Depiction of Intuitionistic Fuzzy Soft Linear Spaces

V. Siva Naga Malleswari, V. Amarendra Babu

***Abstract:** We define Intuitionistic fuzzy soft linear spaces (IFSLS) and its properties and characteristics are studied with examples. We propose thedefinition "Cartesian product of Intuitionistic fuzzy soft linear spaces" and these are illustrated by some examples. Besides that, we define Intuitionistic soft subspaces and given examples.*

***Keywords:** Intuitionistic fuzzy soft linear space (IFSLS), Intuitionistic fuzzy soft subspace (IFSSS).*

I. INTRODUCTION

Molodtsov [2] introduces the definition of soft set. K.Attansov [3][4] introduces the intuitionistic fuzzy set. The hypothesis of intuitionistic fuzzy soft set was proposed by Maji et al [6]. Moumita Chiney and S.K.Samanta describes the concept of Intuitionistic fuzzy vector spaces [5]. A.Sezgin Sezer, A.O.Atagin introduced the concept of soft vector spaces [1].

In this paper, we introduce intuitionistic fuzzy soft linear spaces (IFSLS) and some of its properties and characteristics are studied with examples. We propose the definition "Cartesian product of Intuitionistic fuzzy soft linear spaces" and these are illustrated by some examples. Besides that, we define Intuitionistic soft subspaces and given examples.

II. PRELIMINARIES

Definition 2.1: A continuous t-norm is defined by a binary operation $*: [0,1]X[0,1] \to [0,1]$ if $*$satisfies the below mentioned properties

(i) $s * t = t * s$

(ii) $(s * t) * u = s * (t * u)$

(iii) $*: [0,1]X[0,1] \to [0,1]$ is continuous

(iv) $s * 1 = 1 * s$

(v) $s * t \leq u * v\, if\, s \leq u, t \leq v$ for all $s, t, u, v \in [0,1]$

Some examples are$s * t = st$, $s * t = \min\{s, t\}$, $s * t = \max\{s + t, 1\}$.

Definition 2.2: A continuous t-co norm(s-norm) is defined by a binary operation $\Delta: [0,1]X[0,1] \to [0,1]$ if Δsatisfies the below mentioned properties

(i) $s\Delta t = t\Delta s$

(ii) $(s\Delta t)\Delta u = s\Delta(t\Delta u)$

(iii) $\Delta: [0,1]X[0,1] \to [0,1]$ is continuous

(iv) $s\Delta 0 = 0\Delta s$

(v) $s\Delta t \leq u\Delta v\, if\, s \leq u, t \leq v$ for all $s, t, u, v \in [0,1]$

Some examples are$s\Delta t = s + t - st$, $s\Delta t = \max\{s, t\}, s\Delta t = \min\{s + t, 1\}$.

If $s * s = s$ then $*$ is called an idempotent t-norm and if $s\Delta s = s\, then \Delta$ is called an idempotent s-norm$\forall s \in [0,1]$.

Definition 2.3: Let x be an element in Xwhich is the space of points (objects). An intuitionistic fuzzy set A in X is defined by $T_A: X \to [0,1]$, which is called a truth membership function and$F_A: X \to [0,1]$, which is falsity membership function and 0$\leq T_A(x) + F_A(x) \leq 1$.

Definition 2.4: If S is a mapping defined by S$: B \to P(U)$ then a pair (S, B) is known as a soft set over U for$B \subseteq E$. Here $P(U)$denotes the power set of U, U denotes an initial universal set and the set of parameters is denoted by E.

Definition 2.5: Let F be a mapping given by $F: S \to IF(U)$ then a pair (F, S) is known as an intuitionistic fuzzy soft set over U for S$\subseteq E$. Here the initial universal set is denoted by U and E denotes a set of parameters.

If$F(s)$ is an Intuitionistic fuzzy set of U for every $s \in S$ then it is called an Intuitionistic fuzzy value set of parameter$s$. Then $F(s)$ is an Intuitionistic fuzzy value set such that $F(s) = \{< x, \mu_{F(s)}(x), \lambda_{F(s)}(x) > / x \in U\}$ where $\mu_{F(s)}$ is a membership function and $\lambda_{F(s)}$ is a non membership function. Then intuitionistic fuzzy soft class is defined as set of all intuitionistic fuzzy soft sets over U with parameters from E and is denoted by IFF (U, E).

