An Application to Continuous Probability Distribution

Nirmala Kasturi

***Abstract:** First part of the paper is the brief description of previous work about estimating one of the distributions developed by smoothing techniques of the histograms and second part of the paper is the application of the distribution.*

***Key Words:** Random variable, continuous probability distribution, arrival rate, density function, histograms, frequency.*

I. INTRODUCTION

Part 1: Random variable of interest is "Staying only n number of arrivals in the system in particular time interval". The arrivals stay in the system while going in queue to service and questioning for some data in the system (before leaving the system). Instead of asking "how many arrivals take place in a particular time interval (Poisson)", we ask for "how likely the system have n number of arrivals in particular time interval". Since X is continuous, the PDF should be a function. We have to make inferences about this unknown function. We have surveyed for a considerable period of time. We plot the histograms for different number of arrivals from which we find the density curves [2].

In general for the chosen random variable "Staying only n number of persons in the system in particular interval" the p.d.f. is

$$f(x) = \begin{cases} \dfrac{e^{-\lambda x}(\lambda x)^n \lambda}{n!} & when\, x \geq 0 \\ 0 & otherwise \end{cases}$$

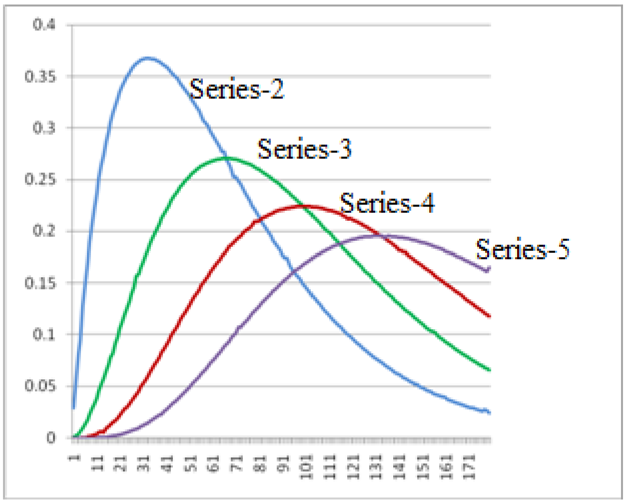

FIGURE – 1

Series 2 represents the probability density function graph when n = 1.

Series 3 represents the probability density function graph when n = 2.

Series 4 represents the probability density function graph when n = 3.

Series 5 represents the probability density function graph when n = 4.

X - Axis represents time; Y - represents f(x).

PART 2: Studying efficiency of queuing system will be useful to make suggestions to management to take good decisions in an optimized way. The efficiency in any queuing system can be measured by no. of arrivals accumulated at the system and queue.

As a result of part 1, by using density function, we can find probability of accumulating number of arrivals in the particular interval. Now it is useful to compare two queuing systems. By using this we can say that in which queuing system there is more probability of more number of customers accumulated in less time

If we consider the random variable that "the no. of people accumulated at the queue in particular interval" then by using p.d.f. function.

$$f(x) = \begin{cases} \dfrac{e^{-\lambda x}(\lambda x)^n \lambda}{n!} & when\, x \geq 0 \\ 0 & otherwise \end{cases},$$

Now this probability of no. of people accumulated in the queue in same time interval for any two queuing system can be compared. The queuing system where we get more probability for accumulating same no. of customers in same time interval, that queuing system referred as less efficient queuing system. We consider this time interval for the comparison i.e. $\left[0, \frac{n}{\lambda}\right]$. A survey was conducted in village and city branches of the same bank. Let us take number of customers n up to n = 1,2,3,4.

$\lambda_1 =$ Arrival rate of village bank = 0.007 customers/sec.

