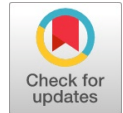


Partial Addition and Ternary Product Based Γ -So-Semirings-2



Bhagyalakshmi Kothuru, V. Amarendra Babu

Abstract: Here we are introducing the notions *i*-system, idempotent, centre of a ternary Γ -SO semiring, Nilpotent are introduced and it is proved that some equivalent conditions. Further it is also proved that (i) if C be a ternary Γ -SO semiring, m is a “strongly regular element”, then $\exists \vartheta, \mu \in \Gamma$ also $n \in C \exists m = m\vartheta n \mu m, n = n \mu m \vartheta n$ (ii) If “ I be an Ideal of A strongly regular ternary Γ -SO semiring R then I is strongly regular and any ideal J of I is an ideal of R ” and many more properties were proved. **Mathematical subject classification:** 16Y60.

Keywords: Idempotent, *i*-system, strongly regular, *m*-system, *n*-system.

I. INTRODUCTION

Sen in 1981 introduced the concepts of Γ -semigroup as generalization of semi group. In 1934 H.S Vandiver develops the theory of semi ring. In 1995 the notion of Γ -semirings was introduced by M.MuralikrishnaRao. In this paper we are introducing some classical notions of ternary Γ -SO semiring.

II. MATHEMATICAL BACKGROUND:

Some of the main definitions and results are as follows. Throughout this paper C is a CTSS means “complete ternary Γ -SO semiring” and “ternary Γ -SO semiring” is denoted by Γ -S-SR.

Definition 2.1: Let R is a Γ -S-SR along with $\varphi \neq S \subseteq R$ then S is known as “*m*-system” if for any $s, t, u \in S$ implies that $R\Gamma R\Gamma s\Gamma R\Gamma R\Gamma t\Gamma R\Gamma R\Gamma u\Gamma R\Gamma R \subseteq S$.

Definition 2.2: Let R be a Γ -S-SR along with $\varphi \neq S \subseteq R$ then S is known as “*n*-system” if any $l \in S$ implies that $R\Gamma R\Gamma l\Gamma R\Gamma R\Gamma R\Gamma l\Gamma R\Gamma R \subseteq S$.

Notation 2.3: For $d \in A$, $CTSSC$ define $(x) = \{d \in C : d \leq x\}$ and for a subset X of a CTSSC. $(x) = \bigcup_{x \in X} (x)$.

Remark 2.4: “Let F be a PO-ternary semiring” and $L \subseteq F, M \subseteq F, N \subseteq F$, then

- (i) $L \subseteq (L)$ (ii) $((L)) = (L)$ (iii)
- $(L)\Gamma(M)\Gamma(N) \subseteq (L\Gamma M\Gamma N)$ (iv) $L \subseteq M \Rightarrow L \subseteq (M)$
- (v) $L \subseteq M \Rightarrow (L) \subseteq (M)$ (vi) $(L \cap M) = (L) \cap (M)$ (vii)
- $(L \cup M) = (L) \cup (M)$

Definition 2.5: F is any proper ideal of a Γ -S-SRM is known as “*prime*” \Leftrightarrow for any ideals R, S, T of $M, R\Gamma S\Gamma T \subseteq F \Rightarrow R \subseteq F$ or $S \subseteq F$ or $T \subseteq F$.

Definition 2.6: F is any proper ideal of a Γ -S-SRM is known as “*semiprime*” \Leftrightarrow for any ideals R of $M, R\Gamma R\Gamma R \subseteq F \Rightarrow R \subseteq F$.

And for more preliminaries the references [12] [13] [14] [15] [16] [17] and [18].

III. EXPERIMENTS AND RESULTS DESCRIPTION

Definition 3.1: Let M be a Γ -S-SR moreover $\varphi \neq S \subseteq M$ then S is known as an “*i*-system” $\Leftrightarrow \forall$

$$r, t, u \in S, \langle r \rangle \cap \langle t \rangle \cap \langle u \rangle \cap S \neq \emptyset$$

Example 3.2: Let $S = \{0, m_1, m_2, m_3, m_4, m_5\}, \Gamma = \{\vartheta, \mu\}$ define Σ on S as

$$\sum_i x_i = \begin{cases} x_j & \text{if } x_i = 0 \text{ for all } i \neq j, \text{ and for some } j \\ \text{not defined, elsewhere} \end{cases}$$

Here S is “ternary SO-monoid”.

ϑ	0	m_1	m_2	m_3	m_4	m_5
0	0	0	0	0	0	0
m_1	0	0	0	0	0	0
m_2	0	0	0	0	0	0
m_3	0	0	0	0	0	0
m_4	0	0	0	0	0	0
m_5	0	0	0	0	0	0

Manuscript published on 30 December 2019.

