊈Z.

Z_1 ⊈Z, Z_2 ⊈Z,Z_3 ⊈Z then there exist a_1, b_1, c_1 such that $a_1 \in Z_1$, a_1∉ Z,$b_1 \in Z_2$, b_1∉ Z and$c_1 \in Z_3$, c_1∉ Z.$a_1 \in Z_1$, $b_1 \in Z_2$, $c_1 \in Z_3$ ⇒< a_1 > Γ< b_1 > Γ< c_1 > ⊆ $Z_1 \Gamma Z_2 \Gamma Z_3$ ⊆Z.

NowRΓRΓ a_1 ΓRΓRΓ b_1 ΓRΓRΓ c_1 ΓRΓR ⊆ $Z_1 \Gamma Z_2 \Gamma Z_3$ ⊆ S⇒ a_1 ∈ Z or b_1 ∈ Z or c_1 ∈ Z. It is a contradiction. Therefore Z_1 ⊆ Z or Z_2 ⊆ Z or Z_3 ⊆ Z.And hence Z is a "prime ideal" of R.

In a similar manner also we can prove **(ii) ⇒ (iv)** as well as **(ii) ⇒ (v).**

(iii) ⇒ (i)

Suppose for any three right ideals Z_1 , Z_2, Z_3 of R, $Z_1 \Gamma Z_2 \Gamma Z_3$ ⊆Z. implies Z_1 ⊆Z or Z_2 ⊆Z or Z_3 ⊆Z. Let F, G, H be any three ideals of C such that FΓGΓH⊆Z.

Then F, G, H are right ideals such that FΓGΓH⊆Z⇒F⊆Z or G⊆Z or H⊆Z.

Hence Z is "prime ideal" of R. In the same way we prove**(iv) ⇒ (i)** and **(v) ⇒ (i)**

Theorem 3.9:"Let C be a CTΓSS and S be a proper ideal of C. Then

(i)S is semiprime ideal of C,

(ii) For *r*∈ C, ∋ (<*r*> Γ)^2<*r*>⊆S implies *x*∈S.

(iii) For *r*∈ C, RΓRΓ*r*ΓRΓRΓ*r*ΓRΓRΓ*r*ΓRΓR⊆S implies *r*∈S.

(iv)For any right ideals T of C, TΓTΓT ⊆S implies T⊆S.

(v) M be any lateral ideal of C, MΓMΓM ⊆S ⇒M⊆S.

(vi)N be any left ideal of C, NΓNΓN⊆S implies N⊆S".

Proof: Theorem 3.9 is as similar as theorem 3.7 and theorem 3.8

Theorem 3.10:"An ideal S of CTΓSSR is a prime ideal iff R\S is an *m*- system of R or empty".

Proof:Suppose Sis a "prime ideal" of aCTΓSSR and R\S≠∅

Let *r, t, u*∈R\S then *r*∉ S,*t*∉ S and *u*∉ S.Suppose if possible RΓRΓ*r*ΓRΓRΓ*t*ΓRΓRΓ*u*ΓRΓR⊈R\S

⇒RΓRΓ*r*ΓRΓRΓ*t*ΓRΓRΓ*u*ΓRΓR⊆S.

Since S is prime, either *r*∈ S or *t*∈ S or *u*∈ S. It is a contradiction.

Therefore RΓRΓ*r*ΓRΓRΓ*t*ΓRΓRΓ*u*ΓRΓR⊆R\S. So"R\S is an*m*-system".

On the other hand if "R\S is either an *m*-system of R or R\S=∅".

If R\S=∅ then R=S ⇒S is a prime ideal of R.Suppose that"R\S is an *m*-system" of R

Let *r*, *t* ,*u*∈R and <*r*>Γ<*t*>Γ<*u*>⊆S. Suppose if possible*r*, t, u∉ S⇒*r*, *t*, u∈R\S

Since R\S is an *m*-system ⇒RΓRΓ*r*ΓRΓRΓ*t*ΓRΓRΓ*u*ΓRΓR⊆R\S

⇒RΓRΓ*r*ΓRΓRΓ*t*ΓRΓRΓ*u*ΓRΓR⊈S ⇒<*r*>Γ<*t*>Γ<*u*>⊈S.