$\lambda_2 =$ Arrival rate of city bank = 0.03

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Nirmala Kasturi, Research Scholar, Department of Computer Science and Engineering, GITAM (Deemed to be University), and Visakhapatnam, India.

customers/sec.
The calculation of the probabilities of accumulating n number of people in the same time interval for both queueing systems.
The density function is

$$f(x)=\frac{e^{-\lambda x}(\lambda x)^{n}\lambda}{n!}$$

For n = 1: Case 1: For the city branch bank.

We take time interval $\left[0,\frac{n}{\lambda}\right]=[0,33]$

For n = 1 the p.d.f. function is

$$f(x)=\frac{e^{-\lambda x}(\lambda x)\lambda}{1!}$$

$$\int_{0}^{33}e^{-(0.03)x}(0.03x).0.03x\,dx$$

u = x

du = dx

v= $e^{-(0.03)}$

$$\int v\ dv=\frac{e^{-0.3x}}{-0.03}$$

$$\int_{0}^{33}e^{-(0.03)x}(0.03x).0.03x\ dx$$

$$=(0.03)^{2}\int_{0}^{33}e^{-(0.03)x}x\ dx$$

$$=\left(-(0.03x+1)e^{-0.03x}\right)_{0}^{33}$$

= 0.260562

Case 2:

For village branch bank:

For n = 1 the p.d.f. function is

$$f(x)=\frac{e^{-\lambda x}(\lambda x)\lambda}{1!}$$

We take the same time interval as in urban branch bank

$$\int_{0}^{33}e^{-(0.007)x}(0.007x)0.007\ dx$$

$$\left[-(0.007x+1)e^{-0.007x}\right]_{0}^{33}$$

= 0.0229067

∴ The probability that the no. of people (n = 1) accumulated at the queue in some time is more for the city branch which we can refer as less efficient.

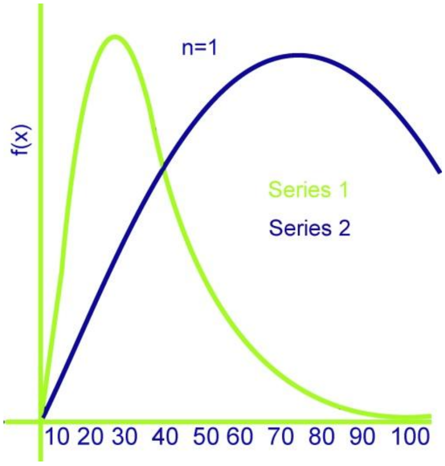

Figure - 2

Series 1 represents n = 1 number of arrivals accumulating at city branch in particular time interval.
Series 2 represents n = 1 number of arrivals accumulating at village branch in particular time interval.
For n = 2: Case 1: For city branch bank.
For n = 2 the p.d.f. function is

$$f(x)=\frac{e^{-\lambda x}(\lambda x)^{2}\lambda}{2!}$$

We take time interval up to the neighborhood of

$$\frac{n}{\lambda}$$

i.e. $\left[0, \frac{n}{\lambda}+1\right]=[0,67]$

$$\int_0^{67} f(x)\,dx$$

$$= \int_0^{67} \frac{e^{-\lambda x}(\lambda x)^2 \lambda}{2} dx$$

$$= \int_0^{67} \frac{e^{-0.03x}(0.03x)^2(0.03)\,dx}{2!}$$

$$= \frac{(0.03)^3}{2}\int_0^{67} e^{-0.03x}(0.03x)^2\,dx$$

$$= \left[-\left(0.00045x^2+0.03x+1\right)e^{-0.03t}\right]_0^{67}$$

$$= \left[-\left(2.02005+0.03\times 67+1\right)e^{-(0.03)67}+1\right]$$

$$= \left[-5.03005\times e^{-2.01}+1\right]$$

= 0.326030

Case 2 For village branch bank

We take the same time interval [0.67]