* Correspondence Author (s)

Bhagyalakshmi Kothuru, Research Scholar, Department of Mathematics, Acharya Nagarjuna University, KKR&KSR Institute of Technology & Sciences Vinjanampadu, Guntur, E-mail: mblakshmi12@gmail.com

V. Amarendra Babu, Department of Mathematics, Acharya Nagarjuna University, Vinjanampadu, Guntur, India. E-mail: amarendrab4@gmail.com

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an open access article under the CC-BY-NC-ND license <http://creativecommons.org/licenses/by-nc-nd/4.0/>



μ	0	m_1	m_2	m_3	m_4	m_5
0	0	0	0	0	0	0
m_1	0	0	0	0	0	m_1
m_2	0	0	0	0	0	m_2
m_3	0	0	0	0	0	m_3
m_4	0	0	0	0	0	m_4
m_5	0	m_1	m_2	m_3	m_4	m_5

Define the mapping $S \times \Gamma \times S \times \Gamma \times S \rightarrow S$ as follows:

Then S is a ternary Γ -SO-semiring. And $\{0, m_1\}$ is an “i-system” where as the subset $\{m_3, m_4, m_5\}$ is not an “i-system” while $\langle m_3 \rangle = \{0, m_3\}$, $\langle m_4 \rangle = \{0, m_4\}$, $\langle m_5 \rangle = \{0, m_5\}$ and $\langle m_3 \rangle \cap \langle m_4 \rangle \cap \langle m_5 \rangle \cap Y = \emptyset$ Where Y is a subset of S .

Theorem: 3.3: Let C be a CTFSS and $u \in C$ implies

$\langle u \rangle_l = \{e \in C / e \leq \sum_m u + \sum r_j \alpha_j s_j \beta_j u, r_j, s_j \in C, \alpha_j, \beta_j \in \Gamma, m \in N\}$ and Σ denotes the finite sum.

Proof: Let $U = \{e \in C / e \leq \sum_m u + \sum r_j \alpha_j s_j \beta_j u, r_j, s_j \in C, \alpha_j, \beta_j \in \Gamma, m \in N\}$

Suppose $e, f \in U$. Then $e \leq \sum_m u + \sum r_j \alpha_j s_j \beta_j u$ and $f \leq \sum_m u + \sum r_i \alpha_i s_i \beta_i u$ for $r_i, s_i, r_j, s_j \in C, m \in N$

Now $e + f \leq \sum_m u + \sum r_j \alpha_j s_j \beta_j u + \sum_m u + \sum r_i \alpha_i s_i \beta_i u$

This implies $e + f \in U$. Therefore U is an additive sub semigroup of C .

$r_1, r_2 \in C, u \in U$ then $r_1 \Gamma r_2 \Gamma u \subseteq r_1 \Gamma r_2 \Gamma (\sum_m e + \sum r_j \alpha_j s_j \beta_j e)$

$= r_1 \Gamma r_2 \Gamma \sum_m e + r_1 \Gamma r_2 \Gamma \sum r_j \alpha_j s_j \beta_j e$
 $= \sum_m (r_1 \Gamma r_2 \Gamma e) + \sum r_j \Gamma r_2 \Gamma r_j \alpha_j s_j \beta_j e \in U$ Therefore $r_1 \Gamma r_2 \Gamma u \subseteq U$

Let $r \in C, u \in U \ni r \leq u$. Because $u \in U$ implies $u \leq \sum_m e + \sum r_j \alpha_j s_j \beta_j e$

Now $r \leq u, u \leq \sum_m e + \sum r_j \alpha_j s_j \beta_j e$. This implies $r \leq e + \sum r_j \alpha_j s_j \beta_j e \Rightarrow r \in U$

Therefore U is a “left ternary Γ -ideal of C ”.

We have $\langle u \rangle_l$ is a “left SO ternary Γ -ideal of C ” containing u . Thus $\langle u \rangle_l \subseteq \langle u \rangle_l$.

On other hand $\langle u \rangle_l$ is also a left SO ternary Γ -ideal containing u .

Hence $U \subseteq \langle u \rangle_l$, thus $\langle u \rangle_l \subseteq U$. Therefore $\langle u \rangle_l = U$.