Example 2.6:

Consider U= $\{v_1, v_2, v_3\}$ be a set of vehicles and E= {e_1(mileage), e_2(cheap),e_3(costly)} be a set of parameters w.r.t which the nature of vehicles are described. Let

Revised Manuscript Received on December 15, 2019.

V. Siva Naga Malleswari, Assistant Professor, PVPSIT, A.P, India. E-mail: vsnm.maths@gmail.com

Dr. V. Amarendra Babu, Department of Mathematics, Acharya Nagarjuna University, Nagarjuna Nagar, India.
E-mail: amarendravelisela@ymail.com

$f(e_1)=\{<v_1(0.5,0.3)>,<v_2(0.4,0.6)>,<v_3(0.6,0.3)>\}$

$f(e_2)=\{<v_1(0.6,0.3)>,<v_2(0.7,0.3)>,<v_3(0.8,0.2)>\}$

$f(e_3)=\{<v_1(0.7,0.3)>,<v_2(0.6,0.2)>,<v_3(0.5,0.2)>\}$

Then $I=\{<e, f_I(e_1)>,<e_2,f_I(e_2)>,<e_3,f_I(e_3)>\}$ is an intuitionistic fuzzy soft set over (U,E).

Definition2.7: The complement of IFSS I is denoted by I^c and it is defined as

$I^c = \{e,<x, F_I(e), T_I(e)>/x\in U, e \in E\}$.

Definition 2.8: Let I_1 and I_2 be two IFS sets over (U, E). Then I_1 is said to be an intuitionistic fuzzy soft subset of I_2 if $T_{f_I(e_1)}(x) \le T_{f_I(e_2)}(x),\ F_{f_I(e_1)}(x) \ge F_{f_I(e_1)}(x) \forall\ e\in E, x \in U$.

Definition 2.9: Let I_1 and I_2 be two IFS sets over (U,E).The union is denoted by $I_1\cup I_2= I_3$ and it is defined as:

$I_3= \{s, <x,T_{f_{I_3(s)}}(x),F_{f_{I_3(s)}}(x)>/x\in U, s \in E\}$

Where $T_{f_{I_3(s)}}(x)=T_{f_{I_1(s)}}(x)\Delta T_{f_{I_2(s)}}(x)$

$F_{f_{I_3(s)}}(x)=F_{f_{I_1(s)}}(x) * F_{f_{I_2(s)}}(x)$

Their intersection is defined by $I_1\cap I_2=I_4$ and it is defined as

$I_4= \{s, <x,T_{f_{I_4(s)}}(x),F_{f_{I_4(s)}}(x)>/x\in U, s \in E\}$

Where $T_{f_{I_4(s)}}(x)=T_{f_{I_1(s)}}(x) * T_{f_{I_2(s)}}(x)$

$F_{f_{I_4(s)}}(x)=F_{f_{I_1(s)}}(x)\Delta F_{f_{I_2(s)}}(x)$

Definition 2.10: Let I_1 and I_2 be two IFSS over (U, E).Then 'and' operation is denoted by

$I_1\wedge I_2=I_5$ and it is defined as

$I_5= \{(s, t)/<x,T_{f_{I_5(s,t)}}(x),F_{f_{I_5(s,t)}}(x)>/x\in U, (s,t) \in EXE\}$

Where $T_{f_{I_5(s,t)}}(x)=T_{f_{I_1(s)}}(x)\Delta T_{f_{I_2(t)}}(x)$

$F_{f_{I_5(s,t)}}(x)=F_{f_{I_1(s)}}(x) * F_{f_{I_2(t)}}(x)$

Definition 2.11: An IFS set I over (U,E) is said to be a null IFS set if $T_{f_I(e)}(x)=0,F_{f_I(e)}(x)=1\quad \forall e\in E, x \in U$ and isdenoted by $\emptyset_I$.

An IFS set I over (U, E) is said to be absolute IFS set if $T_{f_I(e)}(x)=1,F_{f_I(e)}(x)=0\ \forall e\in E, x \in U$ and is denoted by I_A.

