Defuzzification Formula for Modelling and Scheduling A Furniture Fuzzy Project Network

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Abstract: This research article presents a new defuzzification formula for deciding the critical path in a proposed network. Here we introduce an octagonal fuzzy numbers for representing the duration time. It is shown that it is better to use octagonal fuzzy numbers towards determining the critical path. A numerical example is given and the proposed formula as compared with the existing fuzzy numbers.

Keywords: FCPM, fuzzy project network, fuzzy numbers, defuzzification. MSC: 63E72, 65K10, 90B15.

I. INTRODUCTION

The critical path method (CPM), worked out toward the start of the 1960s, Starting with the second part of the 1970s proposed in [5,8,11]. In [10] they calculated the critical path by considering the needed sources, the time of project completion. Zadeh [12] introduced an another path to accord with vague data is to use the perception of fuzziness. Jain [4] proposed the idea of fuzzy numbers and positioning fuzzy numbers. In [1] acquainted a calculation with carryout critical path under fuzzy domain. In [3] they computed fuzzy math operations to calculate the fuzzy earliest starting time of activity in a network. In [2] concentrated on the choice of the centroid strategy to find crisp values which speak to uncertain information. In [9] developed a proposed centroid method for trapezoidal fuzzy number. In [7] displayed an arranging model for a helicopter support focus, it is to ensure a decent administration level to restrict flying machine visit length. In [6] presented a wellbeing related personal satisfaction by utilizing four-advance calculation of positioning fuzzy numbers. This paper, we present an approach, which is not researched so far. We used octagonal fuzzy numbers for centroid method to find the critical path. We compared our proposed fuzzy number with the existing triangular and trapezoidal fuzzy numbers. The conclusion has arrived through numerical example.

II. PRELIMINARIES

Here, we present the most basic concepts and defuzzification.

2.1. Fuzzy set

If X is a collection of objects denoted generically by x, then a fuzzy set A in X is a set of ordered pair $A = \{x, \mu_A(x) : x \in X\}$ where $\mu_A(x)$ is called the membership function of *A*.

2.2. Fuzzy number

The *fuzzy number* A is a fuzzy set of the real number R with membership function $\mu_A(x)$ satisfies

1. $\mu_A(x)$ is continuous

$\mu_A(x)$ is increasing in (a,b)

$\mu_A(x)$ is decreasing in (c,d).

2.3. Trapezoidal fuzzy number

$$\mu_A(x) = \begin{cases} \dfrac{x - b1}{b2 - b1}, & b1 \le x \le b2 \\ 1, & b2 \le x \le b3 \\ \dfrac{x - b4}{b3 - b4}, & b3 \le x \le b4 \\ 0, & otherwise \end{cases}$$

Here $A = (b1,b2,b3,b4)$ is a trapezoidal fuzzy number

2.4. Triangular fuzzy number

$$\mu_A(x) = \begin{cases} \dfrac{x - b1}{b2 - b1}, & b1 \le x \le b2 \\ \dfrac{x - b3}{b2 - b3}, & b2 \le x \le b3 \\ 0, & otherwise \end{cases}$$

Here $A = (b1,b2,b3)$ is a triangular fuzzy number

2.5. Defuzzification

Transformation from a fuzzy set to a crisp number is called a *defuzzification*. defuzzification is required when you want a crisp number as output from a fuzzy system. It can take many forms, but the most standard defuzzification is through computing the centroid method.

2.6. Arithmetic operations

Give A and B be two octagonal fuzzy numbers parameterized by $(A1,A2,A3,A4,A5,A6,A7,A8)$ and $(B1,B2,B3,B4,B5,B6,B7,B8)$ respectively. The simplified fuzzy number arithmetic operations between the fuzzy numbers A and B are as follows: Fuzzy numbers addition ⊕:

$(A1,A2,A3,A4,A5,A6,A7,A8)\oplus$

$(B1,B2,B3,B4,B5,B6,B7,B8)$

$=(A1+B1,A2+B2,A3+B3,A4+B4,A5+B5,A6+B6,A7+B7,A8+B8)$

Fuzzy numbers subtraction Θ:

$(A1,A2,A3,A4,A5,A6,A7,A8)\Theta$

$(B1,B2,B3,B4,B5,B6,B7,B8)$

$=(A1-B8,A2-B7,A3-B6,A4-B5,A5-B4,A6-B3,A7-B2,A8-B1)$

III. PROPOSED METHOD FOR DEFUZZIFICATION

Here, we proposed a defuzzification formula. Authors had researched on ranking methods using fuzzy numbers. And octagonal fuzzy numbers are used to find fuzzy critical path based on defuzzification method which is not researched so far. Motivation behind this study is to find a new formula to calculate fuzzy critical path precisely. Here (a,b,c,d,e,f,g,h) are octagonal fuzzy number, real numbers and its membership functions are as follows.

