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Partial Addition and Ternary Product based Γ-so-semirings-1

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Abstract: *A ternary* Γ *- SO semiring possess a "natural partial ordering", an infinitely "partial addition and a ternary multiplication with a set of laws. In this paper we introduce the notions" of different structures of partially ternary* Γ *- ideals of ternary* Γ *- SO semi ring and characterize them.*

Mathematical subject classification: 16Y60

Keywords: *Partial ternary* Γ *-semi ring, sum ordering, SO monoid, ternary* Γ *- SO semiring partial ternary* Γ *-ideal, ternary* Γ *-ideal.*

I. INTRODUCTION

In1949 tarski investigated cardinal algebras and in 1980 higgs studies $\sum$ structures. Further arbib, manes benson [3] investigated Sum ordered partial semi rings. In 2015 M. Sajani Lavanya, D. MadhusudhanaRao[12-17] introduced the concept of TΓ-SR ("ternary Γ-semirings"). "M.siva mala, K.siva Prasad [19-24]" introduced the notion of Γ - SO rings and prime ideals, semi prime ideals in Γ - SO rings. Here the notions of TΓ-S-SR ("Ternary Γ - SO semi rings") and characterizes them.

II. PRELIMINARIES

In the preliminary wing some important information was collected from [12-17],[19-24].

3. Regular Ternary Γ-SO-Semiring:

Definition3.1: "A be ***partial Γ-monoid*** is a triple (R, Γ, Σ) where R, Γ are non-empty sets and "Σ is a partial addition defined on some but not necessarily all families $(a_i : i \in I)$ in R with the following laws:"

1) **"Unary sum axiom:** If $(a_i : i \in I)$ is a one element family in R and I = {j} then $\sum(a_i : i \in I)$ is defined and equal to a_j."
2) **"Partition Associative axiom:** If $(a_i : i \in I)$ is a family in R and $(a_j : j \in I)$ is a partition of I, then $(a_i : i \in I)$ is sum-able $\Leftrightarrow$ $(a_i : i \in I_j)$ is sum-able for every j in J, $(\sum(a_i : i \in I_j) : j \in J)$ is sum-able and $\sum(a_i : i \in I_j) = \sum(\sum(a_i : i \in I_j) : j \in J)$."

Definition 3.2: "Let M, Γ be partial Γ-monoids then M is known as PTΓ-SR (***partial ternary gamma semiring)*** provided ∃ a mapping $M \times \Gamma \times M \times \Gamma \times M \to M$ satisfying the following conditions:

1) $x\alpha y\beta(z\delta p\gamma q) = x\alpha(y\beta z\delta p)\gamma q = (x\alpha y\beta z)\delta p\gamma q$
2) a family $(a_i : i \in I)$ is sum-able in M $\Rightarrow$ $(x\alpha y\beta a_i :$ for odd $i \in I)$ is sum-able in M , $x\alpha y\beta\left[\sum(a_i : i \in I)\right] = \sum(x\alpha y\beta a_i :$ for odd $i \in I)$

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3) family $(a_i : i \in I)$ is sum able in M $\Rightarrow$ $(x\alpha a_i \beta y :$ for odd $i \in I)$ is sum able in M, $x\alpha\left[\sum(a_i : i \in I)\right]\beta y = \sum(x\alpha a_i \beta\, y :$ for odd $i \in I)$

4) family $(a_i : i \in I)$ is sum able in M $\Rightarrow$ $(a_i \alpha x \beta y :$ for odd $i \in I)$ is sum able in M and $\left[\sum(a_i : i \in I)\right]\alpha x \beta y = \sum(a_i \alpha x \beta y :$ for odd $i \in I)$"

Definition 3.3: A PTΓ-SR said to have a left (lateral, right) unity element provided ∃ a family $(e_i : i \in I)$ of M and $(\alpha_i, \beta_i : i \in I)$ of Γ ∋

$\sum e_i \alpha_i e_i \beta_i a = a(\sum e_i \alpha_i a \beta_i e_i = a, \sum a \alpha_i e_i \beta_i e_i = a)$ for any $a \in$ M

Definition 3.4: The sum ordering relation ≤ in PTΓM ("partially ternary Γ-monoid") M is the binary relation ∋ $d \le e$ iff ∃ an element f in M ∋ $e = d + f$ ∀ $d, e \in$ M.