III. INTUITTIONISTIC FUZZY SOFT LINEAR SPACES

Definition 3.1: An Intuitionistic fuzzy set $B=\{<x, T_B(x), F_B(x) > x \in V\}$ on a vector space V(K) is called an intuitionistic fuzzy sub vector space V(K) if

(i) $T_B(x + y) \ge T_B(x) * T_B(y)$

(ii) $F_B(x + y) \le F_B(x)\Delta F_B(y) \forall x, y \in V$

(iii) $T_B(\lambda x) \ge T_B(x)$

(iv) $F_B(\lambda x) \le F_B(x) \forall x, y \in V, \lambda \in K.$

An intuitionistic fuzzy soft set I on V(K) is said to be an intuitionistic fuzzy soft vector space/ linear space (IFSLS) if f_I(e) is a intuitionistic fuzzy sub vector space on V (K) for all e$\in E$.

Example3.2: If E= {e_1, e_2,....., en} denotes the parametric set and R^n(R) be the n-dimensional Euclidean space. Let us define a mapping$f_I{:}E \to IFS(R^n)$ for any t$\in R^n$ as following:

$$T_{f_I(e)}(t) = \begin{cases} \frac{1}{2}, ifithcoordinateoftiszero \\ \qquad 0, otherwise \end{cases}$$

$$F_{f_I(e)}(t) = \begin{cases} 0, ifithcoordinateoftiszero \\ \qquad 1/10, otherwise \end{cases}$$

If $a * b = \min\{a, b\}, a\Delta b = \max\{a, b\}$.

Then I forms an IFSS as well as IFSLS over R^n(R) w.r.t parametric set E.

For convenience, we take an attempt for the parameter e and Euclidean space R^2(R).

Then the following four cases arise to choose x,y$\in R^2$.

Case I: If x=(0, 4) and y=(0,2) then x+y=(0,6)

Case II: If x=(0, 3) and y=(3,2) then x+y=(3,5)

Case III: If x=(1, 2) and y=(5,1) then x+y=(6,3)

Case IV: If x=(5, 1) and y=(-5,4) then x+y=(0,5)

From these four cases, the first and second set of conditions can be verified.

Example 3.3: Consider a real vector space C= {a+ib/a,b$\in R$,i=$\sqrt{-1}$} and the parametric set E=$\{\alpha, \beta, \gamma\}$.We divide the elements of C into four cases e.g:

(C1) {ib/b$\in R - \{0\}$} when real part is zero

(C2){a/a$\in R - \{0\}$} when imaginary part is zero

(C3){a+ib/a,b$\in R - \{0\}$} when both parts are non zero

(C4){0+i0} the null vector

If x∈C1 and y∈C2 then x+y∈C3.We write C1+ C2= C3.

We define IFSS I over (C, E) is given by $a * b$=max{a+b-1, 0}, a$\Delta b = \{a + b, 1\}$.Then I forms an IFSLS over (C(R),E).

From these four cases, the first and second set of conditions can be verified.

Corollary 3.4: Let I be an IFSLS over (V(K),E).Then for x$\in V$ and

$\lambda(\neq 0) \in K, T_{f_I(e)}(\lambda x)$=$T_{f_I(e)}(x)$,$F_{f_I(e)}(\lambda x)$=$F_{f_I(e)}(x)$ hold.

Proof:$T_{f_I(e)}(x)$=$T_{f_I(e)}(\lambda^{-1}(\lambda x)) \geq T_{f_I(e)}(\lambda x)$

$F_{f_I(e)}(x)$=$F_{f_I(e)}(\lambda^{-1}(\lambda x)) \leq F_{f_I(e)}(\lambda x)$

Now from the second set of conditions in definition of IFSLS , the result follows.

Proposition 3.5:

Let I be an IFSLS over (V (K), E). Then for each$t \in$ V, following hold.

