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Detection of Multi Fuzzy Semipreclosed Sets in Topological Space

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***ABSTRACT:** In this paper, we study some of the properties of interval valued intuitionistic multi fuzzy generalized semi pre-closed sets. Also we have provided the relation between interval valued intuitionistic multi fuzzy generalized semipre closed sets with other interval valued intuitionistic multi fuzzy sets.*

***KEYWORDS:** Fuzzy subset, interval valued intuitionistic multi fuzzy subset, interval valued intuitionistic multi fuzzy topological space, interval valued intuitionistic multi fuzzy interior, interval valued intuitionistic multi fuzzy closure.*

I. INTRODUCTION:

In 1965, Zadeh [16] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Some interesting theorems and results on interval valued intuitionistic multi fuzzy generalized semipreclosed sets are provided in this paper.

II. PRELIMINARIES:

1.1 Definition[16]: Let Z ($\neq \emptyset$) be a set. A **fuzzy subset** F of Z is a function F: Z $\rightarrow$ [0, 1].

1.2 Definition[16]: A **multi fuzzy subset** M of a set Z is defined as an object of the form M = { $\langle$ z, $M_1(z)$, $M_2(z)$, $M_3(z)$, …, $M_n(z)$ $\rangle$ / z$\in$Z}, where M_i : Z$\rightarrow$[0, 1] for all i. It is denoted as M = $\langle M_1, M_2, M_3, \ldots, M_n \rangle$.

1.3 Definition[16]: Let Z ($\neq \emptyset$) be a set. A **interval valued fuzzy subset** I of Z is a function I: Z $\rightarrow$D[0, 1], where $D[0,1]$ denotes the family of all closed subintervals of [0, 1].

1.4 Definition[16]: A **interval valued multi fuzzy subset** A of a set Z is defined as an object of the form A = { $\langle$ z, $A_1(z)$, $A_2(z)$, $A_3(z)$, …, $A_n(z)$ $\rangle$ / z$\in$Z}, where A_i : Z$\rightarrow$D[0, 1] for all i, where $D[0,1]$ denotes the family of all closed subintervals of [0, 1]. It is denoted as A = $\langle A_1, A_2, A_3, \ldots, A_n \rangle$.

1.5 Definition[1]: An **intuitionistic fuzzy subset(IFS**) A of a set Z is defined as an object of the form A = { $\langle$ z, $\lambda_A(z)$, $\delta_A(z)$ $\rangle$ / z$\in$Z }, where λ_A : Z $\rightarrow$ [0, 1] and δ_A : Z $\rightarrow$ [0, 1] define the degree of membership and the degree of non-membership of the element z in Z respectively and for every z in Z satisfying $0 \leq \lambda_A(z) + \delta_A(z) \leq 1$.

1.6 Example: Let Z = { a, b, c } be a set. Then A = { $\langle$ a, 0.521, 0.342 $\rangle$, $\langle$ b, 0.145, 0.713 $\rangle$, $\langle$ c, 0.256, 0.341 $\rangle$ } is an intuitionistic fuzzy subset of Z.

1.7 Definition[1]: A **intuitionistic multi fuzzy subset (IMFS)** A of a set Z is defined as an object of the form A = { $\langle$ z, $\lambda_A(z)$, $\delta_A(z)$ $\rangle$ / z$\in$Z }, where $\lambda_A(z)$ = ($\lambda_{A1}(z)$, $\lambda_{A2}(z)$, … $\lambda_{An}(z)$), λ_{Ai} : Z $\rightarrow$[0, 1] for all i and $\delta_A(z)$ = ($\delta_{A1}(z)$, $\delta_{A2}(z)$, …, $\delta_{An}(z)$), δ_{Ai} : Z $\rightarrow$[0, 1] for all i, define the degree of membership and the degree of non-membership of the element z in Z respectively and for every z in Z satisfying $0 \leq \lambda_{Ai}(z) + \delta_{Ai}(z) \leq 1$ for all i.