Theorem 3.4: Let C is a CTFSS and $u \in C$. Then

$\langle u \rangle_m = \{e \in C / e \leq \sum_n u + \sum_p g'_p \alpha_p u \beta_p g_p + \sum_q g''_q \alpha''_q g'_q \alpha'_q u \beta'_q h'_q \beta''_q h''_q\}$ where $g'_p, g_p, g''_q, g'_q, h'_q, h''_q \in C, \alpha_p, \beta_p, \alpha''_q, \alpha'_q, \beta'_q, \beta''_q \in \Gamma, n, p, q \in N$ ”

Proof: Similar to theorem 3.3.

Theorem 3.5: “Let C be a CTFSS and $u \in C$. Then

$\langle u \rangle_r = \{e \in C / e \leq \sum_n u + \sum_j u \alpha_j r_j \beta_j s_j\}$ where $r_j, s_j \in C, \alpha_j, \beta_j \in \Gamma, n, j \in N$ ”.

Proof: Similar to theorem [3.3].

Theorem 3.6: “Let C be a CTFSS and $u \in C$. Then $\langle u \rangle =$

$\{e \in C / e \leq \sum_n u + \sum_p r_p \alpha_p s_p \beta_p u + \sum_k r'_k \alpha'_k u \beta'_k r_k + \sum_q r''_q \alpha''_q r'_q \alpha'_q u \beta'_q s'_q \beta''_q s''_q + \sum_h u \alpha_h r_h \beta_h s_h\}$ where $r_p, s_p, r'_k, r_k, r''_q, r'_q, s'_q, s''_q, r_h, s_h \in R, \alpha_j, \beta_j, \alpha_k, \beta_k, \alpha'_l, \alpha'_l, \beta'_l, \beta'_l, \alpha_m, \beta_m \in \Gamma, n, p, k, q \in N$ ”

Proof: Similar to theorem [3.3].

Theorem 3.7: “Let C be a CTFSS and S be a proper ideal of C . The following are (i) S is prime (ii) $\check{e}_1, \check{e}_2, \check{e}_3 \in C$ such that $\langle \check{e}_1 \rangle \Gamma \langle \check{e}_2 \rangle \Gamma \langle \check{e}_3 \rangle \subseteq S \Rightarrow \check{e}_1 \in S$ or $\check{e}_2 \in S$ or $\check{e}_3 \in S$ are equivalent.”

(i) \Rightarrow (ii) If S is a “prime ideal” of C . Let $\check{e}_1, \check{e}_2, \check{e}_3 \in C \ni \langle \check{e}_1 \rangle \Gamma \langle \check{e}_2 \rangle \Gamma \langle \check{e}_3 \rangle \subseteq S$.

As $\langle \check{e}_1 \rangle, \langle \check{e}_2 \rangle, \langle \check{e}_3 \rangle$ are ideals of C, S is prime implies that $\langle \check{e}_1 \rangle \subseteq S$ or $\langle \check{e}_2 \rangle \subseteq S$ or $\langle \check{e}_3 \rangle \subseteq S$. We have $\check{e}_1 \in \langle \check{e}_1 \rangle, \check{e}_2 \in \langle \check{e}_2 \rangle, \check{e}_3 \in \langle \check{e}_3 \rangle$. Therefore $\check{e}_1 \in S$ or $\check{e}_2 \in S$ or $\check{e}_3 \in S$.

(ii) \Rightarrow (i) from condition (i) $\check{e}_1, \check{e}_2, \check{e}_3 \in C, \langle \check{e}_1 \rangle \Gamma \langle \check{e}_2 \rangle \Gamma \langle \check{e}_3 \rangle \subseteq S \Rightarrow \check{e}_1 \in S$ or $\check{e}_2 \in S$ or $\check{e}_3 \in S$.

Here E_1, E_2, E_3 are three ideals of $C \ni E_1 \Gamma E_2 \Gamma E_3 \subseteq S$.

Presume $E_1 \not\subseteq S, E_2 \not\subseteq S$ there exists $\check{e}_1 \in E_1 \ni \check{e}_1 \notin S$ & there exists $\check{e}_2 \in E_2 \ni \check{e}_2 \notin S$.

Let $\check{e}_3 \in E_3$ then $\langle \check{e}_1 \rangle \Gamma \langle \check{e}_2 \rangle \Gamma \langle \check{e}_3 \rangle \subseteq S$.

Since by our supposition $\check{e}_1 \in S$ or $\check{e}_2 \in S$ or $\check{e}_3 \in S$. But $\check{e}_1 \notin S$ and $\check{e}_2 \notin S$ there $\check{e}_3 \in S \Rightarrow E_3 \subseteq S$.

Then S is a “prime ideal” of C .

