

Temperature Distribution to Fluid Transport in a Vertical Porous Tube under the Action of Peristalsis

V. Radhakrishna Murthy, P. Sudam Sekhar

Abstract: Peristaltic transport of a Newtonian fluid, with heat transfer, in a vertical porous axisymmetric tube under long wave length approximation is considered. Closed form solution is obtained as an asymptotic expansion in terms of porosity and free convection parameters. Expressions for temperature, coefficient of heat transfer and pressure -flow relationship at the boundary wall of the tube are derived. It is observed that pressure drop increases as amplitude ratio increases. Further, it has been observed that for some specific values of other parameters under consideration the mean flux significantly increases by about 8 to 10 percent as Grashof number increases from 1 to 2.

Key words- Peristalsis, heat transfer, porosity, pressure drop, mean flux.

I. INTRODUCTION

Peristalsis is a mechanism for fluid transport which is achieved by the passage of area contraction and expansion waves along the length of the distensible tube. The peristaltic transport may be useful in processes where it is desirable to avoid using internal moving parts such as piston in the pumping process. It is known to be one of the main mechanisms for fluid transport in physiological systems, in particular, urine passage through Ureter, food mixing and chyme movement in the intestines. Based on this principle, a blood pump in dialysis is designed to prevent the transported fluid from being contaminated. Peristaltic transport of toxic liquid is used in nuclear industries so as not to contaminate outside environment.

In view of its importance, studies of peristaltic transport have been carried out theoretically and experimentally by various authors [1-12]. In particular, Radhakrishnamacharya [1] investigated peristaltic pumping in an axisymmetric tube under long wave length approximation using power law fluid model. Takabatake et al [2] have developed complete numerical solutions for peristaltic pumping and its efficiency in Cartesian and axisymmetric geometries. Recently, the thermo dynamical aspects of peristalsis have received attention [3-7] as it might be relevant in certain processes like oxygenation and hemodialysis.

Translocation of water in tall trees is a phenomenon which is not well understood by scientists for well over centuries. It is speculated that peristalsis might be involved in this process since diameters of trunks of the trees are found to vary with time. Hence some authors [10-11] have investigated the peristalsis phenomena particularly in the case of transport of water in trees.

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Further, it is observed that flow of water takes place through the porous matrix of the tree. Keeping in view of the above observations, in this paper an attempt has been made, to study heat transfer for the Newtonian fluid flow under the action of peristalsis in a vertical axisymmetric porous tube. Assuming long wave length approximation analytical expression for temperature, pressure drop and heat transfer coefficient have been derived and closed form solutions in terms of porosity (σ^2) and free convection (G_m) parameters have been obtained using a perturbation solution. From the analysis it has been observed that for fixed values of other parameters a small change in free convection leads to significant increase in mean flux.

II. MATHEMATICAL FORMULATION

The governing flow of a Newtonian, incompressible fluid through an axisymmetric vertical tube, filled with porous material is considered. Peristaltic waves of very large wave length are assumed to travel down the wall of the tube. In this problem the cylindrical polar coordinate (X, R) is chosen where X represents axial coordinate and R is radial coordinate.

The simplified, zeroth order equations, under long wave approximation, governing the flow [8, 9, 10] are

$$0 = -\frac{\partial p}{\partial X} + \frac{\mu}{R} \frac{\partial}{\partial R} \left(R \frac{\partial W}{\partial R} \right) - \frac{\mu}{k_0} W + \rho g \beta (T - T_0)$$

(1)

$$0 = \frac{\partial W}{\partial X} + \frac{U}{R} + \frac{\partial U}{\partial R}$$

(2)

$$0 = \frac{K}{R} \frac{\partial}{\partial R} \left(R \frac{\partial T}{\partial R} \right) + \mu \left(\frac{\partial W}{\partial R} \right)^2 + \frac{\mu}{k_0} W^2$$

(3)

The equation of the tube wall is given by

$$H(X, t) = a + b \sin \left(\frac{2\pi}{\lambda} \right) (X - ct)$$

(4)

Where the fluid velocity components are W and U in the direction X and R respectively, T is the fluid temperature, T_0 is the boundary temperature, 'p' is the pressure, ' ρ ' is the density, the coefficient of expansion is β , 'k' stands for thermal conductivity of the fluid, ' μ ' is viscosity coefficient, ' g ' is the acceleration due to gravity, ' c ' is the phase speed of the wave, ' k_0 ' is the permeability of the medium, ' a ' is the mean radius of the tube, ' b ' is the amplitude and the wave length is ' λ '. The assumed boundary conditions for the problem under considerations are

$$T = T_0 \text{ and } W = 0 \text{ at } R = \pm H$$

(5)