Def 3.5: "A SPTΓ-M (***sum ordered partially ternary*** Γ***-monoid) (ternary*** Γ***-SO –monoid)*** in which partial sum ordering is a partial ordering".

Def 3.6: "A PTΓ-SR M is said to be SPTΓ-SR (***sum ordered partial ternary*** Γ***-semiring)*** if the partial Γ-monoid is SO-Γ-monoid."

Definition 3.7: "Let M be a PTΓ-SR. A non-empty subset of M is said to be **RPT-ΓI** (***right (Lateral, left) partial ternary Γ-ideal)*** of M provided

(i) $(a_i : i \in I)$ is a sum able family of M and $x_i \in$ A for all $i \in$ I ⇒ $\sum_i x_i \in A$

(ii∀ $x, y \in$ M, $z \in$ A ⇒$z\alpha x\beta y \in$ A ($x\alpha z\beta y \in$ A, $x\alpha y\beta z \in$ A)"

"If A is right, lateral and left partial ternary Γ-ideal of M, then A is called partial ternary Γ-ideal of M".

Definition 3.8: "Let M be a TΓ-S-SR. A non-empty subset A of M is said to be a ***left (lateral, right)*** **TΓI** ***(ternary Γ-ideal)*** of M, if it satisfies the following:

(i) A is a LPT-ΓI (MPT-ΓI, RPT-ΓI**)** of M.

(ii) $x \in$ M and $y \in$ A ∋ $x \le y$ then $x \in$ A".

"If A is left, lateral as well as right ternary Γ-ideal of M, then A is known as ternary Γ-ideal of M."

Def 3.9: "Let M be a TΓ-S-SR & A be a subset of M, then the intersection of all ternary Γ-ideals containing the set A is called ***ternary Γ-ideal generated by A*** and it is denoted by (A)".

Def 3.10: "A TΓ-S-SR M is said to be CTΓSSR (***complete ternary Γ-SO-semiring)*** if every family of elements in M is sum able."

Def 3.11: "An ideal Q of a TΓ-S-SR M is known as ***irreducible*** ⇔ for any ideals R, S, T of M, Q = R ⋂ S⋂ T implies that Q = R or Q = S or Q = T."

Def 3.12: "An ideal Q of a TΓ-S-SR M is known as ***strongly irreducible*** ⇔ for any ideals R, S, T of M, R⋂ S ⋂ T ⊆ Q ⇒ R ⊆ Q or S ⊆ Q or T ⊆ Q."

Ex 3.13: Let M = {$0, p, q, r, s, t$} and Σ on M as

$$\sum_i x_i = \begin{cases} x_j & \text{if } x_i = 0\ \forall i \ne j, \text{for some} \\ d, & \text{if } x_j = a,\ \ x_k = b \text{ or } x_j = b,\ x_k = c \text{ for some } j,k \text{ and } x_i = 0\ \forall i \ne j,k \\ \text{undefined, otherwise} \end{cases}$$

Then M is a ternary SO-monoid. Let Γ = {α, β}.

Define the mapping M ×Γ× M ×Γ× M ⟶ M as follows:

α	0	p	q	r	s	t
0	0	0	0	0	0	0
p	0	0	0	0	0	0

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q	0	0	0	0	0	0
r	0	0	0	0	0	0
s	0	0	0	0	0	0
t	0	0	0	0	0	0

Then M is a TΓ-S-SR. For ideals R = {0, *p*}, S = {0,q}, T = {0, *r*} and U = {0, *s*} of M, S⋂T⋂U = {0, *q*} ⋂ {0, *r*} ⋂ {0, *s*} = {0} ⊆ R and S ⊈ R, T ⊈ R and U ⊈ R. Hence R = {0, *p*} does not "strongly irreducible ideal" of M. But the ideal R = {0, *p*} is an "irreducible ideal" of M.