Theorem 3.8: “Let R be a CTFSS and Z be a “proper ideal” of R . Then the conditions

(i) S is prime

(ii) Let a_1 ,

$b_1, c_1 \in R \Rightarrow R\Gamma R \Gamma a_1 \Gamma R \Gamma R \Gamma b_1 \Gamma R \Gamma R \Gamma c_1 \Gamma R \Gamma R \subseteq Z \Rightarrow a_1 \in Z$ or $b_1 \in Z$ or $c_1 \in Z$.

(iii) If Z_1, Z_2, Z_3 are three right ideals of R such that $Z_1 \Gamma Z_2 \Gamma Z_3 \subseteq Z$ then $Z_1 \subseteq Z$ or $Z_2 \subseteq Z$ or $Z_3 \subseteq Z$

(iv) If T_1, T_2, T_3 are three middle ideals of R such that $T_1 \Gamma T_2 \Gamma T_3 \subseteq Z$ then $T_1 \subseteq Z$ or $T_2 \subseteq Z$ or $T_3 \subseteq Z$.

(v) If Y_1, Y_2, Y_3 are three left ideals of R such that $Y_1 \Gamma Y_2 \Gamma Y_3 \subseteq Z$ then $Y_1 \subseteq Z$ or $Y_2 \subseteq Z$ or $Y_3 \subseteq Z$ are equivalent”.

Proof: (i) \Rightarrow (ii) If Z be a “prime ideal” of R .

Let $a_1, b_1, c_1 \in C$ such that $R\Gamma R \Gamma a_1 \Gamma R \Gamma R \Gamma b_1 \Gamma R \Gamma R \Gamma c_1 \Gamma R \Gamma R \subseteq Z$

Now $\langle a_1 \rangle \Gamma \langle b_1 \rangle \Gamma \langle c_1 \rangle \subseteq (R\Gamma R \Gamma a_1 \Gamma R \Gamma R) \Gamma (R\Gamma R \Gamma b_1 \Gamma R \Gamma R) \Gamma (R\Gamma R \Gamma c_1 \Gamma R \Gamma R) = R\Gamma R \Gamma a_1 \Gamma R \Gamma R \Gamma b_1 \Gamma R \Gamma R \Gamma c_1 \Gamma R \Gamma R \subseteq Z \Rightarrow a_1 \in Z$ or $b_1 \in Z$ or $c_1 \in Z$. (by above theorem)

(ii) \Rightarrow (iii) Suppose that condition (ii) holds. Let Z_1, Z_2, Z_3 are three right ideals of R such that $Z_1 \Gamma Z_2 \Gamma Z_3 \subseteq Z$. Suppose if possible $Z_1 \not\subseteq Z, Z_2 \not\subseteq Z, Z_3 \not\subseteq Z$.

$Z_1 \not\subseteq Z, Z_2 \not\subseteq Z, Z_3 \not\subseteq Z$ then there exist a_1, b_1, c_1 such that $a_1 \in Z_1, a_1 \notin Z, b_1 \in Z_2, b_1 \notin Z$ and $c_1 \in Z_3, c_1 \notin Z$. $a_1 \in Z_1, b_1 \in Z_2, c_1 \in Z_3 \Rightarrow \langle a_1 \rangle \Gamma \langle b_1 \rangle \Gamma \langle c_1 \rangle \subseteq Z_1 \Gamma Z_2 \Gamma Z_3 \subseteq Z$.

Now $R\Gamma R \Gamma a_1 \Gamma R \Gamma R \Gamma b_1 \Gamma R \Gamma R \Gamma c_1 \Gamma R \Gamma R \subseteq Z_1 \Gamma Z_2 \Gamma Z_3 \subseteq S \Rightarrow a_1 \in Z$ or $b_1 \in Z$ or $c_1 \in Z$. It is a contradiction. Therefore $Z_1 \subseteq Z$ or $Z_2 \subseteq Z$ or $Z_3 \subseteq Z$. And hence Z is a “prime ideal” of R .

In a similar manner also we can prove (ii) \Rightarrow (iv) as well as (ii) \Rightarrow (v).

(iii) \Rightarrow (i)

Suppose for any three right ideals Z_1, Z_2, Z_3 of $R, Z_1 \Gamma Z_2 \Gamma Z_3 \subseteq Z$. implies $Z_1 \subseteq Z$ or $Z_2 \subseteq Z$ or $Z_3 \subseteq Z$. Let F, G, H be any three ideals of C such that $F \Gamma G \Gamma H \subseteq Z$.

Then F, G, H are right ideals such that $F \Gamma G \Gamma H \subseteq Z \Rightarrow F \subseteq Z$ or $G \subseteq Z$ or $H \subseteq Z$.