For n = 2 the p.d.f. function is

$$f(x)=\frac{e^{-\lambda x}(\lambda x)^2 \lambda}{2!}$$

$$= \int_0^{67} f(x)dx$$

$$= \int_0^{67} \frac{e^{-\lambda x}(\lambda x)^2 \lambda}{2!} dx$$

$$= \int_0^{67} \frac{e^{-(0.007)t}(0.007\times 67)^2(0.007)}{2!} dt$$

$$= -\left(0.00897+0.04087+0.0874635\right)e^{-0.469}+0.087446$$

$$= -\left(0.1373035\right)e^{-0.469}+0.087446$$

$$= -\left(0.1373035\right)0.625627+0.087446$$

$$= -0.085732+0.087446$$

= 0.00171320

The number (n=2) of people accumulated at the queue in same time interval is more for the city branch bank that village branch bank.

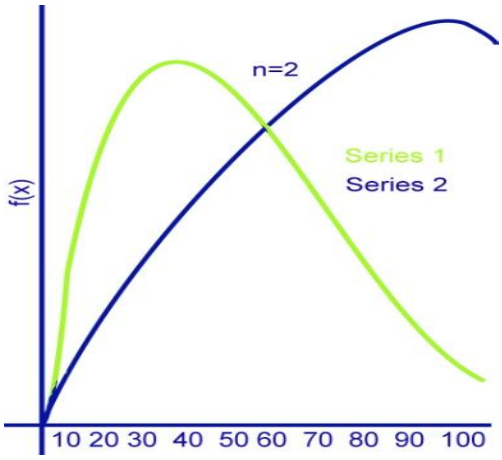

Figure - 3

Series 1 represents n = 2 number of arrivals accumulating in city branch in particular time interval.
Series 2 represents n = 2 number of arrivals accumulating in village branch in particular time interval.

For n = 3 Case 1: for city branch bank

For n = 3 the p.d.f. function is

$$f(x)=\frac{e^{-\lambda x}(\lambda x)^3 \lambda}{3!}$$

We consider the interval up to in the neighborhood of $\frac{n}{\lambda}$ i.e., $\left[0, \frac{n}{\lambda}+1\right]=[0,100]$

$$\int_0^{100} f(x)dx$$

$$\int_0^{100} \frac{e^{-\lambda x}(\lambda x)^3 \lambda}{3!} dx$$

$$\int_0^{100} \frac{e^{-0.03x}(0.03x)^3}{3!} 0.03dx$$

$$\frac{(0.03)^4}{3!}\int_0^{100} e^{-0.03x} x^3 dx$$

-1.209327891+1.8685= 0.6591721093

We got the probability of accumulating 3 customers in the queue in the time period of [0,100] is 0.659/72 for urban branch bank

Case 2: For village branch bank

For n = 3 the p.d.f. function is

$$f(x) = \frac{e^{-\lambda x}(\lambda x)^3 \lambda}{3!}$$

We consider the same interval [0,100]

$$\int_0^{100} \frac{e^{-\lambda x}(\lambda x)^3 \lambda}{3!} dx$$

$$\int_0^{100} \frac{e^{(-0.007)x}(0.007x)^3 0.007}{3!} dx$$

$$\frac{(0.007)^4}{6}\int_0^{100} e^{(-0.007)x} x^3 dx$$

$$\left[-\left(5.6938\times10^{-8}x^3 + 2.4402\times10^{-5}x^2 + 0.00697x + 0.99\right)e^{-0.007x}\right]_0^{100}$$

$$= -(0.98969) + 0.996$$

= 0.0063

We got the probability of accumulating 3 customers in the queue in the time period of [0,100] is 0.0063 for village branch bank

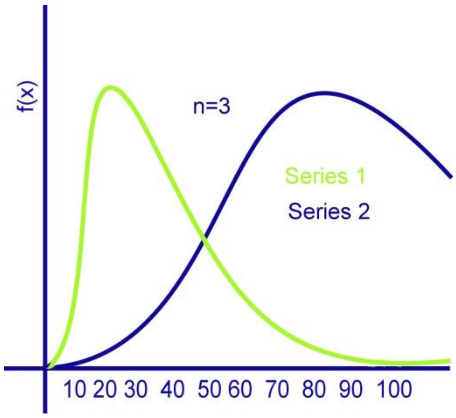

Figure - 4

Series 1 represents (n = 3) n number of customers accumulating at the city branch in particular time interval.
Series 2 represents (n = 3) n number of customers accumulating at the village branch in particular time interval.