$$\mu_A(x)=\begin{cases} \dfrac{x-a}{b-a} & ,a\le x<b \\ 1 & ,b\le x\le c \\ \dfrac{d-x}{c-d} & ,c<x\le d \\ 0 & ,otherwise \end{cases}, \mu_B(x)=\begin{cases} \dfrac{x-c}{d-c} & ,c\le x<d \\ 1 & ,d\le x\le e \\ \dfrac{f-x}{f-e} & ,e<x\le f \\ 0 & ,otherwise \end{cases}, \mu_C(x)=\begin{cases} \dfrac{x-e}{f-e} & ,e\le x<f \\ 1 & ,f\le x\le g \\ \dfrac{h-x}{h-g} & ,g<x\le h \\ 0 & ,otherwise \end{cases}$$
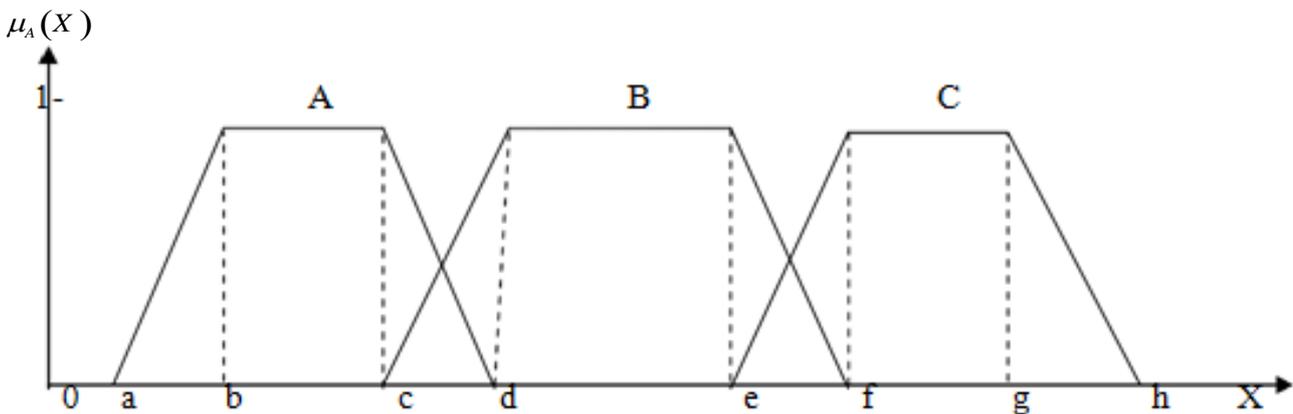

$\mu_A(X)$

Figure 1. Representation of an octagonal fuzzy number

By fig.(1) and from the above membership function we may write the defuzzification formula as

$$Centroid(A)=\frac{\int_a^b \frac{x-a}{b-a}x\,dx+\int_b^c x\,dx+\int_c^d \frac{d-x}{d-c}x\,dx+\int_c^d \frac{x-c}{d-c}x\,dx+\int_d^e x\,dx+\int_e^f \frac{f-x}{f-e}x\,dx+\int_e^f \frac{x-e}{f-e}x\,dx+\int_f^g x\,dx+\int_g^h \frac{h-x}{h-g}x\,dx}{\int_a^b \frac{x-a}{b-a}dx+\int_b^c dx+\int_c^d \frac{d-x}{d-c}dx+\int_c^d \frac{x-c}{d-c}dx+\int_d^e dx+\int_e^f \frac{f-x}{f-e}dx+\int_e^f \frac{x-e}{f-e}dx+\int_f^g dx+\int_g^h \frac{h-x}{h-g}dx}$$

3.1. Fuzzy Critical Path Algorithm

Consider the project network, here octagonal fuzzy numbers are utilized to speak to the movement time.

Step 1: Compute E_i^s's and E_i^f's using

$$E_i^s = \underset{j\in P(i)}{Max}\left\{E_j^s \oplus \tilde{t}_j\right\} \text{ and } E_i^f = E_i^s \oplus \tilde{t}_i$$

Step 2: Compute L_i^f's and L_i^s's using $L_i^f = \underset{j\in P(i)}{Min}\left\{L_j^f \Theta \tilde{t}_j\right\}$

and $L_i^s = L_i^f \Theta \tilde{t}_i$

Step 3: Compute T_i^F's for all the activities (i,j) using

$$T_i^F = L_i^s \Theta E_i^s \text{ or } T_i^F = L_i^f \Theta E_i^f \quad ~$$

Step 4: Compute the total fuzzy slack time for every path.

Step 5: The path which is having minimum value is the critical path.