Def 3.14: "A proper ideal P of a TΓ-S-SR M is known as ***prime*** ⇔for any ideals R, S, T of M, RΓSΓT ⊆ P ⟹ R ⊆ P or S ⊆ P or T ⊆ P.**"**

Def 3.15: "A proper ideal P of a TΓ-S-SR M is known as ***semiprime*** ⇔ for any ideals R of M, RΓRΓR ⊆ P ⟹ R ⊆ P**"**.

Th 3.16: "An ideal R of a complete TΓ-S-SR **M is prime ⇔ it is semiprime and strongly irreducible".**

***Proof*:** Assume R is "prime ideal" of M. Then R is a "semiprime ideal" of M. Let S, T, U ideals of M such that S⋂T⋂U ⊆ R. Since SΓTΓU ⊆S⋂T⋂U ⊆ R and R is prime implies that S ⊆ R or T ⊆ R or U ⊆ R and hence R is a "strongly irreducible ideal" of M.

On the other hand, suppose that R is semiprime and "strongly irreducible" ideal of M. Let S, T, U ideals of M such that SΓTΓU ⊆ R, subsequently (S⋂T⋂U)Γ(S⋂T⋂U)Γ(S⋂T⋂U) ⊆ SΓTΓU ⊆ R. Since R is semiprime and strongly irreducible and hence S ⊆ R or T ⊆ R or U ⊆ R. So, R is "prime ideal" of M.

Remark 3.17: "Let $\{A_j \,/\, j \in J\}$ be a family of ideals in TΓ-S-SR M. Then we denote $<\bigcup_{j\in J} A_j>$ as $<\bigvee_{j\in J} A_j>$ and thus $<\bigvee_{j\in J} A_j> = \{x \in M : x \le \sum_j x_j, x_j \in \bigcup_{j\in J} A_j\}$."

Def: 3.18: "Let M be a TΓ-S-SR. Then *m*∈ M is said to be a ***Regular*** if $m \le m\alpha n\beta m$ for *n*∈ M, *α, β*∈Γ if every element of M is regular then M is called a Regular ternary Γ- SO –semiring".

Def: 3.19: "Let M be a TΓ-S-SR. Then *m*∈ M is said to be a strongly Regular if there exist for *n*∈ M, α, β∈Γ such that $m = m\alpha n\beta m$. If every element of M is strongly Regular then M is called a ***strongly regular ternary Γ-SO-semiring***."

III. EXPERIMENTS AND RESULTS DESCRIPTION

Note: Let M be a TΓ-S-SR. Then *m*∈ M is said to be a ***strongly Regular*** if there exist for *n*∈ M, *α, β, γ, δ*∈Γ such that $m = m\alpha n\beta m\gamma o\delta m$.

Ex: Let K= [0, 1] and Γ=N ∪{0} for any family(x_i: i∈ *I*) of K defined as $\sum_i x_i$=Sup {x_i: i∈ *I*} then K is a ternary Γ- SO monoid. i.e., (x, α, y, β, z) → inf{x, α, y, β, z} of K×Γ× K ×Γ× K → K. Now K is a TΓ-S-SR. For any *k*∈ K ∃ 1,1∈ Γ ∋ *k1k1k*= inf{*k,1,k,1,k*}=*k* .

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Th: “Let M be a complete ternary Γ- SO semiring with left unity. Then M is regular iff C ΓB ΓA=A∩ $B \cap C$ for left ideal A, lateral ideal B and right ideal C of M”.

Proof: “Suppose M is Regular for left ideal A, Lateral ideal B and right ideal C of M”.

