

Fuzzy Soft Ternary Γ -Semirings-II

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Abstract: In this paper we are introducing the notions of “fuzzy soft quasi $T\Gamma$ -ideal(FSQTTI), Fuzzy soft bi- $T\Gamma$ -ideal(FSBTTI)” are introduced. It is proved that (1) A “fuzzy soft set (q, P_1, Γ) over a ternary Γ -semiring($T\Gamma$ -SR)”T is a FSQTTI over T iff $\forall a \in P_1, q(a) \neq \emptyset$ is a “quasi-ideal of T”. (2) Every “Fuzzy soft left (right, lateral) $T\Gamma$ -ideal[FSLTTI(FSMTTI, FSRTTI)] over a $T\Gamma$ -SR T is a FSQTTI over T”: (3)Every “FSQTTI is a fuzzy soft $T\Gamma$ -SR over T”: (4) Let “ $(r, A, \Gamma), (l, B, \Gamma)$ and (m, C, Γ) be FSLTTI, FSMTTI, FSRTTI over T”, respectively. Then “ $(r, A, \Gamma) \wedge (l, B, \Gamma) \wedge (m, C, \Gamma)$ is a FSQTTI over T”. (5) Let “ (f, A, Γ) be a FSQTTI and (g, B, Γ) a Fuzzy soft ternary Γ -semiring(FST Γ SR) over T”. Then “ $(f, A, \Gamma) \text{ I}_R (g, B, \Gamma)$ is a FSQTTI of (g, B, Γ) ”. (6) Let “ (f, A, Γ) and (g, B, Γ) be two non-empty fuzzy soft sets over a $T\Gamma$ -SR T”.Then the “fuzzy soft set $(h, C, \Gamma) = (f, A, \Gamma) \circ (T, E, \Gamma) \circ (g, B, \Gamma)$ is a FSBTTI over T”. Further many more properties are proved and some examples are given.

Index Terms: $T\Gamma$ -SR, FST Γ -SR, FSTTI, FSQTTI.

I. INTRODUCTION

Molodtsov introduced the new tool as soft set dealing with uncertainties. D. MadhusudhanaRao and SajaniLavanya introduced the concept of $T\Gamma$ -SR. Further that MadhusudhanaRao and Revathi developed the $T\Gamma$ -SR in

terms fuzzy structures. MadhusudhanaRao and Ravi kumar developed the notion of “soft $T\Gamma$ -ideals in $T\Gamma$ -SR”. MadhusudhanaRao, Revathi studied about “fuzzy $T\Gamma$ -ideals in $T\Gamma$ -SR”. Further, they introduced the concept of fuzzy completely prime and prime ternary gamma ideals in $T\Gamma$ -SR.

II. PRELIMINARIES

A “ternary Γ -semiring” $(T, \Gamma, +, [,])$ is known as commutative if “ $f_1 \Gamma f_2 \Gamma f_3 = f_2 \Gamma f_3 \Gamma f_1 = f_3 \Gamma f_1 \Gamma f_2 = f_2 \Gamma f_1 \Gamma f_3 = f_3 \Gamma f_2 \Gamma f_1 = f_1 \Gamma f_3 \Gamma f_2 \forall f_1, f_2, f_3 \in T$ ”. A non-empty subset S of a ternary Γ -semiring T is known as a ternary Γ -subsemiring of T if $S \Gamma S \Gamma S \subseteq S$. By a L (R, M) ideal of a ternary Γ -subsemiring $V, V_l \subseteq V$ such that $V \Gamma V \Gamma V_l \subseteq V_l (V_l \Gamma V \Gamma V \subseteq V_l, V \Gamma V_l \Gamma V \subseteq V_l)$. By a “two sided ideal”, we mean a subset A which is both a L as well as a R -ideal of T . If a non-empty subset A is a L, R and a M -ideal of T ; then it is known as an ideal of T . A L ($R, M, two sided$) ideal P_l of a ternary Γ -semiring T is idempotent if $P_l \Gamma P_l \Gamma P_l = P_l$. For more information refer to the references [6, 8].

