

Partial Addition and Ternary Product based Γ -so-semirings-1

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Abstract: A ternary Γ - SO semiring possess a “natural partial ordering”, an infinitely “partial addition and a ternary multiplication with a set of laws. In this paper we introduce the notions” of different structures of partially ternary Γ - ideals of ternary Γ - SO semi ring and characterize them.

Mathematical subject classification: 16Y60

Keywords: Partial ternary Γ -semi ring, sum ordering, SO monoid, ternary Γ - SO semiring partial ternary Γ -ideal, ternary Γ -ideal.

In 1949 tarski investigated cardinal algebras and in 1980 higgs studies Σ structures. Further arbib, manes benson [3] investigated Sum ordered partial semi rings. In 2015 M. Sajani Lavanya, D. MadhusudhanaRao[12-17] introduced the concept of Γ -SR (“ternary Γ -semirings”). “M.siva mala, K.siva Prasad [19-24]” introduced the notion of Γ - SO rings and prime ideals, semi prime ideals in Γ - SO rings. Here the notions of Γ -S-SR (“Ternary Γ - SO semi rings”) and characterizes them.

I. INTRODUCTION

II. PRELIMINARIES

In the preliminary wing some important information was collected from [12-17],[19-24].

3. Regular Ternary Γ -SO-Semiring:

Definition 3.1: “A be *partial Γ -monoid* is a triple (R, Γ, Σ) where R, Γ are non-empty sets and “ Σ is a partial addition defined on some but not necessarily all families $(a_i : i \in I)$ in R with the following laws:”

- 1) “**Unary sum axiom:** If $(a_i : i \in I)$ is a one element family in R and $I = \{j\}$ then $\sum(a_i : i \in I)$ is defined and equal to a_j .”
- 2) “**Partition Associative axiom:** If $(a_i : i \in I)$ is a family in R and $(a_j : j \in I)$ is a partition of I , then $(a_i : i \in I)$ is sum-able $\Leftrightarrow (a_i : i \in I_j)$ is sum-able for every j in J , $(\sum(a_i : i \in I_j) : j \in J)$ is sum-able and $\sum(a_i : i \in I) = \sum(\sum(a_i : i \in I_j) : j \in J)$.”

Definition 3.2: “Let M, Γ be partial Γ -monoids then M is known as P Γ -SR (*partial ternary gamma semiring*) provided \exists a mapping $M \times \Gamma \times M \times \Gamma \times M \rightarrow M$ satisfying the following conditions:

- 1) $x\alpha y\beta(z\delta p\gamma q) = x\alpha(y\beta z\delta p)\gamma q = (x\alpha y\beta z)\delta p\gamma q$
- 2) a family $(a_i : i \in I)$ is sum-able in $M \Rightarrow (x\alpha y\beta a_i : \text{for odd } i \in I)$ is sum-able in M , $x\alpha y\beta \left[\sum(a_i : i \in I) \right] = \sum(x\alpha y\beta a_i : \text{for odd } i \in I)$
- 3) family $(a_i : i \in I)$ is sum able in $M \Rightarrow (x\alpha a_i \beta y : \text{for odd } i \in I)$ is sum able in M , $x\alpha \left[\sum(a_i : i \in I) \right] \beta y = \sum(x\alpha a_i \beta y : \text{for odd } i \in I)$
- 4) family $(a_i : i \in I)$ is sum able in $M \Rightarrow (a_i \alpha x \beta y : \text{for odd } i \in I)$ is sum able in M and $\left[\sum(a_i : i \in I) \right] \alpha x \beta y = \sum(a_i \alpha x \beta y : \text{for odd } i \in I)$ ”

Revised Manuscript Received on December 06, 2020.

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Definition 3.3: A PTT-SR said to have a left (lateral, right) unity element provided \exists a family $(e_i : i \in I)$ of M and $(\alpha_i, \beta_i : i \in I)$ of $\Gamma \ni$

$$\sum e_i \alpha_i e_i \beta_i a = a (\sum e_i \alpha_i a \beta_i e_i = a, \sum a \alpha_i e_i \beta_i e_i = a) \text{ for any } a \in M$$

Definition 3.4: The sum ordering relation \leq in PTTM (“partially ternary Γ -monoid”) M is the binary relation $\ni d \leq e$ iff \exists an element f in $M \ni e = d + f \forall d, e \in M$.