Relative to the laboratory frame the wave frame of reference moves with a constant speed 'c'. In the wave frame the measurement of the variables 'x' and 'r' are defined by $x = X - ct, r = R$

$$(6)$$

The corresponding velocity components of the fluid are

$$w = W - c, u = U \quad (7)$$

The governing system of equations of the fluid flow in the wave frame of reference are written as

$$0 = -\frac{\partial p}{\partial x} + \frac{\mu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) - \frac{\mu}{k_0} (w + c) + \rho g \beta (T - T_0) \quad (8)$$

$$0 = \frac{\partial w}{\partial x} + \frac{u}{r} + \frac{\partial u}{\partial r} \quad (9)$$

$$0 = \frac{k}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \mu \left(\frac{\partial w}{\partial r} \right)^2 + \frac{\mu}{k_0} (w + c)^2 \quad (10)$$

The boundary conditions are

$$w = -c \text{ and } T = T_0 \text{ at } r = a + b \sin \left(\frac{2\pi}{\lambda} x \right) \quad (11)$$

The non-dimensional quantities can be introduced as

$$x' = \frac{x}{\lambda}, r' = \frac{r}{a}, w' = \frac{w}{c}, u' = \frac{\lambda}{ac} u, \theta = \frac{T - T_0}{T_0}, p' = \frac{p}{\mu c \lambda / a} \quad (12)$$

in to the equations (8) - (11), we get (after dropping the primes)

$$0 = -\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial w}{\partial r} \right) - \sigma^2 (w + 1) G_m \theta \quad (13)$$

$$0 = \frac{\partial w}{\partial x} + \frac{u}{r} + \frac{\partial u}{\partial r} \quad (14)$$

$$0 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \theta}{\partial r} \right) + E_m \left(\frac{\partial w}{\partial r} \right)^2 + \sigma^2 E_m (w + 1)^2 \quad (15)$$

The boundary conditions for the dimensionless quantities are

$$w = -1 \text{ and } \theta = 0 \text{ at } r = \pm \eta(x) \quad (16)$$

$$\text{Where } \eta(x) = 1 + \varepsilon \sin 2\pi x \quad (17)$$

$$\sigma^2 = \frac{a^2}{k_0} (\text{Porosity parameter}), G_m = \frac{g \beta T_0 a^3}{v^2} (\text{Grashof number})$$

$$E_m = \frac{\mu c^2}{KT_0} (\text{Eckert number}) \text{ and } \varepsilon = \frac{b}{a} (\text{amplitude ratio})$$

III. ANALYSIS

Equations (13) and (15) are simultaneous non-linear equations hence for arbitrary values of all the parameters it is quite impossible to get an exact solution. So, we opt perturbation method in the form of a series as

$$F = (F_{00} + G_m F_{01} + \dots) + \sigma^2 (F_{10} + \dots) + \dots \quad (18)$$

Where, the flow variable is denoted as F.

By solving the resulting equations which are obtained by using (18) in equation (13), (15) and (16), the solutions for velocity component 'w' and temperature 'θ' can be obtained under the suitable boundary conditions. However, the expressions for velocity component w is not presented here and the expressions for temperature θ is

$$\theta = (\theta_{00} + G_m \theta_{01} + \dots) + \sigma^2 (\theta_{10} + \dots) + \dots \quad (19)$$

Where

$$\theta_{00} = \frac{-1}{64} E_m \alpha^2 (r^4 - \eta^4) \theta_{01} = \frac{-1}{32} E_m [\alpha \tau (r^4 - \eta^4) + \frac{E_m}{768} (r^8 - 12r^4 \eta^4 + 11\eta^8)]$$

$$\theta_{10} = \frac{-1}{32} E_m [\alpha \gamma (r^4 - \eta^4) + \frac{\alpha^2}{36} (2r^6 - 9r^4 \eta^2 + 7\eta^6)] - \frac{\alpha^2}{72} (2r^6 - 9r^4 \eta^2 + 18r^4 \eta^4 - 20\eta^6)$$

$$\alpha = \frac{8(\overline{Q}_{00} + \pi)}{\pi(1 - 2\eta^2)}$$

$$\tau = \frac{1}{\pi(1 - 2\eta^2)} [32\overline{Q}_{01} - \frac{1}{9} E_m \alpha^2 (140\eta^6 - 18\eta^4) + 1]$$

$$\gamma = \frac{1}{(1 - 2\eta^2)} \left[\frac{8}{\pi} \overline{Q}_{10} - \frac{\alpha}{24} (1 - 21\eta^4 - 6\eta^2) \right]$$

The pressure drop over one wave length is defined by

$$\Delta p_\lambda = \int_0^\lambda \frac{\partial p}{\partial x} dx \quad (20)$$