Hence Z is “prime ideal” of R . In the same way we prove (iv) \Rightarrow (i) and (v) \Rightarrow (i)

Theorem 3.9: “Let C be a $CT\Gamma SS$ and S be a proper ideal of C . Then

- (i) S is semiprime ideal of C ,
- (ii) For $r \in C, \exists \langle r \rangle \Gamma^2 \langle r \rangle \subseteq S$ implies $r \in S$.
- (iii) For $r \in C, R\Gamma R \Gamma r \Gamma R \Gamma R \Gamma r \Gamma R \Gamma R \Gamma r \Gamma R \Gamma R \subseteq S$ implies $r \in S$.

(iv) For any right ideals T of $C, T\Gamma T\Gamma T \subseteq S$ implies $T \subseteq S$.

(v) M be any lateral ideal of $C, M\Gamma M\Gamma M \subseteq S \Rightarrow M \subseteq S$.

(vi) N be any left ideal of $C, N\Gamma N\Gamma N \subseteq S$ implies $N \subseteq S$ ”.

Proof: Theorem 3.9 is as similar as theorem 3.7 and theorem 3.8

Theorem 3.10: “An ideal S of $CT\Gamma SSR$ is a prime ideal iff $R \setminus S$ is an m -system of R or empty”.

Proof: Suppose S is a “prime ideal” of a $CT\Gamma SSR$ and $R \setminus S \neq \emptyset$. Let $r, t, u \in R \setminus S$ then $r \notin S, t \notin S$ and $u \notin S$. Suppose if possible $R\Gamma R \Gamma r \Gamma R \Gamma R \Gamma t \Gamma R \Gamma R \Gamma u \Gamma R \Gamma R \not\subseteq R \setminus S \Rightarrow R\Gamma R \Gamma r \Gamma R \Gamma R \Gamma t \Gamma R \Gamma R \Gamma u \Gamma R \Gamma R \subseteq S$.

Since S is prime, either $r \in S$ or $t \in S$ or $u \in S$. It is a contradiction.

Therefore $R\Gamma R \Gamma r \Gamma R \Gamma R \Gamma t \Gamma R \Gamma R \Gamma u \Gamma R \Gamma R \subseteq R \setminus S$. So “ $R \setminus S$ is an m -system”.

On the other hand if “ $R \setminus S$ is either an m -system of R or $R \setminus S = \emptyset$ ”.

If $R \setminus S = \emptyset$ then $R = S \Rightarrow S$ is a prime ideal of R . Suppose that “ $R \setminus S$ is an m -system” of R

Let $r, t, u \in R$ and $\langle r \rangle \Gamma \langle t \rangle \Gamma \langle u \rangle \subseteq S$. Suppose if possible, $r, t, u \notin S \Rightarrow r, t, u \in R \setminus S$

Since $R \setminus S$ is an m -system $\Rightarrow R\Gamma R \Gamma r \Gamma R \Gamma R \Gamma t \Gamma R \Gamma R \Gamma u \Gamma R \Gamma R \subseteq R \setminus S \Rightarrow R\Gamma R \Gamma r \Gamma R \Gamma R \Gamma t \Gamma R \Gamma R \Gamma u \Gamma R \Gamma R \not\subseteq S \Rightarrow \langle r \rangle \Gamma \langle t \rangle \Gamma \langle u \rangle \not\subseteq S$.

We have absurdity. Thus $r \in S$ or $t \in S$ or $u \in S$. Hence S is a “prime ideal” of R .

Theorem 3.11: “Let R be a $CT\Gamma SS$ and S be a proper ideal of R . then S is semi prime $\Leftrightarrow R \setminus S$ is an n -system or empty”.

Proof: Similar to theorem 3.9

Definition 3.12: An ideal Q of a $T\Gamma S$ -SRM is known as “strongly irreducible” iff for any ideals R, S, T of $M, R \cap S \cap T \subseteq Q \Rightarrow R \subseteq Q$ or $S \subseteq Q$ or $T \subseteq Q$.

Theorem 3.13: “Let M is an ideal of a $CT\Gamma SS R$. Then

- (i) M is strongly irreducible
- (ii) If $m_1, m_2, m_3 \in R$ such that $\langle m_1 \rangle \cap \langle m_2 \rangle \cap \langle m_3 \rangle \subseteq M$ then $m_1 \in M$ or $m_2 \in M$ or $m_3 \in M$
- (iii) $R \setminus M$ is an i -system of R ”.

Proof: (i) \Rightarrow (ii)

Let M is a strongly irreducible ideal of R .