III. “Fuzzy Soft Quasi- $t\Gamma$ -Ideal”:

Def 3.1: A “fuzzy soft set (FSS) (u, V_l, Γ) over a ternary Γ -semiring($T\Gamma$ -SR) T ” is called a “fuzzy soft quasi- $t\Gamma$ -ideal(FSQTTI) over T ” if (1) “ $(u, V_l, \Gamma) \circ (q, V, \Gamma) \circ (q, V, \Gamma) \text{ I}_R (q, V, \Gamma) \circ (u, V_l, \Gamma) \circ (q, V, \Gamma) \text{ I}_R (q, V, \Gamma) \circ (q, V, \Gamma) \circ (u, V_l, \Gamma) \subseteq (u, V_l, \Gamma)$ ”.

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From equation (8) and (10) we have, $u(d_i)$ is QT Γ I.

Conversely, if $u(d_i) \neq \emptyset$ is a QT Γ I of T for all $d_i \in V_1$.

Now we claim that (u, V_1, Γ) is a FSQT Γ I over T. By equations (1), (2) and (3)

$$“(u, V_1, \Gamma) \circ (q, P, \Gamma) \circ (q, P, \Gamma) \text{I}_R (q, P, \Gamma) \circ (u, V_1, \Gamma) \circ (q, P, \Gamma) \text{I}_R (q, P, \Gamma) \circ (q, P, \Gamma) \circ (u, V_1, \Gamma) = (g, V_1, \Gamma) \text{I}_R (h, V_1, \Gamma) \text{I}_R (i, V_1, \Gamma) = (w, V_1, \Gamma)”.$$

By the definition “ $u(d_i) \alpha q(d_i) \beta q(d_i) \text{I} q(d_i) \alpha u(d_i) \beta q(d_i) \text{I} q(d_i) \alpha q(d_i) \beta u(d_i) = w(d_i)$ for all $d_i \in V_1$ ”. Since $u(d_i)$ is a QT Γ I of T”.

$$\therefore, “w(d_i) = u(d_i) \alpha q(d_i) \beta q(d_i) \text{I} q(d_i) \alpha u(d_i) \beta q(d_i) \text{I} q(d_i) \alpha q(d_i) \beta u(d_i) \subseteq u(d_i)”$$

$$\Rightarrow “w(d_i) \subseteq u(d_i) \Rightarrow (w, V_1, \Gamma) \subseteq (u, V_1, \Gamma)”.$$

$$\text{Thus “}(u, V_1, \Gamma) \circ (q, P, \Gamma) \circ (q, P, \Gamma) \text{I}_R (q, P, \Gamma) \circ (u, V_1, \Gamma) \circ (q, P, \Gamma) \text{I}_R (q, P, \Gamma) \circ (q, P, \Gamma) \circ (u, V_1, \Gamma) \subseteq (u, V_1, \Gamma)” \rightarrow (11)$$

Now form equation (1), (3) and (4)

$$“(u, V_1, \Gamma) \circ (q, P, \Gamma) \circ (q, P, \Gamma) \text{I}_R (q, P, \Gamma) \circ (q, P, \Gamma) \circ (u, V_1, \Gamma) \circ (q, P, \Gamma) \circ (q, P, \Gamma) \text{I}_R (q, P, \Gamma) \circ (q, P, \Gamma) \circ (u, P_1, \Gamma) = (g, V_1, \Gamma) \text{I}_R (i, V_1, \Gamma) \text{I}_R (j, V_1, \Gamma) = (l, V_1, \Gamma)”$$

$$\Rightarrow “u(d_i) \alpha q(d_i) \beta q(d_i) \text{I} q(d_i) \gamma q(d_i) \alpha u(d_i) \beta q(d_i) \delta q(d_i) \text{I} q(d_i) \alpha q(d_i) \beta u(d_i) = l(d_i)$$

$$\forall d_i \in V_1”. \Rightarrow “l(d_i) = u(d_i) \alpha q(d_i) \beta q(d_i) \text{I} q(d_i) \gamma q(d_i) \alpha u(d_i) \beta q(d_i) \delta q(d_i) \text{I} q(d_i) \alpha q(d_i) \beta u(d_i) \subseteq u(d_i)” \Rightarrow “(l, V_1, \Gamma) \subseteq (u, V_1, \Gamma)”$$

$$\Rightarrow “(u, V_1, \Gamma) \circ (q, P, \Gamma) \circ (q, P, \Gamma) \text{I}_R (q, P, \Gamma) \circ (q, P, \Gamma) \circ (u, V_1, \Gamma) \circ (q, P, \Gamma) \circ (q, P, \Gamma) \text{I}_R (q, P, \Gamma) \circ (q, P, \Gamma) \circ (u, P_1, \Gamma) \subseteq (u, V_1, \Gamma)” \rightarrow (12).$$

Therefore, from equations (11) and (12) we can say that (u, V_1, Γ) is a FSQT Γ I over T.