We have CΓB ΓA⊆ M ΓM ΓA⊆A, C ΓB ΓA⊆ M ΓB ΓM⊆B and CΓB ΓA⊆ CΓM ΓM⊆C and hence CΓBΓA⊆A⋂B⋂C. Let $x \in A \cap B \cap C$. Since M is regular implies that x=$\sum_i x\alpha_i\, y_i\beta_i x\gamma_i z_i\delta_i x$ for some $\alpha_i, \beta_i, \gamma_i, \delta_i \in \Gamma$,$y_i, z_i \in M$.Since x∈B and B is a lateral ideal of M, $y_i\beta_i x\gamma_i z_i \in B \Rightarrow x=\sum_i x\alpha_i\, (y_i\beta_i x\gamma_i z_i)\delta_i x$, x∈ $B \Rightarrow y_i\beta_i x\gamma_i z_i \in A$ *and* $y_i\beta_i x\gamma_i z_i \in C$

⇒x∈ C ΓB ΓA *and* hence A∩ $B \cap C \subseteq$ C ΓB ΓA. Therefore CΓB ΓA=A∩ $B \cap C$

On the contrary CΓB ΓA=A∩ $B \cap C$for left ideal A, lateral ideal B and right ideal C of M. Let x∈ M. Take A=<x>), B=<x> and C=<x>.

Then A= A∩ $B \cap C$= M ΓM ΓA⊆M ΓM Γx, B= M ΓB ΓM⊆M Γx ΓM and

C= C ΓM ΓM⊆x ΓM ΓM. Now x∈ A∩ $B \cap C \subseteq$M ΓM Γx∩ M Γx ΓM ∩x ΓMΓM

= x ΓM Γx ΓM Γx. Therefore x is regular in M. Hence M is regaular ternary Γ-SO- semiring.

Theorem: “Let M be a complete regular ternary Γ-SO-semiring with left unity and Q be an ideal of M. Then Q is prime if and only if it is irreducible”.

***Proof*: “**Suppose Q is prime, then by theorem 3.16, Q is strongly irreducible and hence Q is reducible”. On the other hand, presume that “Q is irreducible”. Let R, S, T are three ideals of M such that RΓSΓT ⊆ Q. Let *x*∈ (RVQ)⋂(SVQ)⋂(TVQ). Since M is regular, by theorem 3.21, (RVQ)Γ(SVQ)Γ(TVQ)= (RVQ)⋂(SVQ)⋂(TVQ) and hence *x*∈ (RVQ)Γ(SVQ)Γ(TVQ) implies that $x \leq \sum_i p_i\alpha_i q_i\beta_i r_i$ where $p_i \in R \vee Q, q_i \in S \vee Q, r_i \in T \vee Q$ and $\alpha_i, \beta_i \in \Gamma, i \in I$ implies that

$$x \leq \sum_i [\sum_j (q_{i_j} + r_{i_j})]\alpha_i [\sum_k (q'_{i_k} + s_{i_k})]\beta_i [\sum_l (q''_{i_l} + t_{i_l})]$$

for some $q_{i_j}, q'_{i_k}, q''_{i_l} \in Q$, $r_{i_j} \in R, s_{i_k} \in S, t_{i_l} \in T$ and $\alpha_i, \beta_i \in \Gamma$

$$\Rightarrow \begin{array}{l} x \leq \sum_i \sum_j \sum_k \sum_l [q_{i_j}\alpha_i q'_{i_k}\beta_i q''_{i_l} + q_{i_j}\alpha_i q'_{i_k}\beta_i t_{i_l} + q_{i_j}\alpha_i s_{i_k}\beta_i q''_{i_l} + q_{i_j}\alpha_i s_{i_k}\beta_i t_{i_l} + r_{i_j}\alpha_i q'_{i_k}\beta_i q''_{i_l} + r_{i_j}\alpha_i q'_{i_k}\beta_i t_{i_l} \\ + r_{i_j}\alpha_i s_{i_k}\beta_i q''_{i_l} + r_{i_j}\alpha_i s_{i_k}\beta_i t_{i_l} \end{array}$$

$\Rightarrow x \in Q \vee (R\Gamma S\Gamma T) = Q \Rightarrow$(RVQ)Γ(SVQ)Γ(TVQ) = Q.

Since Q is irreducible. (RVQ) = Q or (SVQ) = Q or (TVQ) = Q $\Rightarrow R \subseteq Q$ or $S \subseteq Q$ or $T \subseteq Q$. Thus “Q is a prime ideal of M”.

IV. CONCLUSION

Mainly we introduced in this paper about regular ternary Γ-SO-Semirings and characterized ternary Γ-SO-Semiring.

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