Let $m_1, m_2, m_3 \in R$ such that $\langle m_1 \rangle \cap \langle m_2 \rangle \cap \langle m_3 \rangle \subseteq M$

Since M is strongly irreducible $\langle m_1 \rangle \subseteq M$ or $\langle m_2 \rangle \subseteq M$ or $\langle m_3 \rangle \subseteq M \Rightarrow m_1 \in M$ or $m_2 \in M$ or $m_3 \in M$.

(ii) \Rightarrow (iii)

If $m_1, m_2, m_3 \in R$ such that $\langle m_1 \rangle \cap \langle m_2 \rangle \cap \langle m_3 \rangle \subseteq M \Rightarrow m_1 \in M$ or $m_2 \in M$ or $m_3 \in M$.

Suppose $m_1, m_2, m_3 \in R \setminus M \Rightarrow \langle m_1 \rangle \cap \langle m_2 \rangle \cap \langle m_3 \rangle \not\subseteq M \Rightarrow \langle m_1 \rangle \cap \langle m_2 \rangle \cap \langle m_3 \rangle \not\subseteq M \Rightarrow R \setminus M \neq \emptyset$

Therefore $R \setminus M$ is an “ i -system” of R .

(iii) \Rightarrow (i)

Assume “ $R \setminus M$ is an i -system of R ”.

Assume that P, Q, T the ideals of $R \ni P \not\subseteq M, Q \not\subseteq M$ and $T \not\subseteq M \exists m_1 \in P, m_2 \in Q$ and $m_3 \in T$ such that $m_1, m_2, m_3 \in R \setminus M \Rightarrow$ By condition (iii)

$\exists \langle m_1 \rangle \cap \langle m_2 \rangle \cap \langle m_3 \rangle \cap R \setminus M \neq \emptyset \Rightarrow$ There exists $m_4 \in \langle m_1 \rangle \cap \langle m_2 \rangle \cap \langle m_3 \rangle$ and $m_4 \notin M$. Therefore $m_4 \in P \cap Q \cap T$ and $m_4 \notin M \Rightarrow P \cap Q \cap T \not\subseteq M$. Thus “ M is a strongly irreducible ideal of R ”.

Theorem 3.14: “Let R be a Γ -S-SR. If x is a strongly regular element in $R \Rightarrow \exists y \in R \ni x \in x\Gamma y\Gamma x$ and $y \in y\Gamma x\Gamma y$ ”.

Proof: If “ x is a strongly regular element of R ” then $x \in x\Gamma e\Gamma x \Rightarrow x = x\alpha e\beta x$ for $\alpha, \beta \in \Gamma$

Let us take $y \in e\Gamma x\Gamma e \Rightarrow y = e\beta x\alpha e$ then $y \in R$.

And $x\Gamma y\Gamma x = x\Gamma(e\Gamma x\Gamma e)\Gamma x = (x\Gamma e\Gamma x)\Gamma e\Gamma x = x\Gamma e\Gamma x$. Since $x \in x\Gamma e\Gamma x \Rightarrow x \in x\Gamma y\Gamma x$.

Again $y\Gamma x\Gamma y = (e\Gamma x\Gamma e)\Gamma x\Gamma(e\Gamma x\Gamma e) = e\Gamma x\Gamma e \Rightarrow y \in e\Gamma x\Gamma e \Rightarrow y = e\beta x\alpha e \Rightarrow y \in y\Gamma x\Gamma y$

Definition 3.15: “An element x of a Γ -S-SRR is known as an α -idempotent if $x\alpha x\alpha x = x$ for some $\alpha \in \Gamma$ ”.

Example 3.16: Every identity and zero elements of a ternary Γ -SO semiring are α -idempotents.

Definition 3.17: “Let M be a Γ -S-SR. Then $m \in M$ is said to be a strongly Regular if there exist for $n \in M, \alpha, \beta \in \Gamma$ such that $m = m\alpha n\beta m$ if every element of M is **strongly Regular** then M is called a strongly Regular ternary Γ -SO – semiring”.

Definition 3.18: “Let M be a ternary - Γ SO semi ring. Then $m \in M$ is said to be a **strongly Regular** if there exist for $n \in M, \alpha, \beta, \gamma, \delta \in \Gamma$ such that $m = m\alpha n\beta m\gamma\delta m$ ”.

Note 3.19: Definition 4.7 and definition 4.8 both are equal.

Definition 3.20: “An element x ” of a Γ -S-SR R is known as an (α, β) -idempotent if $x\alpha x\beta x = x$ for some $\alpha, \beta \in \Gamma$.

Note 3.21: (α, β) -idempotent of an element simply called idempotent.

Remark 3.22: Every idempotent element is a strongly regular element.