Def 3.5: “A SPTM (*sum ordered partially ternary Γ -monoid*) (*ternary Γ -SO –monoid*) in which partial sum ordering is a partial ordering”.

Def 3.6: “A PTT-SR M is said to be SPTM-SR (*sum ordered partial ternary Γ -semiring*) if the partial Γ -monoid is SO- Γ -monoid.”

Definition 3.7: “Let M be a PTT-SR. A non-empty subset of M is said to be **RPT- Γ I** (*right (Lateral, left) partial ternary Γ -ideal*) of M provided

(i) $(a_i : i \in I)$ is a sum able family of M and $x_i \in A$ for all $i \in I \Rightarrow \sum_i x_i \in A$

(ii) $\forall x, y \in M, z \in A \Rightarrow z \alpha x \beta y \in A (x \alpha z \beta y \in A, x \alpha y \beta z \in A)$ ”

“If A is right, lateral and left partial ternary Γ -ideal of M , then A is called partial ternary Γ -ideal of M ”.

Definition 3.8: “Let M be a Γ -S-SR. A non-empty subset A of M is said to be a *left (lateral, right) Γ I* (*ternary Γ -ideal*) of M , if it satisfies the following:

(i) A is a LPT- Γ I (MPT- Γ I, RPT- Γ I) of M .

(ii) $x \in M$ and $y \in A \ni x \leq y$ then $x \in A$ ”.

“If A is left, lateral as well as right ternary Γ -ideal of M , then A is known as ternary Γ -ideal of M .”

Def 3.9: “Let M be a Γ -S-SR & A be a subset of M , then the intersection of all ternary Γ -ideals containing the set A is called *ternary Γ -ideal generated by A* and it is denoted by (A) ”.

Def 3.10: “A Γ -S-SR M is said to be CTSSR (*complete ternary Γ -SO-semiring*) if every family of elements in M is sum able.”

Def 3.11: “An ideal Q of a Γ -S-SR M is known as *irreducible* \Leftrightarrow for any ideals R, S, T of $M, Q = R \cap S \cap T$ implies that $Q = R$ or $Q = S$ or $Q = T$.”

Def 3.12: “An ideal Q of a Γ -S-SR M is known as *strongly irreducible* \Leftrightarrow for any ideals R, S, T of $M, R \cap S \cap T \subseteq Q \Rightarrow R \subseteq Q$ or $S \subseteq Q$ or $T \subseteq Q$.”

Ex 3.13: Let $M = \{0, p, q, r, s, t\}$ and Σ on M as

$$\sum_i x_i = \begin{cases} x_j & \text{if } x_i = 0 \forall i \neq j, \text{ for some} \\ d, & \text{if } x_j = a, x_k = b \text{ or } x_j = b, x_k = c \text{ for some } j, k \text{ and } x_i = 0 \forall i \neq j, k \\ \text{undefined,} & \text{otherwise} \end{cases}$$

Then M is a ternary SO-monoid. Let $\Gamma = \{\alpha, \beta\}$.

Define the mapping $M \times \Gamma \times M \times \Gamma \times M \rightarrow M$ as follows:

β	α	β	α	β	α	β	t
0	0	0	0	0	0	0	0
p	0	0	0	0	0	β	0
q	α	0	0	0	0	α	0
r	0	0	0	0	0	β	0
s	0	0	0	0	0	β	0
t	0	β	α	β	α	β	0

Then M is a Γ -S-SR. For ideals $R = \{0, p\}$, $S = \{0, q\}$, $T = \{0, r\}$ and $U = \{0, s\}$ of M, $S \cap T \cap U = \{0, q\} \cap \{0, r\} \cap \{0, s\} = \{0\} \subseteq R$ and $S \not\subseteq R$, $T \not\subseteq R$ and $U \not\subseteq R$. Hence $R = \{0, p\}$ does not “strongly irreducible ideal” of M. But the ideal $R = \{0, p\}$ is an “irreducible ideal” of M.

Def 3.14: “A proper ideal P of a Γ -S-SR M is known as *prime* \Leftrightarrow for any ideals R, S, T of M, $R \cap S \cap T \subseteq P \Rightarrow R \subseteq P$ or $S \subseteq P$ or $T \subseteq P$.”

Def 3.15: “A proper ideal P of a Γ -S-SR M is known as *semiprime* \Leftrightarrow for any ideals R of M, $R \cap R \cap R \subseteq P \Rightarrow R \subseteq P$ ”.