Th 3.3: “Let (r, P_1, Γ) ; (l, P_2, Γ) and (m, P_3, Γ) be FSRT Γ I, FSLT Γ I and FSMT Γ I s over T, respectively. Then $(r, P_1, \Gamma) \text{I}_R (m, P_3, \Gamma) \text{I}_R (l, P_2, \Gamma)$ is a FSQT Γ I over T”.

Proof: Straightforward.

Th 3.4: “Let (r, P_1, Γ) ; (l, P_2, Γ) and (m, P_3, Γ) be FSRT Γ I, FSLT Γ I and FSMT Γ I s over T, respectively, such that $P_1 \cap P_2 \cap P_3 \neq \emptyset$. Then $(r, P_1, \Gamma) \text{I}_E (m, P_3, \Gamma) \text{I}_E (l, P_2, \Gamma)$ is a FSQT Γ I over T”.

Proof: By the definition we have “ $(h, P_4, \Gamma) = (r, P_1, \Gamma) \text{I}_E (m, P_2, \Gamma) \text{I}_E (l, P_3, \Gamma)”$,

$$\text{Where, “} P_4 = P_1 \cup P_2 \cup P_3, P_1 \cap P_2 \cap P_3 = \emptyset \text{” and “} h(v) = \begin{cases} r(v) & \text{if } v \in P_1 - P_2 \text{I } P_3 \\ m(v) & \text{if } v \in P_3 - P_1 \text{I } P_2 \\ l(v) & \text{if } v \in P_2 - P_1 \text{I } P_3. \end{cases} \text{”}$$

For any $v \in P_4$. In each of the case $h(v)$ is a QT Γ I of T. As every LT Γ I, MT Γ I, RT Γ I of TFSR T is QT Γ I of T, \Rightarrow by definition “ $(h, P_4, \Gamma) = (r, P_1, \Gamma) \cap_E (m, P_2, \Gamma) \cap_E (l, P_3, \Gamma)$ is a FSQT Γ I over T”.

IV. EXPERIMENTS AND RESULTS DESCRIPTION

Proof: Let (l, P_1, Γ) be a FSLT Γ I over T. Then $l(p_i)$ is a LT Γ I of T. As each LT Γ I of T is a QT Γ I of T, $\therefore l(a)$ is a QT Γ I of T. Hence (L, P, Γ) is a FSQT Γ I over T.

Note: “The converse of the th 4.5, is not true”.

Ex: “Let $T = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ and $\Gamma = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ be a $T\Gamma$ -SR under usual addition and matrix ternary multiplication. Let $P_1 = \{a\}$ and $q: P_1 \rightarrow P(T)$ defined as $q(a) = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right\}$. Then (q, P_1, Γ) be a FSQTFI over T , but it is not a LSTFI, MSTFI as well as RSTFI over T ”.

Th: Every FSLTFI (FSMTFI, FSRTFI) over T is a FSTF-SR over T .

Proof: Straight forward.

Th: Every FSQTFI over T is a FSTF-SR over T .

Proof: Straight forward.

Th: “Let $(l, P_1, \Gamma), (m, P_2, \Gamma)$ and (r, P_3, Γ) be FSLTFI, FSMTFI as well as FSRTFIs over T , respectively. Then $(l, P_1, \Gamma) \wedge (m, P_2, \Gamma) \wedge (r, P_3, \Gamma)$ is a FSQTFI over T ”.

Proof: By the definition “ $(h, P_4, \Gamma) = (l, P_1, \Gamma) \wedge (m, P_2, \Gamma) \wedge (r, P_3, \Gamma)$ where $P_4 = P_1 \times P_2 \times P_3$ ”, and for any “ $(f_1, f_2, f_3) \in P_1 \times P_2 \times P_3$, $h(f_1, f_2, f_3) = l(f_1) \cap m(f_2) \cap r(f_3)$ is a QTFI of T ”. Since the “intersection of a LTFI, RTFI, MTFI is a QTFI of T ”, thus, $(l, P_1, \Gamma) \wedge (m, P_2, \Gamma) \wedge (r, P_3, \Gamma)$ is a FSQTFI over T ”.