(i) $\mathrm{T}_{\mathrm{f_I(e)}}(-t) = \mathrm{T}_{\mathrm{f_I(e)}}(t)$, $\mathrm{F}_{\mathrm{f_I(e)}}(-t) = \mathrm{F}_{\mathrm{f_I(e)}}(t)$

(ii) $\mathrm{T}_{\mathrm{f_I(e)}}(\theta) \geq \mathrm{T}_{\mathrm{f_I(e)}}(t), \mathrm{F}_{\mathrm{f_I(e)}}(\theta) \leq \mathrm{F}_{\mathrm{f_I(e)}}(t)$

if $a * b = \min\{a, b\}, a\Delta b = \max\{a, b\}$ and θ is null vector of V.

Proof :(i) For $\lambda = -1$, the result directly follows from above corollary.

(ii)For the null vector $\theta \in V$

$\mathrm{T}_{\mathrm{f_I(e)}}(\theta)$=$\mathrm{T}_{\mathrm{f_I(e)}}(t + (-t)) \geq$

$\mathrm{T}_{\mathrm{f_I(e)}}(t) * \mathrm{T}_{\mathrm{f_I(e)}}(-t)$=$\mathrm{T}_{\mathrm{f_I(e)}}(t) * \mathrm{T}_{\mathrm{f_I(e)}}(t)$=$\mathrm{T}_{\mathrm{f_I(e)}}(t)$

$\mathrm{F}_{\mathrm{f_I(e)}}(\theta)$=$\mathrm{F}_{\mathrm{f_I(e)}}(t + (-t)) \leq$

$\mathrm{F}_{\mathrm{f_I(e)}}(t)\Delta\mathrm{F}_{\mathrm{f_I(e)}}(-t)$=$\mathrm{F}_{\mathrm{f_I(e)}}(t)\Delta\mathrm{F}_{\mathrm{f_I(e)}}(t)$=$\mathrm{F}_{\mathrm{f_I(e)}}(t)$

Hence proved.

Proposition 3.6:

An IFSS I on (V, K) is said to be an IFSLS with respect to the set E if and only if the following equations hold

$$T_{f_I(e)}(\lambda s + \mu t)) \geq T_{f_I(e)}(s) * T_{f_I(e)}(t)$$

$$F_{f_I(e)}(\lambda s + \mu t)) \leq F_{f_I(e)}(s)\Delta F_{f_I(e)}(t) \forall s, t \in V, \lambda, \mu \in F, e \in E$$

If $a * b = \min\{a, b\}, a\Delta b = \max\{a, b\}$

Proof: First suppose I be an IFSLS onV(K) w.r.t.E.

Then $T_{f_I(e)}(\lambda s + \mu t)) \geq T_{f_I(e)}(\lambda s) * T_{f_I(e)}(\mu t) \geq T_{f_I(e)}(s) * T_{f_I(e)}(t)$

$$F_{f_I(e)}(\lambda s + \mu t)) \leq F_{f_I(e)}(\lambda s)\Delta F_{f_I(e)}(\mu t) \leq F_{f_I(e)}(s)\Delta F_{f_I(e)}(t)$$

Conversely by proposition 3.5,

$$T_{f_I(e)}(\lambda s) = T_{f_I(e)}(\theta + \lambda s) \geq T_{f_I(e)}(\theta) * T_{f_I(e)}(s)$$
$$\geq T_{f_I(e)}(s) * T_{f_I(e)}(s)$$
$$= T_{f_I(e)}(s) F_{f_I(e)}(\lambda s) = F_{f_I(e)}(\theta + \lambda s)$$
$$\leq F_{f_I(e)}(\theta)\Delta F_{f_I(e)}(s)$$
$$\leq F_{f_I(e)}(s)\Delta F_{f_I(e)}(s) = F_{f_I(e)}(s)$$

$$T_{f_I(e)}(s + t) = T_{f_I(e)}(s + (-1)(-t))$$
$$\geq T_{f_I(e)}(s) * T_{f_I(e)}(-t)$$
$$\geq T_{f_I(e)}(s) * T_{f_I(e)}(t)$$

$$F_{f_I(e)}(s + t) = F_{f_I(e)}(s + (-1)(-t))$$
$$\leq F_{f_I(e)}(s)\Delta F_{f_I(e)}(-t) \leq F_{f_I(e)}(s)\Delta F_{f_I(e)}(t)$$

Hence the proof.