Th 3.16: “An ideal R of a complete Γ -S-SR M is prime \Leftrightarrow it is semiprime and strongly irreducible”.

Proof: Assume R is “prime ideal” of M. Then R is a “semiprime ideal” of M. Let S, T, U ideals of M such that $S \cap T \cap U \subseteq R$. Since $S \cap T \cap U \subseteq R$ and R is prime implies that $S \subseteq R$ or $T \subseteq R$ or $U \subseteq R$ and hence R is a “strongly irreducible ideal” of M.

On the other hand, suppose that R is semiprime and “strongly irreducible” ideal of M. Let S, T, U ideals of M such that $S \cap T \cap U \subseteq R$, subsequently $(S \cap T \cap U) \cap (S \cap T \cap U) \cap (S \cap T \cap U) \subseteq S \cap T \cap U \subseteq R$. Since R is semiprime and strongly irreducible and hence $S \subseteq R$ or $T \subseteq R$ or $U \subseteq R$. So, R is “prime ideal” of M.

Remark 3.17: “Let $\{A_j / j \in J\}$ be a family of ideals in Γ -S-SR M. Then we denote $\langle \bigcup_{j \in J} A_j \rangle$ as

$\langle \bigvee_{j \in J} A_j \rangle$ and thus $\langle \bigvee_{j \in J} A_j \rangle = \{x \in M : x \leq \sum_j x_j, x_j \in \bigcup_{j \in J} A_j\}$.”

Def: 3.18: “Let M be a Γ -S-SR. Then $m \in M$ is said to be a *Regular* if $m \leq \alpha n \beta m$ for $n \in M$, $\alpha, \beta \in \Gamma$ if every element of M is regular then M is called a Regular ternary Γ -SO-semiring”.

Def: 3.19: “Let M be a Γ -S-SR. Then $m \in M$ is said to be a strongly Regular if there exist for $n \in M$, $\alpha, \beta \in \Gamma$ such that $m = \alpha n \beta m$. If every element of M is strongly Regular then M is called a *strongly regular ternary Γ -SO-semiring*.”

III. EXPERIMENTS AND RESULTS DESCRIPTION

Note: Let M be a Γ -S-SR. Then $m \in M$ is said to be a *strongly Regular* if there exist for $n \in M$, $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $m = \alpha n \beta m \gamma \delta m$.

Ex: Let $K = [0, 1]$ and $\Gamma = \mathbb{N} \cup \{0\}$ for any family $(x_i : i \in I)$ of K defined as $\sum_i x_i = \text{Sup} \{x_i : i \in I\}$ then K is a ternary Γ -SO monoid. i.e., $(x, \alpha, y, \beta, z) \rightarrow \inf\{x, \alpha, y, \beta, z\}$ of $K \times \Gamma \times K \times \Gamma \times K \rightarrow K$. Now K is a Γ -S-SR. For any $k \in K \exists 1, 1 \in \Gamma \ni k 1 k 1 k = \inf\{k, 1, k, 1, k\} = k$.

Th: “Let M be a complete ternary Γ -SO semiring with left unity. Then M is regular iff $C \cap \Gamma B \cap \Gamma A = A \cap B \cap C$ for left ideal A, lateral ideal B and right ideal C of M”.

Proof: “Suppose M is Regular for left ideal A, Lateral ideal B and right ideal C of M”.

We have $C \cap \Gamma B \cap \Gamma A \subseteq M \cap \Gamma M \cap \Gamma A \subseteq A$, $C \cap \Gamma B \cap \Gamma A \subseteq M \cap \Gamma B \cap \Gamma M \subseteq B$ and $C \cap \Gamma B \cap \Gamma A \subseteq C \cap \Gamma M \cap \Gamma M \subseteq C$ and hence $C \cap \Gamma B \cap \Gamma A \subseteq A \cap B \cap C$. Let $x \in A \cap B \cap C$. Since M is regular implies that $x = \sum_i x \alpha_i y_i \beta_i x \gamma_i z_i \delta_i x$ for some $\alpha_i, \beta_i, \gamma_i, \delta_i \in \Gamma, y_i, z_i \in M$. Since $x \in B$ and B is a lateral ideal of M, $y_i \beta_i x \gamma_i z_i \in B \Rightarrow x = \sum_i x \alpha_i (y_i \beta_i x \gamma_i z_i) \delta_i x, x \in B \Rightarrow y_i \beta_i x \gamma_i z_i \in A$ and $y_i \beta_i x \gamma_i z_i \in C$

$\Rightarrow x \in C \Gamma B \Gamma A$ and hence $A \cap B \cap C \subseteq C \Gamma B \Gamma A$. Therefore $C \Gamma B \Gamma A = A \cap B \cap C$

On the contrary $C \Gamma B \Gamma A = A \cap B \cap C$ for left ideal A, lateral ideal B and right ideal C of M. Let $x \in M$. Take $A = \langle x \rangle$, $B = \langle x \rangle$ and $C = \langle x \rangle$.