We have absurdity.Thus *r*∈ S or *t*∈ S or *u*∈ S.Hence S is a "prime ideal" of R.

Theorem 3.11:"Let R be a CTΓSSand S be a proper ideal of R. then S is semi prime ⇔ R\S is an *n*-system or empty".

Proof: Similar to theorem 3.9

Definition3.12: An ideal Q of a TΓ-S-SRM is known as ***"strongly irreducible"***iff for any ideals R, S, T of M, R∩ S ∩ T ⊆ Q ⇒ R ⊆ Q or S ⊆ Q or T ⊆ Q.

Theorem3.13:"Let M is an ideal of a CTΓSS R. Then

(i) M is strongly irreducible

(ii)If m_1, m_2, m_3 ∈R such that <m_1>∩<m_2>∩<m_3>⊆M then m_1 ∈ M or m_2 ∈ M or m_3 ∈ M

(iii) R\M is an *i*- systemof R".

Proof: **(i) ⇒ (ii)**

Let M is a strongly irreducible ideal of R.

Let m_1, m_2, m_3 ∈R such that $<m_1>\cap<m_2>\cap<m_3>\subseteq M$

Since M is strongly irreducible $<m_1>\subseteq M$ or $<m_2>\subseteq M$ or $<m_3>\subseteq M.\Rightarrow m_1 \in M$ or $m_2 \in M$ or $m_3 \in M$.

(ii) ⇒ (iii)

If $m_1, m_2, m_3 \in R$ such that $<m_1>\cap<m_2>\cap<m_3>\subseteq M$ $\Rightarrow m_1 \in M$ or $m_2 \in M$ or $m_3 \in M$.

Suppose m_1, m_2, $m_3 \in R\backslash M \Rightarrow <m_1>\cap<m_2>\cap<m_3>\not\subseteq M \Rightarrow <m_1>\cap<m_2>\cap<m_3>\cap R\backslash M\neq\emptyset$

Therefore R\M is an "*i*-system" of R.

(iii) ⇒ (i)

Assume"R\M is an *i*-system of R".

Assume that P, Q, T the ideals of R ∋ P⊈M, Q⊈Mand T ⊈ M ∃$m_1 \in P$, $m_2 \in Q$ and $m_3 \in T$ such that m_1, m_2, $m_3 \in R\backslash M$⇒By condition (iii) $\exists <m_1>\cap<m_2>\cap<m_3>\cap R\backslash M\neq\emptyset$

⇒There exists $m_4 \in <m_1>\cap<m_2>\cap<m_3>$ and $m_4 \notin M$. Therefore$m_4 \in P\cap Q\cap T$ and $m_4 \notin M$

⇒P∩Q∩T⊈M. Thus"M is a strongly irreducible ideal of R".

Theorem3.14:"Let R be a TΓ-S-SR. If x is a strongly regular element in R ⇒∃ y∈R ∋ $x \in x\Gamma y\Gamma x$ and $y \in y\Gamma x\Gamma y$".

Proof: If "x is a strongly regular element of R" then $x \in x\Gamma e\Gamma x \Rightarrow x = x\alpha e\beta x$ for $\alpha,\beta \in \Gamma$

Let us take $y \in e\Gamma x\Gamma e \Rightarrow y = e\beta x\alpha e$ then y∈R.

And $x\Gamma y\Gamma x = x\Gamma(e\Gamma x\Gamma e)\Gamma x = (x\Gamma e\Gamma x)\Gamma e\Gamma x = x\Gamma e\Gamma x$.Since $x \in x\Gamma e\Gamma x \Rightarrow x \in x\Gamma y\Gamma x$.

Again $y\Gamma x\Gamma y = (e\Gamma x\Gamma e)\Gamma x\Gamma(e\Gamma x\Gamma e) = e\Gamma x\Gamma e \Rightarrow y \in e\Gamma x\Gamma e \Rightarrow y = e\beta x\alpha e \Rightarrow y \in y\Gamma x\Gamma y$

Definition3.15:"An element x of a TΓ-S-SRR is known as an ***α-idempotent*** if $x\alpha x\alpha x = x$ for some $\alpha \in \Gamma$".