Theorem 3.7: Let I_1 and I_2 be two IFSLSs over (V (K), E).Then $I_1 \cap I_2$ is also an IFSLS over (V (K), E).

Proof: Let $I_1 \cap I_2 = P$.Now for $s,t \in V$

$T_{f_p(e)}(s + t) = T_{f_{I_1}(e)}(s + t) * T_{f_{I_2}(e)}(s + t) \geq$

$[T_{f_{I_1}(e)}(s) * T_{f_{I_1}(e)}(t)]$ $*[T_{f_{I_2}(e)}(s) * T_{f_{I_2}(e)}(t)]$

$=[T_{f_{I_1}(e)}(s) * T_{f_{I_2}(e)}(t)]$ $*[T_{f_{I_1}(e)}(t) * T_{f_{I_2}(e)}(s)]$

$=T_{f_{I_1}(e)}(s) * [T_{f_{I_1}(e)}(t)]$ $*[T_{f_{I_2}(e)}(t)] * T_{f_{I_2}(e)}(s)$

$=T_{f_{I_1}(e)}(s) * T_{f_P(e)}(t) * T_{f_{I_2}(e)}(s)$

$=T_{f_{I_1}(e)}(s) * T_{f_{I_2}(e)}(s) * T_{f_P(e)}(t)$(as $*$ is commutative)

$=T_{f_P(e)}(s) * T_{f_P(e)}(t)$

Hence $T_{f_p(e)}(s + t) \geq T_{f_P(e)}(s) * T_{f_P(e)}(t)$

Also, $T_{f_P(e)}(\lambda s) = T_{f_{I_1}(e)}(\lambda s) * T_{f_{I_2}(e)}(\lambda s) \geq T_{f_{I_1}(e)}(s) * T_{f_{I_2}(e)}(s) = T_{f_P(e)}(s)$

Thus, $T_{f_P(e)}(\lambda s) \geq T_{f_P(e)}(s)$ for $\lambda \in K$.

Similarly $F_{f_p(e)}(s + t) \leq F_{f_P(e)}(s)\Delta F_{f_P(e)}(t)$ and $F_{f_P(e)}(\lambda s) \geq F_{f_P(e)}(t)$ for $\lambda \in K$.

Hence proved.

Remark 3.8: For two IFSLS I_1 and I_2 over (V (K), E), $I_1 \cup I_2$ is not generally an IFSLS over (V (K), E).It is possible if anyone is contained in another. For instance, let us consider two IFSLS s I_1 and I_2 over the real vector space V=R^2 and the parametric set E= [e$_i$/i=1, 2} as following:

$$T_{f_{I_1}(e_i)}(x)= \begin{cases} \frac{1}{2} ifithcoordinateofx \in R^2 isnonzeroonly \\ 0\ otherwise \end{cases}$$

$$F_{f_{I_1}(e_i)}(x)= \begin{cases} \frac{2}{5} ifithcoordinateofx \in R^3 isnonzeroonly \\ 1\ otherwise \end{cases}$$

$$T_{f_{I_2}(e_i)}(x)= \begin{cases} \frac{2}{5} ifithcoordinateofx \in R^2 isnonzeroonly \\ 1/10\ otherwise \end{cases}$$

$$F_{f_{I_2}(e_i)}(x)= \begin{cases} 0\ ifithcoordinateofx \in R^2 isnonzeroonly \\ 1/5\ otherwise \end{cases}$$

If a$* b = \min\{a, b\}$, a$\Delta b = max\{a, b\}$.

Let $I_1 \cup I_2 = P$

$T_{f_P(e_1)}(x + y) = T_{f_{I_1}(e_1)}(1{,}1) = \max\{0, \frac{1}{10}\}$=$\frac{1}{10}$

$T_{f_P(e_1)}(x) *$

$T_{f_P(e_1)}(y)$=$\{T_{f_{I_1}(e_1)}(x)\Delta T_{f_{I_1}(e)}(x)\}$$*$$\{T_{f_{I_1}(e_1)}(y)\Delta T_{f_{I_1}(e)}(y)$

}

=min{max{1/2,1/10}},max{0,2/5}}=min{1/2,2/5}=2/5

Hence $T_{f_{P(e_1)}}(x+y) < T_{f_{P(e_1)}}(x) * T_{f_{P(e_1)}}(y)$i.e $I_1 \cup I_2$ is not an IFSLS here.