1.8 Definition: A **interval valued intuitionistic multi fuzzy subset (IVIMFS)** M of a set Z is defined as an object of the form M = { $\langle$ z, $\lambda_M(z)$, $\delta_M(z)$ $\rangle$ / z$\in$Z }, where $\lambda_M(z)$ = ($\lambda_{M1}(z)$, $\lambda_{M2}(z)$, … $\lambda_{Mn}(z)$), λ_{Mi} : Z $\rightarrow$D[0, 1] for all i and $\delta_M(z)$ = ($\delta_{M1}(z)$, $\delta_{M2}(z)$, …, $\delta_{Mn}(z)$), δ_{Mi} : Z $\rightarrow$D[0, 1] for all i, define the degree of membership and the degree of non-membership of the element z in Z respectively and for every z in Z satisfying $0 \leq \sup \lambda_{Mi}(z) + \sup \delta_{Mi}(z) \leq 1$ for all i, where $D[0,1]$ denotes the family of all closed subintervals of [0, 1].

1.9 Definition: Let M and N be any two interval valued intuitionistic multi fuzzy subsets of a set Z. We define the following relations and operations:

(i) M $\subseteq$ N iff $\lambda_M(z) \leq \lambda_N(z)$ and $\delta_M(z) \geq \delta_N(z)$, for all z in Z.

(ii) M = N iff $\lambda_M(z) = \lambda_N(z)$ and $\delta_M(z) = \delta_N(z)$, for all z in Z.

(iii) M^c = { $\langle$ z, $\delta_M(z)$, $\lambda_M(z)$ $\rangle$ / z$\in$Z }.

(iv) M$\cap$N = { $\langle$ z, min{ $\lambda_M(z)$, $\lambda_N(z)$ }, max{ $\delta_M(z)$, $\delta_N(z)$ } $\rangle$ / z$\in$Z }.

(v) M$\cup$N = { $\langle$ z, max {$\lambda_M(z)$, $\lambda_N(z)$ }, min{ $\delta_M(z)$, $\delta_N(z)$ } $\rangle$ / z$\in$Z }.

1.10 Definition: Let Z be a set and $\Im$ be a family of interval valued intuitionistic multi fuzzy subsets of Z. The family $\Im$ is called an interval valued intuitionistic multi fuzzy topology(IVIMFT) on Z iff $\Im$ satisfies the following axioms

(i) 0_Z, 1_Z $\in\Im$,

(ii) If { M_i ; i$\in$I } $\subseteq \Im$, then $\bigcup_{i\in I} M_i \in \Im$,

(iii) If $M_1, M_2, M_3, \ldots M_n \in \Im$, then $\bigcap_{i=1}^{i=n} M_i \in \Im$. The pair (Z, $\Im$) is called an interval valued intuitionistic multi fuzzy topological space (IVIMFTS). The members of $\Im$ are called interval valued intuitionistic multi fuzzy open sets(IVIMFOSs) in Z. An interval valued intuitionistic multi fuzzy set M in Z is said to be interval valued intuitionistic multi fuzzy closed set(IVIMFCS) in Z iff if M^c is an interval valued intuitionistic multi fuzzy open set in Z.

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1.11 Definition: Let (Z, ℑ) be an IVIMFTS and M be an IVIMFS in Z. Then the interval valued intuitionistic multi fuzzy interior and interval valued intuitionistic multi fuzzy closure are defined by

ivimfint(M) = ∪ {G : G is an IVIMFOS in Z and G ⊆ M }

ivimfcl(M) = {K: K is an IVIMFCS in Z and M⊆ K}.

For any IVIMFS A in (Z, ℑ), we have

ivimfcl(M^C) = (*ivimfint*(M))^C and *ivimfint*(M^C) = (*ivimfcl*(M))^C .