Example3.16: Every identity and zero elements of a ternary Γ- SO semiring are α-idempotents.

Definition3.17:"Let M be a TΓ-S-SR. Then $m \in$ M is said to be a strongly Regular if there exist for $n \in$ M, α, β∈ Γ such that $m = man\beta m$ if every element of M is ***strongly Regular*** then M is called a strongly Regular ternary Γ-SO – semiring".

Definition3.18:"Let M be a ternary -Γ SO semi ring. Then $m \in$ M is said to be a ***strongly Regular*** if there exist for $n \in$ M, α, β, $\gamma,\delta \in \Gamma$ such that $m = man\beta m\gamma o\delta m$".

Note3.19: Definition4.7 and definition 4.8 both are equal.

Definition3.20:"An element x" of a TΓ-S-SR R is known as an ***(α,β)-idempotent***if $x\alpha x\beta x = x$ for some $\alpha,\beta \in \Gamma$.

Note:*3.21* : (α,β)-idempotent of an element simply called idempotent.

Remark3.22: Every idempotent element is a strongly regular element.

Def 3.23:"The ***centre***of a ternary Γ- SO semiring R is defined as the $\{ a \in R / a\alpha x\beta y = x\beta y\alpha a = y\alpha a\beta x \ \forall$ x, y∈ $R, \alpha,\beta \in \Gamma\}$. And it is denoted by C(R)".

Therefore "C(R) $=\{ a \in R / a\alpha x\beta y = x\beta y\alpha a = y\alpha a\beta x \ \forall$ x, y∈ $R, \alpha,\beta \in \Gamma\}$".

Th 3.24:"The centreC(R) of a TΓ-S-SRR is a ternary Γ-SO sub semiring of R".

Proof:"Let $\{ b_j : j \in I \}$ be a summable family in R"∋ $b_j \in$C(R) ∀j∈ I. Then $\sum_j b_j \in R$

And $b_j\alpha x\beta y = x\beta y\alpha b_j = y\alpha b_j\beta x \ \forall$ x, $y \in R, \alpha,\beta \in \Gamma$

for $\alpha,\beta \in \Gamma$, x, $y \in R (\sum_j b_j)\alpha x\beta y = \sum_j (b_j\alpha x\beta y) = \sum_j (x\beta y\alpha b_j) = x\beta y\alpha(\sum_j b_j)$

$= \sum_j (y\alpha b_j\beta x) = y\alpha(\sum_j b_j)\beta x \Rightarrow (\sum_j b_j)\alpha x\beta y = x\beta y\alpha(\sum_j b_j) = y\alpha(\sum_j b_j)\beta x$. $\Rightarrow \sum_j b_j \in$C(R).

let a, b, $c \in$C(R) and $a \in R \Rightarrow a\alpha x\beta y = x\beta y\alpha a = y\alpha a\beta x$ ∀x, y∈ $R, \alpha,\beta \in \Gamma$.

$\forall$ x, $y \in R, \alpha,\beta,\gamma,\delta \in \Gamma$, $(a\alpha b\beta c)\gamma x\delta y = a\alpha b\beta(c\gamma x\delta y) = a\alpha b\beta(x\delta y\gamma c) = a\alpha b\beta(y\gamma c\delta x)$

$= a\alpha(b\beta y\gamma c)\delta x = a\alpha(y\gamma c\beta b)\delta x = (a\alpha y\gamma c)\beta b\delta x = (y\gamma c\alpha a)\beta b\delta x = y\gamma(c\alpha a\beta b)\delta x$

$= y\gamma(a\beta b\alpha c)\delta x$

Similarly, we can show that $(a\alpha b\beta c)\gamma x\delta y = x\delta y\gamma(a\alpha b\beta c)$.

Therefore $a\alpha b\beta c \in$C(R) and thus"C(R) is a ternary Γ-SO semiring R".

IV. CONCLUSION

Mainly we introduced in this paper about regular ternary Γ-So-Semirings and characterized ternary Γ-SO-Semiring.

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