Now if define I over (R^3, E) as following

$$T_{f_{I_1(e_1)}}(x)=\begin{cases}\frac{1}{6} ifithcoordianateofx \in R^2 isnonzeroonly \\ 0\ otherwise\end{cases}$$

$$F_{f_{I_1(e_1)}}(x)=\begin{cases}\frac{7}{10} ifithcoordianateofx \in R^2 isnonzeroonly \\ 1\ otherwise\end{cases}$$

It can be easily verified that $I_2 \subseteq I_1$ and $I_1 \cup I_2$ is an IFSLS over $((R^2(R), E)$.

Theorem 3.9: Let I_1 and I_2 be two IFSLSs over (V (K), E).Then $I_1 \wedge I_2$ is also an IFSLS over (V (K), E).

Proof: Let $I_1 \wedge I_2 = Q$.Now for x, y$\in V$ and (s, t)$\in EXE$

$T_{f_{Q(s,t)}}(x+y) = T_{f_{I_1(s)}}(x+y) * T_{f_{I_2(t)}}(x+y) \geq$

$[T_{f_{I_1(s)}}(x) * T_{f_{I_1(s)}}(y)] *[T_{f_{I_2(t)}}(x) * T_{f_{I_2(t)}}(y)]$

$= [T_{f_{I_1(s)}}(x) * T_{f_{I_1(s)}}(y)] *[T_{f_{I_2(t)}}(y) * T_{f_{I_2(t)}}(x)]$(as $*$ is commutative)

$= T_{f_{I_1(s)}}(x) * [T_{f_{I_1(s)}}(y)] *[T_{f_{I_2(t)}}(y)] * T_{f_{I_2(t)}}(x)$(as $*$ is associative)

$=T_{f_{I_1(s)}}(x) * T_{f_{Q(s,t)}}(y) * T_{f_{I_2(s,t)}}(x)$

$=T_{f_{I_1(s)}}(x) * T_{f_{I_2(t)}}(x) * T_{f_{Q(s,t)}}(y)$(](as $*$ is commutative)

$=T_{f_{Q(s,t)}}(x) * T_{f_{Q(s,t)}}(y)$

Hence $T_{f_{q(s,t)}}(x+y) \geq T_{f_{Q(s,t)}}(x) * T_{f_{Q(s,t)}}(y)$

Also, $T_{f_{Q(s,t)}}(\lambda x) = T_{f_{I_1(s)}}(\lambda x) * T_{f_{I_2(t)}}(\lambda x) \geq T_{f_{I_1(s)}}(x) * T_{f_{I_2(t)}}(x) = T_{f_{Q(s,t)}}(x)$

Thus, $T_{f_{Q(s,t)}}(\lambda x) \geq T_{f_{Q(s,t)}}(x)$ for $\lambda \in K$.

Similarly $F_{f_{Q(s,t)}}(x+y) \leq F_{f_{Q(s,t)}}(x) \Delta F_{f_{Q(s,t)}}(y)$ and $F_{f_{Q(s,t)}}(\lambda x) \leq F_{f_{Q(s,t)}}(x)$ for $\lambda \in K$.

Hence proved.

IV. CARTESIAN PRODUCT OF INTUITIONISTIC FUZZY SOFT LINEAR SPACES

Definition4.1: Let I_1 and I_2 be two IFSLSs over (V (K), E) and (W (K), E) respectively. Then their Cartesian product is $I_1 X I_2 = C$ where $f_C(s,t) = f_{I_1}(s) X f_{I_2}(t)$ for (s,t)$\in EXE$.

Analytically

$$f_C(s,t) = \{< (x,y), T_{f_{C(s,t)}}(x,y), F_{f_{C(s,t)}}(y) > /(x,y) \in VXW\}$$

With $T_{f_{C(s,t)}}(x,y) = T_{f_{I_1(s)}}(x) * T_{f_{I_2(t)}}(y)$

$F_{f_{C(s,t)}}(x,y) = F_{f_{I_1(s)}}(x) \Delta F_{f_{I_2(t)}}(y)$

This can be extended for more than two IFSLS s.