1.12 Definition: An IVIMFS M of an IVIMFTS (Z, ℑ) is said to be an

(i) interval valued intuitionistic multi fuzzy regular closed set (IVIMFRCS) if M = *ivimfcl(ivimfint*(M))

(ii) interval valued intuitionistic multi fuzzy semiclosed set (IVIMFSCS) if *ivimfint(ivimfcl*(M)) ⊆ M

(iii) interval valued intuitionistic multi fuzzy preclosed set (IVIMFPCS) if *ivimfcl(ivimfint*(M)) ⊆ M

(iv) interval valued intuitionistic multi fuzzy α closed set (IVIMF**α**CS) if *ivimfcl(ivimfint(ivimfcl* (M))) ⊆ M

(v) interval valued intuitionistic multi fuzzy β closed set (IVIMF**β**CS) if *ivimfint(ivimfcl(ivimfint*(M))) ⊆ M

1.13 Definition: An IVIMFS M of an IVIMFTS (Z, ℑ) is said to be an

(i) interval valued intuitionistic multi fuzzy generalized closed set (IVIMFGCS) if

ivimfcl(M) = U whenever M⊆ U and U is an IVIMFOS

(ii) interval valued intuitionistic multi fuzzy regular generalized closed set (IVIMFRGCS) if *ivimfcl*(M) ⊆ U, whenever M⊆ U and U is an IVIMFROS.

1.14 Definition: An IVIMFS M of an IVIMFTS (Z, ℑ) is said to be an

(i) interval valued intuitionistic multi fuzzy semipreclosed set (IVIMFSPCS) if there exists an IVIMFPCS N such that *ivimfint*(N) ⊆ M ⊆N.

(ii) interval valued intuitionistic multi fuzzy semipreopen set (IVIMFSPOS) if there exists an IVIMFPOS N such that N ⊆ M ⊆ *ivimfcl* (N)

1.15 Definition: Let A be an IVIMFS in an IVIMFTS (Z, ℑ). Then the interval valued intuitionistic multi fuzzy semipre interior of M (*ivimfspint*(M)) and the interval valued intuitionistic multi fuzzy semipre closure of M (*ivimfspcl*(M)) are defined by *ivimfspint*(M) = ∪ {G : G is an IVIMFSPOS in Z and G ⊆ M }

ivimfspcl(M) = ∩ {K: K is an IVIMFSPCS in Z and M ⊆ K}.

For any IVIMFS M in (X, ℑ), we have *ivimfspcl*(M^C) = (*ivimfspcl*(M))^C and *ivimfspint*(M^C) = (*ivimfspint*(M))^C .

1.16 Definition: An IVIMFS M in IVIMFTS (Z, ℑ) is said to be an interval valued intuitionistic multi fuzzy generalized semipreclosed set (IVIMFGSPCS) if *ivimfspcl*(M)) ⊆ U whenever M⊆ U and U is an IVIMFOS in (Z,ℑ).

1.17 Example: Let $X = \{ a, b \}$ and G = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] ⟩ }. Then τ = {0_Z, G, 1_Z} is an IVIMFTS on Z.

Let M = { ⟨ a, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] ⟩, ⟨ b, [0.2, 0.2], [0.25, 0.25], [0.3, 0.3], [0.7, 0.7], [0.65, 0.65], [0.6, 0.6] ⟩ } is an IVIMFGSPCS in (Z,ℑ).

III. EXPERIMENTS AND RESULTS DESCRIPTION

2.1 Theorem: Every IVIMFCS in (Z,ℑ) is an IVIMFGSPCS in (Z,ℑ).

Proof: Let A be an IVIMFCS in (Z,ℑ). Assume that A ⊆ U and U is an IVIMFOS in (Z,ℑ).Then $ivimfspcl(A) \subseteq ivimfcl(A) = A \subseteq U$, by hypothesis. Hence A is an IVIMFGSPCS in (Z,ℑ).

2.2 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{ a, b \}$ and G = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] ⟩ }. Then τ = {0_Z, G, 1_Z} is an IVIMFTS on Z. Let A = { ⟨ a, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] ⟩, ⟨ b, [0.2, 0.2], [0.25, 0.25], [0.3, 0.3], [0.7, 0.7], [0.65, 0.65], [0.6, 0.6] ⟩ }be an IVIMFS in Z. Then A is an IVIMFGSPCS but not an IVIMFCS in Z.

2.3 Theorem: Every IVIMFRCS in (Z,ℑ) is an IVIMFGSPCS in (Z,ℑ).

Proof: Since every IVIMFRCS is an IVIMFCS, the proof is obvious from Theorem 2.1.