Def 3.23: “The **centre** of a ternary Γ -SO semiring R is defined as the $\{a \in R / a\alpha x\beta y = x\beta y\alpha a = y\alpha a\beta x \forall x, y \in R, \alpha, \beta \in \Gamma\}$. And it is denoted by $C(R)$ ”.

Therefore “ $C(R) = \{a \in R / a\alpha x\beta y = x\beta y\alpha a = y\alpha a\beta x \forall x, y \in R, \alpha, \beta \in \Gamma\}$ ”.

Th 3.24: “The centre $C(R)$ of a Γ -S-SRR is a ternary Γ -SO sub semiring of R ”.

Proof: “Let $\{b_j : j \in I\}$ be a summable family in $R \ni b_j \in C(R) \forall j \in I$. Then $\sum_j b_j \in R$

And $b_j\alpha x\beta y = x\beta y\alpha b_j = y\alpha b_j\beta x \forall x, y \in R, \alpha, \beta \in \Gamma$ for $\alpha, \beta \in \Gamma, x, y \in R(\sum_j b_j)\alpha x\beta y = \sum_j (b_j\alpha x\beta y) = \sum_j (x\beta y\alpha b_j) = x\beta y\alpha(\sum_j b_j) = y\alpha(\sum_j b_j)\beta x \Rightarrow (\sum_j b_j)\alpha x\beta y = x\beta y\alpha(\sum_j b_j) = y\alpha(\sum_j b_j)\beta x \Rightarrow \sum_j b_j \in C(R)$.

let $a, b, c \in C(R)$ and $a \in R \Rightarrow a\alpha x\beta y = x\beta y\alpha a = y\alpha a\beta x \forall x, y \in R, \alpha, \beta \in \Gamma$.

$\forall x, y \in R, \alpha, \beta, \gamma, \delta \in \Gamma, (a\alpha b\beta c)\gamma x\delta y = a\alpha b\beta(c\gamma x\delta y) = a\alpha b\beta(x\delta y\gamma c) = a\alpha b\beta(y\gamma c\delta x) = a\alpha(b\beta y\gamma c)\delta x = a\alpha(y\gamma c\beta b)\delta x = (a\alpha y\gamma c)\beta b\delta x = (y\gamma c\alpha a)\beta b\delta x = y\gamma(c\alpha a\beta b)\delta x = y\gamma(a\beta b\alpha c)\delta x$

Similarly, we can show that $(a\alpha b\beta c)\gamma x\delta y = x\delta y\gamma(a\alpha b\beta c)$.

Therefore $a\alpha b\beta c \in C(R)$ and thus “ $C(R)$ is a ternary Γ -SO semiring R ”.

IV. CONCLUSION

Mainly we introduced in this paper about regular ternary Γ -So-Semirings and characterized ternary Γ -SO-Semiring.

ACKNOWLEDGEMENT

we are very thank full to Dr. D. MadhusudhanaRao, Professor in Mathematics, VSR & NVR College, Tenali and Dr. P. Siva Prasad, Associate Professor, Vignan University, Vadlamudi, Guntur(Dt).