Substituting the relation for $\frac{\partial p}{\partial x}$ from equation (13) in (20) and using expressions for velocity (which is not presented) and temperature from equation (19), the non-dimensional pressure drop can finally be obtained as

$$\Delta p = \frac{\Delta p_\lambda}{\mu c \lambda / a} = (\Delta p_{00} + G_m \Delta p_{01} + \dots) + \sigma^2 (\Delta p_{10} + \dots) + \dots \quad (21)$$

Where,

$$\Delta p_{00} = \frac{32\varepsilon^2 \left(1 - \frac{\varepsilon^2}{16} \right) - 8 \overline{Q}_{00} \left(1 + \frac{3\varepsilon^2}{2} \right)}{(1 - \varepsilon^2)^{7/2}}$$

$$\Delta p_{01} = \frac{5}{6} E_m \left[1 + \left(\overline{Q}_{00} - \left(1 + \frac{\varepsilon^2}{2} \right)^2 \right) \left(\frac{1 + \frac{3\varepsilon^2}{2}}{(1 - \varepsilon^2)^{7/2}} + \frac{\overline{Q}_{00} - \left(1 + \frac{\varepsilon^2}{2} \right)}{(1 - \varepsilon^2)^{9/2}} \right) - 8 \overline{Q}_{01} \frac{1 + \frac{3\varepsilon^2}{2}}{(1 - \varepsilon^2)^{7/2}} \right]$$

$$\Delta p_{10} = -6 \overline{Q}_{10} \left[\frac{2 + \varepsilon^2}{2(1 - \varepsilon^2)^{5/2}} \right] - \frac{7}{20} \left[1 + \frac{\overline{Q}_{00} - 1}{(1 - \varepsilon^2)^{9/2}} \right]$$

And \overline{Q} is the dimensionless mean flux.

The non-dimensional form of heat transfer coefficient Z on the boundary of the tube is given by

$$Z = \left(\frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial r} \frac{\partial \eta}{\partial x} \right) \text{ at } r = \eta \quad (22)$$

Which in view of (18), can be expressed as



$$Z = (Z_{00} + G_m Z_{01} + \dots) + \sigma^2 (Z_{10} + \dots) + \dots \quad (23)$$

Where

$$Z_{00} = \frac{\partial \theta_{00}}{\partial r} \frac{\partial \eta}{\partial x}, Z_{01} = \frac{\partial \theta_{01}}{\partial r} \frac{\partial \eta}{\partial x}, Z_{10} = \frac{\partial \theta_{10}}{\partial r} \frac{\partial \eta}{\partial x}$$

IV. RESULTS AND DISCUSSION

Analytical expressions for temperature, pressure drop and the coefficient of heat transfer are given by the equations (19), (21) and (22) respectively. To study explicitly these quantities are numerically evaluated and the results are presented in fig: 1-to fig: 4 and Table-1 to Table -4. Here the effects of various parameters on these flow variables are depicted clearly.

Figs 1 -4 show the variation of temperature vs. X with respect to various parameters. It is observed from Figs.1 – 4 that for a fixed values of certain parameters, temperature first increases down the tube and then decreases. This may be due to the effects of peristalsis. From figs. 1 and 2, we can see that, the temperature increases as Eckert number (E_m) or Grashof number(G_m) increases when other parameters are made fixed. Further, the temperature increases as σ^2 or ε increases i.e. the temperature increases as the tube becomes more porous or peristaltic wave amplitude increases (Figs 3 and 4).

It has been observed that the mean flux, \bar{Q} , increases by about 8 to 10 percent as G_m increases from 1 to 2 when some other parameters are given a fixed value during the computational process.

Heat transfer coefficient, Z, on the boundary of the tube, is evaluated numerically and the results are presented in tables 1- 4. Z increases down the tube and then decreases, in the case of temperature, which may be due to peristalsis.

From table 1 and 2, we can see that, by making some parameters fixed during the computation, there is an increase in the heat transfer coefficient as Eckert number or Grashof number increases. Tables3 and 4 show that Z increase with ε while it decreases with porosity [11-12].

V. CONCLUSION

In the above results it has been observed that temperature is increases for some specific values of different parameters under consideration which is shown in fig-1 to fig-4. Further it has been observed that the absolute value of the temperature significantly increases by about 8 to 10 percent as increase in 1 to 2 in Grashof number (G_m) which is shown in table-1 to table-4.