2.4 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{ a, b \}$ and G = { ⟨ a, [0.4, 0.8], [0.45, 0.85], [0.5, 0.9], [0, 0], [0, 0], [0, 0] ⟩, ⟨ b, [0.3, 0.6], [0.35, 0.65], [0.4, 0.7], [0, 0], [0, 0], [0, 0] ⟩ }. Then τ = {0_Z, G, 1_Z} is an IVIMFTS on Z. Let A = { ⟨ a, [0.3, 0.6], [0.35, 0.65], [0.4, 0.7], [0, 0], [0, 0], [0, 0] ⟩, ⟨ b, [0.2, 0.4], [0.25, 0.45], [0.3, 0.5], [0, 0], [0, 0], [0, 0] ⟩ } be an IVIMFS in Z. Then A is an IVIMFGSPCS but not an IVIMFRCS in Z.

2.5 Theorem: Every IVIMFGCS in $(X, ℑ)$ is an IVIMFGSPCS in (Z,ℑ). **Proof:** Let A be an IVIMFGCS in (Z,ℑ). Then assume that A ⊆ U and U is an IVIMFOS in (Z,ℑ). Since $ivimfspcl(A) \subseteq ivimfcl(A)$ and $ivimfcl(A) \subseteq U$, by hypothesis, A is an IVIMFGSPCS in Z.

2.6 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{ a, b \}$ and G = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] ⟩ }. Then τ = {0_Z, G, 1_Z} is an IVIMFTS on Z. Let A = { ⟨ a, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [06, 0.6], [0.55, 0.55], [0.5, 0.5] ⟩, ⟨ b, [0.2, 0.2], [0.25, 0.25], [0.3, 0.3], [0.7, 0.7], [0.65, 0.65], [0.6, 0.6] ⟩ } be an

IVIMFS in Z. Then A is an IVIMFGSPCS but not an IVIMFGCS in Z.

2.7 Theorem: Every IVIMFSPCS in (Z,ℑ) is an IVIMFGSPCS in (Z,ℑ).

Proof: Let A be an IVIMFSPCS in Z. Assume that A $\subseteq U$ and U is an IVIMFOS in (Z,ℑ). Then since $ivimfspcl(A) = A$, we have $ivimfspcl(A) \subseteq U$. Hence A is an IVIMFGSPCS in (Z,ℑ).

2.8 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{ a, b \}$ and G = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩ }. Then τ = {0_Z, G, 1_Z} is an IVIMFTS on Z. Let A = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] ⟩ } be an IVIMFS in Z. Then A is an IVIMFGSPCS but not an IVIMFSPCS in Z.

2.9 Theorem: Every IVIMFαCS in (Z,ℑ) is an IVIMFGSPCS in (Z,ℑ).

Proof: Since every IVIMFαCS is an IVIMFSPCS, the proof is obvious from Theorem 2.7.

2.10 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{ a, b \}$ and G = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩ }. Then τ = {0_Z, G, 1_Z} is an IVIMFTS on Z. Let A = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] ⟩ } be an IVIMFS in Z. Then A is an IVIMFGSPCS but not an IVIMFαCS in (Z,ℑ).

2.11 Theorem: Every IVIMFβCS in (Z,ℑ) is an IVIMFGSPCS in (Z,ℑ).

Proof: Let A be an IVIMFβCS in Z. Assume that A ⊆ U, U is an IVIMFOS in (Z,ℑ).
Since $ivimf\beta cl(A) = A$, we have $ivimf\beta cl(A) \subseteq U$. Hence A is an IVIMFGSPCS in (Z,ℑ).

2.12 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{ a, b \}$ and G = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩ }. Then τ = {0_Z, G, 1_Z} is an IVIMFTS on Z. Let A = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] ⟩ } be an IVIMFS in Z. Then A is an IVIMFGSPCS but not an IVIMFβCS in Z.

2.13 Theorem: Every IVIMFSCS in (Z,ℑ) is an IVIMFGSPCS in (Z,ℑ).

Proof: Let A be an IVIMFSCS in (Z,ℑ). Since every IVIMFSCS is an IVIMFSPCS and by theorem 2.7, we have A is an IVIMFGSPCS in (Z,ℑ).

2.14 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{ a, b \}$ and G = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩ }. Then τ = {0_Z, G, 1_Z} is an IVIMFTS on Z. Let A = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] ⟩ } be an IVIMFS in Z. Then A is an IVIMFGSPCS but not an IVIMFSCS in Z.