Theorem 4.2: Let I_1 and I_2 be two IFSLSs over (V (K), E) and (W(K),E) respectively. Then their Cartesian product I_1X $I_2$is an IFSLS over ([VXW](k),EXE).

Proof: Let $I_1 X I_2 = C$ where $f_C(s,t) = f_{I_1}(s) X f_{I_2}(t)$ for(s, t)$\in EXE$

Then for(x_1,y_1), $(x_2,y_2) \in VXW$

$T_{f_{C(s,t)}}[(x_1,y_1)+(x_2,y_2)]=T_{f_{C(s,t)}}[(x_1+x_2, y_1+y_2)]=T_{f_{I_1(s)}}(x_1+x_2) * T_{f_{I_2(t)}}(y_1+y_2)$

$\geq \left[T_{f_{I_1(s)}}(x_1) * T_{f_{I_1(s)}}(x_2)\right] * [T_{f_{I_2(t)}}(y_1) * T_{f_{I_2(t)}}(y_2)]$

$=\left[T_{f_{I_1(s)}}(x_1) * T_{f_{I_2(t)}}(y_1)\right] * [T_{f_{I_1(s)}}(x_2) * T_{f_{I_2(t)}}(y_2)]=T_{f_C}(s,t)(x_1,y_1) * T_{f_C}(s,t)(x_2,y_2)$

Similarly

$F_{f_C}(s,t)[(x_1,y_1)+(x_2,y_2)] \leq F_{f_C}(s,t)(x_1,y_1) \Delta F_{f_C}(s,t)(x_2,y_2)$

Next,$T_{f_C}(s,t)[\lambda(x_1,y_1)]=T_{f_C}(s,t)[(\lambda x_1, \lambda y_1)]=T_{f_{I_1(s)}}(\lambda x_1) * T_{f_{I_2(t)}}(\lambda y_1) \geq T_{f_{I_1(s)}}(x_1) * T_{f_{I_2(t)}}(y_1)=T_{f_s}(s,t)(x_1,y_1)$

Similarly $F_{f_s}(s,t)[\lambda(x_1,y_1)] \leq F_{f_s}(s,t)(x_1,y_1)$

V. EXPERIMENTS AND RESULTS DESCRIPTION

Definition 5.1: Let I_1 and I_2 be two IFSLS over (V (K), E).Then I_1 is IFSS of I_2 if $I_1 \subseteq I_2$ i.e. $T_{f_{I_1(e)}}(u) \leq T_{f_{I_2(e)}}(u)$

$F_{f_{I_1(e)}}(u) \geq F_{f_{I_2(e)}}(u) \forall u \in V, e \in E$

Example 5.2: Let us consider two IFSLS s I_1 and I_2 over real vector space V=R^3 and parametric set E= {e} as following:

$$T_{f_{I_1(e)}}(x) = \begin{cases}\frac{1}{4} if x \in (a,b,c) \in R^3, a+b+c=0 \\ 0, otherwise\end{cases}$$

$$F_{f_{I_1(e)}}(x) = \begin{cases}0, if x \in (a,b,c) \in R^3, a+b+c=0 \\ 1/6, otherwise\end{cases}$$

$$T_{f_{I_2(e)}}(x) = \begin{cases}\frac{1}{2} if x \in (a,b,c) \in R^3, a+b+c=0 \\ 2/7, otherwise\end{cases}$$

$$F_{f_{I_2(e)}}(x) = \begin{cases}0, if x \in (a,b,c) \in R^3, a+b+c=0 \\ 1/9, otherwise\end{cases}$$

If a$* b = \max\{a+b-1, 0\}$ and a$\Delta b = \min\{a+b, 1\}$.Then $I_1$is an intuitionistic fuzzy soft subspace of I_2 over $(R^3(R), E)$.

Corollary 5.3:Let I be an IFSLS over (V(K),E).Then for arbitrary but fixed $\lambda \in K$,$\lambda I = \{e, \frac{\lambda f_I(e)}{e} \in E\}$ is also a IFSLs over (V(K),E) where $\lambda f_I(e)$={<$\lambda x, T_{f_I(e)}(x)$,$F_{f_I(e)}(x) >$ $/x \in V\}$.Moreover λI is a intuitionistic fuzzy soft subspace of I.