Th: Let $(t, P_1, \Gamma), (w, P_2, \Gamma)$ be two FSQTFI over a $T\Gamma$ -SR T . Then

- (1) $(t, P_1, \Gamma) \overset{I}{\underset{R}{\cap}} (w, P_2, \Gamma)$ is a FSQTFI over T .
- (2) $(t, P_1, \Gamma) \overset{I}{\underset{E}{\cap}} (w, P_2, \Gamma)$ is a FSQTFI over T .
- (3) $(t, P_1, \Gamma) \wedge (w, P_2, \Gamma)$ is a FSQTFI over T .
- (4) $(t, P_1, \Gamma) \overset{U}{\underset{E}{\cup}} (w, P_2, \Gamma)$ is a FSQTFI over T , if $P_1 \cap P_2 = \emptyset$ are hold.

Th: Let (t, P_1, Γ) be a FSQTFI as well as (w, P_2, Γ) a FSTF-SR T . Then, $(t, P_1, \Gamma) \overset{I}{\underset{R}{\cap}} (w, P_2, \Gamma)$ is a FSQTFI of (w, P_2, Γ) .

Proof: By definition “ $(m, P, \Gamma) = (t, P_1, \Gamma) \overset{I}{\underset{R}{\cap}} (w, P_2, \Gamma)$, where $P = P_1 \cap P_2 \neq \emptyset$, $m(p) = t(p) \cap w(p) \forall p \in P$, as $m(p) \subseteq t(p)$, $m(p) \subseteq w(p)$. We show that $m(p)$ is a QTFI of $w(p)$. $\therefore m(p) \subseteq w(p)$ ”.

$$\begin{aligned} & “m(p)\Gamma w(p)\Gamma w(p) \cap w(p)\Gamma m(p)\Gamma w(p) \cap w(p)\Gamma w(p)\Gamma m(p) \\ & \subseteq w(p)\Gamma w(p)\Gamma w(p) \cap w(p)\Gamma w(p)\Gamma w(p) \cap w(p)\Gamma w(p)\Gamma w(p) \\ & \subseteq w(p)\Gamma w(p)\Gamma w(p) \subseteq w(p) ” \end{aligned}$$

Because, $w(p)$ is a $T\Gamma$ -SSR of T .

$$\Rightarrow “m(p)\Gamma w(p)\Gamma w(p) \cap w(p)\Gamma m(p)\Gamma w(p) \cap w(p)\Gamma w(p)\Gamma m(p) \subseteq w(p) ” \rightarrow (1)$$

Also $m(p) \subseteq t(p)$. So

$$\begin{aligned} & “m(p)\Gamma w(p)\Gamma w(p) \cap w(p)\Gamma m(p)\Gamma w(p) \cap w(p)\Gamma w(p)\Gamma m(p) \\ & \subseteq t(p)\Gamma w(p)\Gamma w(p) \cap w(p)\Gamma t(p)\Gamma w(p) \cap w(p)\Gamma w(p)\Gamma t(p) \\ & \subseteq t(p)\Gamma q(p)\Gamma q(p) \cap q(p)\Gamma t(p)\Gamma q(p) \cap q(p)\Gamma q(p)\Gamma t(p) \subseteq t(p) ” \end{aligned}$$

Because, $t(p)$ is a QTFI of T . Thus

$$\Rightarrow “m(p)\Gamma w(p)\Gamma w(p) \cap w(p)\Gamma m(p)\Gamma w(p) \cap w(p)\Gamma w(p)\Gamma m(p) \subseteq t(p) ” \rightarrow (2)$$

From equation (1) and (2) we have

$$\begin{aligned} & “m(p)\Gamma w(p)\Gamma w(p) \cap w(p)\Gamma m(p)\Gamma w(p) \cap w(p)\Gamma w(p)\Gamma m(p) \subseteq t(p) \cap w(p) \\ & m(p) ” \rightarrow (3) \end{aligned} =$$

Similarly, we can show that

$$“m(p)\Gamma w(p)\Gamma w(p)\cap w(p)\Gamma w(p)\Gamma m(p)\Gamma w(p)\Gamma w(p)\cap w(p)\Gamma w(p)\Gamma m(p)\subseteq m(p)” \rightarrow (4)$$

From equation (3) and (4) we have $m(p)$ is a QTFI of $w(p)$. “ $(t, P_1, \Gamma) \mathbf{I}_R(w, P_2, \Gamma)$ is a FSQTFI of (w, P_2, Γ) ”.

V. CONCLUSION

By using FSS theory one can develop further algebraic structures of TF-SR.

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