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IV. PROPOSED CENTROID METHOD USING OCTAGONAL FUZZY NUMBERS

Let A= (a,b,c,d,e,f,g,h) be an octagonal fuzzy number. From fig. (1) and the above membership function the formula for centroid is

$$Centroid(A) = \frac{\int_a^b \frac{x-a}{b-a}x\,dx + \int_b^c x\,dx + \int_c^d \frac{d-x}{d-c}x\,dx + \int_c^d \frac{x-c}{d-c}x\,dx + \int_d^e x\,dx + \int_e^f \frac{f-x}{f-e}x\,dx + \int_e^f \frac{x-e}{f-e}x\,dx + \int_f^g x\,dx + \int_g^h \frac{h-x}{h-g}x\,dx}{\int_a^b \frac{x-a}{b-a}\,dx + \int_b^c dx + \int_c^d \frac{d-x}{d-c}\,dx + \int_c^d \frac{x-c}{d-c}\,dx + \int_d^e dx + \int_e^f \frac{f-x}{f-e}\,dx + \int_e^f \frac{x-e}{f-e}\,dx + \int_f^g dx + \int_g^h \frac{h-x}{h-g}\,dx}$$

By solving the above centroid equation, we get the centroid octagonal formula as given in the following equation (1).

$$Centroid(A) = \frac{\left(c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + cd + ef + gh\right) - \left(a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + ab + cd + eh\right)}{3\left[(c+d+e+f+g+h) - (a+b+c+d+e+h)\right]} \qquad (1)$$

V. EXPERIMENTS AND RESULTS DESCRIPTION

To delineate the proposed formula, consider the accompanying framework graph.Node.1-Wooden arms and legs, Node.2- Wooden back, Node.3-Wooden base, Node.4-foam for back and base, Node.5- Fit covers, Node.6- Set everything together.
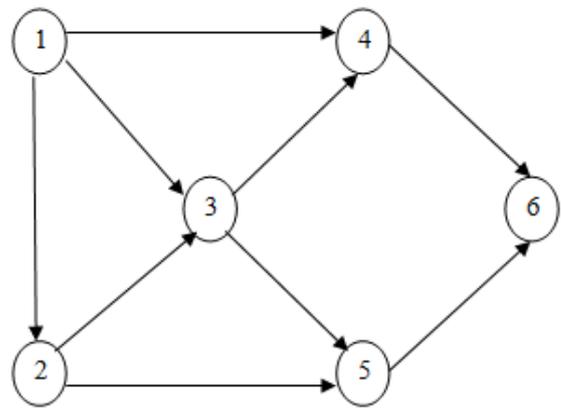

Figure.5.1. Network Project

5.1.Fuzzy Activity Time for Trapezoidal and Triangular Fuzzy Numbers

The venture given in Fig.(2) fuzzy action times are spoken to by trapezoidal and triangular fuzzy numbers.

Table 1: Total float value

Activity	ivityTime in Trapezoidal	Total float	ActivityTime in Triangular	Total float
1-2	(3,5,6,9)	(-24,-4,4,24)	(3,6,9)	(-24,0,24)
1-3	(6,8,9,12)	(-18,0,7,25)	(6,9,12)	(-18,4,25)
1-4	(8,10,11,14)	(-4,11,17,32)	(8,11,14)	(-4,15,32)
2-3	(4,6,7,9)	(-24,-4,4,24)	(4,7,9)	(-24,0,24)
2-5	(4,7,7,10)	(-8,10,16,35)	(4,7,10)	(-8,14,35)
3-4	(9,12,13,16)	(-24,-4,4,24)	(9,13,16)	(-24,0,24)
3-5	(10,13,14,17)	(-24,-4,4,25)	(10,14,17)	(-24,0,25)
4-6	(14,16,17,20)	(-24,-4,4,24)	(14,17,20)	(-24,0,24)
5-6	(12,15,16,19)	(-24,-4,4,25)	(12,16,19)	(-24,0,25)

These are the paths from the Fig.(2) 1-4-6, 1-2-5-6, 1-3-4-6, 1-2-3-5-6, 1-3-5- 6 and 1-2-3-4-6.

$$Centroid(A) = \frac{a+b+c}{3} \qquad (2)$$

5.2.Centroid Method for Triangular Fuzzy Number

Fuzzy activity and slack time for each path is given below. And defuzzified values are calculated using triangular centroid formula.

Table 2: Defuzzified values of each path

Path	FuzzyActivityTime	SlackTime (a, b, c)	DefuzzifiedValue using eqn.(2)
1-4-6	(22, 28, 34)	(-28,15,56)	14.333
1-2-5-6	(19,29, 38)	(-56,14,83)	13.666
1-3-4-6	(29, 39, 48)	(-66,4,73)	3.666
1-2-3-5-6	(29, 43, 54)	(-96,0,97)	0.666
1-3-5-6	(28, 39, 48)	(-66,4,74)	4
1-2-3-4-6	(30, 43,54)	(-96,0,96)	0

We used defuzzified formula and we are getting the minimum value is 0. The path 1-2-3-4-6 is the critical path for the network. This is the critical path for the considered fuzzy project network.