Proof: Clearly $\lambda x \in V$ for $x \in V, \lambda \in K$

Since I be an IFSLS over (V (K), E) so by construction of λI

$T_{f_I(e)}(\lambda x + \lambda y) \geq T_{f_I(e)}(\lambda x) * T_{f_I(e)}(\lambda y)$

$F_{f_I(e)}(\lambda x + \lambda y) \leq F_{f_I(e)}(\lambda x) * F_{f_I(e)}(\lambda y) \forall \lambda x, \lambda y \in V$,e∈ E

$T_{f_I(e)}(\mu(\lambda x)) \geq T_{f_I(e)}(\lambda x)$

$F_{f_I(e)}(\mu(\lambda x)) \leq F_{f_I(e)}(\lambda x) \forall \lambda x \in V$,e∈ $E, \mu \in K$

Hence λI is IFSLS over (V (K), E)

Next $T_{f_I(e)}(x) = T_{f_I(e)}(\lambda^{-1}(\lambda x)) \geq T_{f_I(e)}(\lambda x)$

$F_{f_I(e)}(x) = F_{f_I(e)}(\lambda^{-1}(\lambda x)) \geq F_{f_I(e)}(\lambda x)(\forall \lambda \neq 0) \in$ $K, x \in V$,e∈ E

Then $\lambda I is$IFS subspace of I.

Corollary 5.4: Let I_1 be an IFSLS over (V (K), E).Then for arbitrary but fixed$\lambda, \mu \in K.\lambda I_1 + \mu I_1$= {e,$(\lambda f_{I_1} + \mu f_{I_1})(e)/$ e∈ $E\}$ is again in IFSLS over (V(K),E) where ,$(\lambda f_{I_1} +$ $\mu f_{I_1})(e)$={<$(\lambda x + \mu y), T_{f_{I_2}(e)}(\lambda x + \mu y)$,$F_{f_{I_2}(e)}(\lambda x +$ μy>/x, y∈V}

Moreover $\lambda I_1 + \mu I_1$ is an IF soft subspace of I_1

Proof: Since V (K) is a vector space.

So x+y,$\lambda x + \mu y \in V$ for x, y∈ V and$\lambda, \mu \in F$.

Hence the proof is completed.

Corollary 5.5: Let $f_{I_1}(e), e \in E$ be a IF subspace on V (K) where I_1 is an IFSLS over (V (K), E).Then $\lambda f_{I_1}(e) =$ $\{< \lambda x, T_{f_{I_1}(e)}(\lambda x), F_{f_{I_1}(e)}(\lambda x) >$/x∈ $V\}$

Proof: It is obvious

For instance, if V={x, y, z} and k= $\{\lambda, \mu\}$ then $\lambda x, \lambda y, \lambda z, \mu x, \mu y, \mu z \in V$ and $\lambda x + \lambda x, \lambda x + \lambda y, \lambda x + \mu x, \mu y +$ μy.........∈ V.

Since$f_{I_1}(e)$, e∈ E is an IFS subspace on V (K) so all the inequalities hold good.

REFERENCES

1. A.sezgin sezer, A.O.Atagin, A new kind of vector space: soft vector space, southest asian Bulletin of Mathematics(2016040:753-770.
2. D. Molodtsov, Soft set theory-First results, Computers Math. Applic. 37 (4/5), 19-31, (1999).
3. K. Atanassov and S. Stoeva, Intuitionistic fuzzy sets, in: Polish Symp.
4. K. Atanassov, Intuitionistic fuzzy sets, in: V. Sgurev, Ed., VII ITKR's Session, Sofia, June 1983 (Central Sci. and Techn. Library, Bulg. Academy of Sciences, 1984).
5. Moumita Chiney,S.K.Samanta,Intuitionistic fuzzy dimension of an intuitionistic fuzzy vector space,Notes onintuitionistic fuzzy sets,vol 24,2018,No 1,21-29.
6. P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl. 45 (2003) 555–562.