REFERENCES

- G.V.S. Acharyulu, Matrix representable So-rings, Semigroup Forum, Springer-Verlag,46(1993), 31-47, doi: 10.1007/BF02573542.
- G.V.S. Acharyulu, A Study of Sum-Ordered Partial Semirings, Doctoral thesis, AndhraUniversity, 1992.
- M.A. Arbib, E.G. Manes, Partially Additive Categories and Flow-diagram Semantics,Journal of Algebra, 62(1980), 203-227.
- T.K. Dutta, S.K. Sardar, Semiprime ideals and irreducible ideals of Γ -semirings, NoviSad J. Math., 30(1)(2000), 97-108.
- T.K. Dutta, S.K. Sardar, On the Operator Semirings of a Γ -semiring, Southeast Asian Bulletin of Mathematics, 26(2)(2002), 215-225.
- Jonathan S. Golan. : Semirings and their Applications, Kluwer Academic Publishers,1999.
- E.G. Manes, D.B. Benson, The Inverse Semigroup of a Sum-Ordered Partial Semiring, Semigroup Forum, 31(1985), 129-152, doi: 10.1007/BF02572645.
- M. Murali Krishna Rao, Γ -semirings-I, Southeast Asian Bulletin of Mathematics,19(1)(1995), 49-54.
- M. Murali Krishna Rao, Γ -semirings-II, Southeast Asian Bulletin of Mathematics,21(1997), 281-287.
- M. Murali Krishna Rao, The Jacobson Radical of a Γ -semiring, Southeast Asian Bulletin of Mathematics, 23(1999), 127-134.
- M. Murali Krishna Rao, Γ -Semirings, Doctoral thesis, Andhra University, 1995.
- SajaniLavanya. M., MadhusudhanaRao. D., and V. Syam Julius Rajendra, *On Quasi-Ternary Γ -Ideals and Bi-Ternary Γ -Ideals in Ternary Γ -Semirings*-International Journal of Mathematics and Statistics Invention, Volume 6, Issue 3, (September-2015), PP 05-14.
- SajaniLavanya. M., MadhusudhanaRao. D., and V. Syam Julius Rajendra, *On Lateral Ternary Γ -Ideals of Ternary Γ -Semirings*-American International Journal of Research in Science, Technology, Engineering & Mathematics (AIJRSTEM), 12(1), September-November, 2015, pp: 11-14.
- SajaniLavanya. M., MadhusudhanaRao. D., and V. Syam Julius Rajendra, *Prime Bi-ternary Γ -Ideals in Ternary Γ -Semirings*-British Journal of Research, Volume 2, Issue 6, November-December, 2015, pp: 156-166.
- SajaniLavanya. M., MadhusudhanaRao. D., and V. Syam Julius Rajendra, *A Study on Jacobson Radical of a Ternary Γ -Semieing*-International Journal of Mathematics and Computer Applications Research, Volume 6, Issue 1, (Feb-2016), PP 17-30.
- SajaniLavanya. M., MadhusudhanaRao. D., and VB. SubrahmanyeswaraRaoSeetamraju, *On Right Ternary Γ -ideals of Ternary Γ -Semiring*, IMPACT-International Journal of Applied, Natural and Social Sciences, Vol. 4, Issue 5, May 2016, 107-114.
- SajaniLavanya. M., MadhusudhanaRao.D., and Vasantha. M, *A Study on Inverses Strongly Ternary Gamma Semiring*-Accepted for publication in Associated Asia Research Foundation.
- SajaniLavanya. M., MadhusudhanaRao. D., and VB. SubrahmanyeswaraRaoSeetamraju, *Properties of Right Strongly Prime Ternary Gamma Semiring*- Accepted for publication in International Organization of Scientific Research Journal of Mathematics.
- M. Siva Mala, K. Siva Prasad, Partial Γ -Semirings, Southeast Asian Bulletin of Mathematics, 38(6)(2014), 873-885.
- M. Siva Mala, K. Siva Prasad, (ϕ, ρ) -Representation of Γ -So-Rings, Iranian Journal of Mathematical Sciences and Informatics, 10(1)(2015), 103-119, :10.7508/ijmsi.2015.01.008.
- M. Siva Mala, K. Siva Prasad, Ideals of Sum-Ordered partial Γ -Semirings, SoutheastAsian Bulletin of Mathematics, 40(2016), 413-426.
- K. Siva Prasad, M. Siva Mala, P.V. SrinivasaRao, Green's Relations in Partial Γ -Semirings, International Journal of Algebra and Statistics(IJAS), 2(2)(2013), 21-28, doi:10.20454/ijas.2013.755.
- M. Siva Mala, K. Siva Prasad, Prime Ideals of Γ -So-rings, International Journal of Algebra and Statistics(IJAS), 3(1)(2014), 1-8, doi: 10.20454/ijas.2014.774.
- M. Siva Mala, K. Siva Prasad, Semiprime Ideals of Γ -So-rings, International Journal of Algebra and Statistics(IJAS), 3(1)(2014), 26-33, doi: 10.20454/ijas.2014.814.
- M.E. Streenstrup, Sum-Ordered Partial Semirings, Doctoral thesis, Graduate school ofthe University of Massachusetts, Feb 1985 (Department of Computer and Information Science).
- P.V. SrinivasaRao, Ideals of Sum Ordered Partial semirings, International Journal of ComputationalCognition(IJCC), 7(2)(2009), 59-64.
- P.V. SrinivasaRao, Ideal Theory of Sum-Ordered Partial Semirings, Doctoral thesis, AcharyaNagarjuna University, 2011.
- Syamjuliusrajendra. V, Dr. Madhusudhanarao. D, SajaniLavanya. M- A Study on Completely Regular po-Ternary Γ -Semiring, Asian Academic Research Journal of Multidisciplinary, Volume 2, Issue 4, September 2015, PP: 240-246.
- Syam Julius Rajendra. V, Dr.MadhusudhanaRao. D and SajaniLavanya. M-On Pure PO-Ternary Γ -Ideals in Ordered Ternary Γ -Semirings, IOSR Journal of Mathematics (IOSR-JM), Volume 11, Issue 5 Ver. IV (Sep. - Oct. 2015), PP 05-13.