5.3. Centroid Method for Trapezoidal Fuzzy Numbers

Fuzzy activity and slack time for each path given below.

And defuzzified values are calculated using trapezoidal centroid formula.

$$Centroid(A) = \frac{(c^2 + d^2 + cd) - (a^2 + b^2 + ab)}{3[(c+d) - (b+a)]} \quad (3)$$

Table 3: Defuzzified values of each path

Path	FuzzyActivity time	SlackTime (a,b,c,d)	$\dfrac{a+b+c+d}{4}$	Values using eqn.(3)
1-4-6	(22, 26,28, 34)	(-28,7,21,56)	14	14
1-2-5-6	(19, 27,29, 38)	(-56,2,24, 84)	13.5	13.625
1-3-4-6	(29, 36,39, 48)	(-66,-8,15, 73)	3.5	3.5
1-2-3-5-6	(29, 39,43, 54)	(-96,-16,16, ,98)	0.5	0.619
1-3-5-6	(28, 36,39, 48)	(-66,-8,15,74)	3.75	3.809
1-2-3-4-6	(30, 39,43,54)	(-96,-16,16,96)	0	0

We used defuzzified formulae and we are getting the minimum value is 0. This is the critical path 1-2-3-4-6 for the furniture network.

Table 4: Total float of each activity

Activities	FuzzyAactivity time	Total float
1-2	(3,3,4,5,6,7,8,9)	(-24,-18,-11,-4,4,11,18,24)
1-3	(6,7,8,8,9,10,11,12)	(-18,-13,-6,0,7,12,18,25)
1-4	(8,9,10,10,11,12,13,14)	(-4,0,6,11,17,21,26,32)
2-3	(4,4,5,6,7,8,8,9)	(-24,-18,-11,-4,4,11,18,24)
2-5	(4,5,6,7,7,8,9,10)	(-8,-3,4,10,16,23,29,35)
3-5	(10,11,12,13,14,15,16,17)	(-24,-18,-11,-4,4,11,18,25)
3-4	(9,10,11,12,13,14,15,16)	(-24,-18,-11,-4,4,11,18,24)
4-6	(14,15,16,16,17,18,19,20)	(-24,-18,-11,-4,4,11,18,24)
5-6	(12,13,14,15,16,17,18,19)	(-24,-18,-11,-4,4,11,18,25)

The possible paths for the project network and fuzzy slack times are calculated and given below

5.4.Proposed Centroid method using Octagonal FuzzyNumbers

The project network given in Fig.(2) octagonal fuzzy numbers are used to represent the activity times

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Table 5: Defuzzified values of each path

Path	Slack time (a,b,c,d)	$\dfrac{a+b+c+d+e+f+g+h}{8}$	Proposed Defuzzified Value using eqn. (1)
1-4-6	(-28,-18,-5,7,21,32,44,56)	13.625	12.374
1-2-5-6	(-56,-39,-18,2,24,46,66,84)	13.625	12.523
1-3-4-6	(-66,-49,-28,-8,15,34,54,73)	3.125	2.219
1-2-3-5-6	(-96,-72,-44,-16,16,46,74,98)	0.75	0.184
1-3-5-6	(-66,-49,-28,-8,15,35,55,74)	3.5	3.031
1-2-3-4-6	(-96,-72,-44,-16,16,44,72,96)	0	0

We used defuzzified formulae and we are getting the minimum value is 0. This is the critical path 1-2-3-4-6 for the furniture network.

5.5.Comparison of the proposed method with the existingmethods

Here triangular, trapezoidal and octagonal fuzzy numbers are compared. For finding the critical path in the project network octagonal fuzzy number is the better method.

Table 6.Comparison of fuzzy numbers

Path	Triangular FuzzyNumber	Trapezoidal FuzzyNumber	ProposedOctagonal Number
1-4-6	14.333	14	12.374
1-2-5-6	13.666	13.625	12.523
1-3-4-6	3.666	3.5	2.219
1-2-3-5-6	0.666	0.619	0.184
1-3-5-6	4	3.809	3.031
1-2-3-4-6	0	0	0

VI.   CONCLUSION

This research article we have presented a proposed centroid method for fuzzy project network to find the critical path using octagonal fuzzy numbers. We have computed numerical example to compare with the existing methods and the illustrated method. This method will enable to meet the complex fuzzy project time line by channelizing resource on the Critical Path activities.

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