Then $A = A \cap B \cap C = M \Gamma M \Gamma A \subseteq M \Gamma M \Gamma x$, $B = M \Gamma B \Gamma M \subseteq M \Gamma x \Gamma M$ and

$C = C \Gamma M \Gamma M \subseteq x \Gamma M \Gamma M$. Now $x \in A \cap B \cap C \subseteq M \Gamma M \Gamma x \cap M \Gamma x \Gamma M \cap x \Gamma M \Gamma M$

$= x \Gamma M \Gamma x \Gamma M \Gamma x$. Therefore x is regular in M. Hence M is regular ternary Γ -SO- semiring.

Theorem: “Let M be a complete regular ternary Γ -SO-semiring with left unity and Q be an ideal of M. Then Q is prime if and only if it is irreducible”.

Proof: “Suppose Q is prime, then by theorem 3.16, Q is strongly irreducible and hence Q is reducible”.

On the other hand, presume that “Q is irreducible”. Let R, S, T are three ideals of M such that $R \Gamma S \Gamma T \subseteq Q$. Let $x \in (RVQ) \cap (SVQ) \cap (TVQ)$. Since M is regular, by theorem 3.21, $(RVQ) \Gamma (SVQ) \Gamma (TVQ) = (RVQ) \cap (SVQ) \cap (TVQ)$ and hence $x \in (RVQ) \Gamma (SVQ) \Gamma (TVQ)$ implies that $x \leq \sum_i p_i \alpha_i q_i \beta_i r_i$ where

$p_i \in R \vee Q, q_i \in S \vee Q, r_i \in T \vee Q$ and $\alpha_i, \beta_i \in \Gamma, i \in I$ implies that

$$x \leq \sum_i [\sum_j (q_{i_j} + r_{i_j})] \alpha_i [\sum_k (q'_{i_k} + s_{i_k})] \beta_i [\sum_l (q''_{i_l} + t_{i_l})]$$

for some $q_{i_j}, q'_{i_k}, q''_{i_l} \in Q, r_{i_j} \in R, s_{i_k} \in S, t_{i_l} \in T$ and $\alpha_i, \beta_i \in \Gamma$

$$\Rightarrow x \leq \sum_i \sum_j \sum_k \sum_l [q_{i_j} \alpha_i q'_{i_k} \beta_i q''_{i_l} + q_{i_j} \alpha_i q'_{i_k} \beta_i t_{i_l} + q_{i_j} \alpha_i s_{i_k} \beta_i q''_{i_l} + q_{i_j} \alpha_i s_{i_k} \beta_i t_{i_l} + r_{i_j} \alpha_i q'_{i_k} \beta_i q''_{i_l} + r_{i_j} \alpha_i q'_{i_k} \beta_i t_{i_l} + r_{i_j} \alpha_i s_{i_k} \beta_i q''_{i_l} + r_{i_j} \alpha_i s_{i_k} \beta_i t_{i_l}]$$

$\Rightarrow x \in Q \vee (R \Gamma S \Gamma T) = Q \Rightarrow (RVQ) \Gamma (SVQ) \Gamma (TVQ) = Q$.

Since Q is irreducible. $(RVQ) = Q$ or $(SVQ) = Q$ or $(TVQ) = Q \Rightarrow R \subseteq Q$ or $S \subseteq Q$ or $T \subseteq Q$. Thus “Q is a prime ideal of M”.

IV. CONCLUSION

Mainly we introduced in this paper about regular ternary Γ -SO-Semirings and characterized ternary Γ -SO-Semiring.

V. ACKNOWLEDGEMENT

we are very thank full to Dr. D. MadhusudhanaRao, Professor in Mathematics, VSR & NVR College, Tenali and Dr. P. Siva Prasad, Associate Professor, Vignan University, Vadlamudi, Guntur(Dt).

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