For n = 4: Case 1

For city branch Bank

We take time interval up to the neighborhood of

$$\frac{n}{\lambda} i.e\left[0, \frac{n}{\lambda}+1\right] = [0,133]$$

$$\int_0^{133} f(x)dx$$

For n = 4 the p.d.f. function is

$$f(x) = \frac{e^{-\lambda x}(\lambda x)^4 \lambda}{4!}$$

$$\int_0^{133} \frac{e^{-0.03x}(0.03x)^4 0.003}{24} dt$$

$$\frac{(0.03)^3}{24}\int_0^{133} e^{-0.03t} x^4 dt$$

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$$=\left[(-3.24\times10^{-8}x^4+4.32\times10^{-6}x^3+0.000432x^2+0.0288x+0.96)e\right]_0^{100}$$

= 0.587232+0.96

= 0.3727

We got the probability of accumulating 4 customers in the queue in the time period of [0,133] is 0.3727 for city branch bank.

Case 2: For village branch bank

We consider the same interval [0.133] as in city branch bank case

i.e. [0.133]

For n = 4 the p.d.f. function is

$$f(x)=\frac{e^{-\lambda x}(\lambda x)^4\lambda}{4!}$$

$$\int_0^{133} f(x)dx$$

$$\int_0^{133}\frac{e^{-\lambda x}(\lambda x)^4\lambda}{4!}dx$$

$$\int_0^{133}\frac{e^{-(0.007x)}(0.007x)^4\,0.007}{4!}dx$$

$$=\frac{(0.007)^5}{24}\int_0^{133}e^{-0.007x}x^4dx$$

$$=\left[-\left(9.604\times10^{-9}x^4+5.488\times10^{-8}x^3+2.352\times10^{5}x^2+0.00672x+0.96\right)e^{-0.007x}\right]_0^{133}$$

$$=\left[-(2.19)e^{-0.007x}\right]_0^{133}$$

= 0.34

We got the probability of accumulating customers in queue in the time period of [0.133] is 0.3400 for village branch bank.

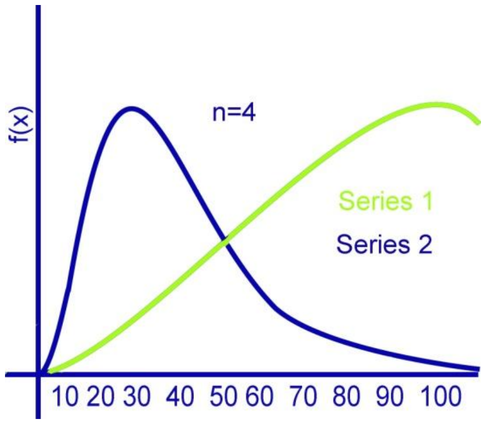

Figure - 5

Series 1 represents (n = 4) n number of customers accumulating at the city branch in particular time interval.
Series 2 represents (n = 4) n number of customers accumulating at the village branch in particular time interval.
This represents that probability of an arriving customer has to wait more in a city branch queuing system than that of a village branch.

II. CONCLUSION

This paper proposed the application of the p.d.f. function.

$$f(x)=\begin{cases}\dfrac{e^{-\lambda x}(\lambda x)^n\lambda}{n!} & when\ \ x\geq 0\,.\\ 0 & otherwise\end{cases}$$

By measuring the probability of accumulating n number of customers we made the inference that the quantum of work is not equal and may vary in different locations. Therefore, we can conclude an employee in a rural branch is prone to more idle time. Various graphs and data authenticate the same.

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