TABLE -1

Heat transfer variation with E_m
($G_m = 3, \sigma^2 = 2, \varepsilon = 0.1$)

x	$E_m = 1$	$E_m = 3$	$E_m = 5$
0.0	2.42945	22.0037	61.17046
0.4	2.84512	25.72656	71.150905
0.8	0.39168	3.56543	9.91642

TABLE -2

Variation of heat transfer with respect to G_m
($\sigma^2 = 2, \varepsilon = 0.1, E_m = 3$)

x	$G_m = 1$	$G_m = 3$	$G_m = 5$
0.0	7.30205	22.0037	36.70534

0.4	8.54303	25.72656	42.910105
0.8	1.18121	3.56543	5.94964

TABLE -3

Variation of heat transfer with respect to σ^2
($E_m = 3, G_m = 3, \varepsilon = 0.1$)

x	$\sigma^2 = 1$	$\sigma^2 = 2$	$\sigma^2 = 3$
0.0	22.42945	22.0037	21.99219
0.4	25.94013	25.72656	25.451301
0.8	3.76532	3.56543	3.47135

TABLE -4

Heat transfer variation with ε
($E_m = 3, G_m = 3, \sigma^2 = 2$)

x	$\varepsilon = 0$	$\varepsilon = 0.1$	$\varepsilon = 0.2$
0.0	0.0	22.0037	44.0073
0.4	0.0	25.72656	72.89057
0.8	0.0	3.56543	3.763

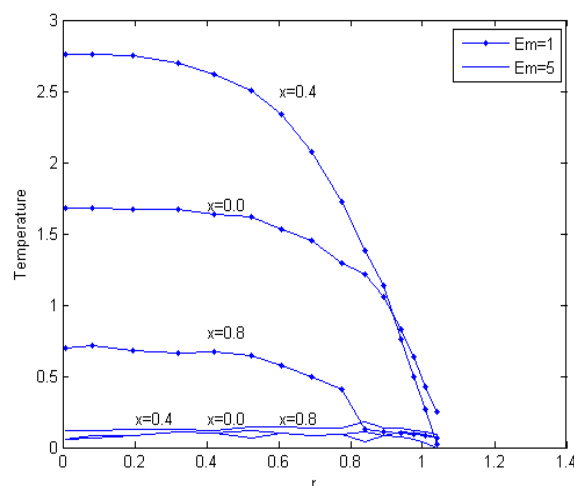


Fig-1, Temperature variation with E_m for the fixed values of $G_m = 3, \sigma^2 = 2$ and $\varepsilon = 0.1$

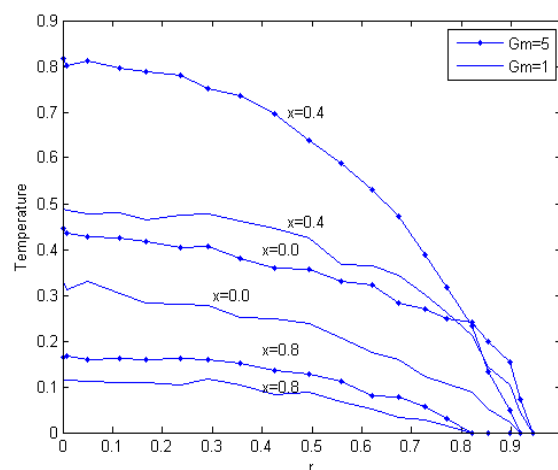


Fig-2, Temperature variation with G_m for the fixed values of $\varepsilon = 0.1, \sigma^2 = 2$ and $E_m = 3$

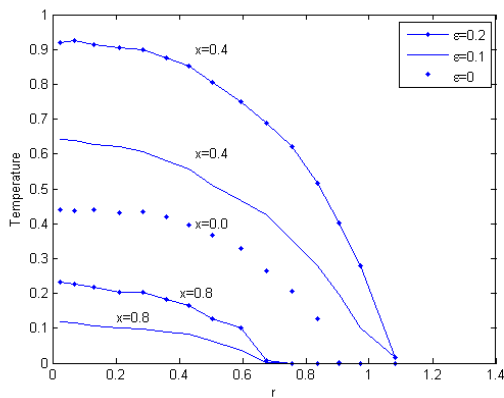


Fig-3, Temperature variation with ε for the fixed values of $G_m = 3, \sigma^2 = 2$ and $E_m = 3$

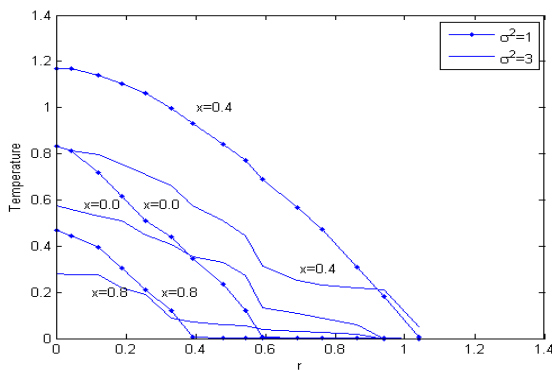


Fig-4, Temperature variation with σ^2 for the fixed values of $G_m = 3, \varepsilon = 0.1$ and $E_m = 3$
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