2.15 Theorem: Every IVIMFPCS in (Z,ℑ) is an IVIMFGSPCS in (Z,ℑ).

Proof: Since every IVIMFPCS is an IVIMFSPCS, the proof is obvious from Theorem 2.7.

2.16 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{ a, b \}$ and G = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩ }. Then τ = {0_Z, G, 1_Z} is an IVIMFTS on Z. Let A = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] ⟩ } be an IVIMFS in Z. Then A is an IVIMFGSPCS but not an IVIMFPCS in Z.

2.17 Remark: The union of any two IVIMFGSPCS in (Z,ℑ) is not an IVIMFGSPCS in (Z,ℑ).

Proof: Consider the example, let $X = \{ a, b \}$ and A_1 = { ⟨ a, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] ⟩, ⟨ b, [0.8, 0.8], [0.85, 0.85], [0.9, 0.9], [0.2, 0.2], [0.15, 0.15], [0.1, 0.1] ⟩ } and

A_2 = { ⟨ a, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩, ⟨ b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] ⟩ }. Then τ = {0_Z, A_1, A_2, 1_Z} is an IVIMFTS on Z. Let A = { ⟨ a, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩, ⟨ b, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] ⟩ } and B = { ⟨ a, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩, ⟨ b, [0.8, 0.8], [0.85, 0.85], [0.9, 0.9], [0.2, 0.2], [0.15, 0.15], [0.1, 0.1] ⟩ } be two IVIMFSs in Z. Then A and B are IVIMFGSPCS but $A \cup B$ is not an IVIMFGSPCS in Z, since $A \cup B$ = { ⟨ a, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3]⟩, ⟨b, [0.8, 0.8], [0.85, 0.85], [0.9, 0.9], [0.2, 0.2], [0.15, 0.15], [0.1, 0.1] ⟩ } ⊆ A_1 but $ivimfspcl(A \cup B)$ = $1_Z \not\subseteq A_1$.

2.18 Remark: The intersection of two IVIMFGSPCS in (Z,ℑ) is not an IVIMFGSPCS in (Z, ℑ).

Proof: Consider the example, let $X = \{ a, b \}$ and G = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩ }. Then τ = {0_Z, G, 1_Z} is an IVIMFTS on Z. Let A = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] ⟩ } and B = { ⟨ a, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩, ⟨ b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩ } be IVIMFSs in Z. Then A and B are IVIMFGSPCS but A∩B is not an IVIMFGSPCS in Z, since A∩B = { ⟨ a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] ⟩, ⟨ b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] ⟩ } ⊆ G but $ivimfspcl(A \cap B) = 1_Z \not\subset G$.

2.19 Theorem: Let (Z,ℑ) be an IVIMFTS. Then for every A∈ IVIMFGSPC(Z) and for every B∈IVIMFS(Z), A $\subseteq B \subseteq ivimfspcl(A)$ implies B∈ IVIMFGSPC(Z).

Proof: Let B $\subseteq U$ and U be an IVIMFOS in (Z,ℑ). Then since A$\subseteq B, A \subseteq U$. By hypothesis, $B \subseteq ivimfspcl(A)$.

Therefore

$$ivimfspcl(B) \subseteq ivimfspcl(ivimfspcl(A)) = ivimfspcl(A) \subseteq U$$

, since A is an IVIMFGSPCS in (Z,ℑ). Hence B∈ IVIMFGSPC(Z).

2.20 Theorem: Let $(X, \Im)$ be an IVIMFTS. Then every IVIMFS in $(X, \Im)$ is an IVIMFGSPCS in $(X, \Im)$ if and only if IVIMFSPO(Z) = IVIMFSPC(Z).

Proof: Necessity: Suppose that every IVIMFS in $(X, \Im)$ is an IVIMFGSPCS in $(X, \Im)$. Let U∈ $IVIMFO(X)$. Then U∈ $IVIMFSPO(X)$ and by hypothesis, $ivimfspcl(U) \subseteq U \subseteq ivimfspcl(U)$. This implies $ivimfspcl(U) = U$. Therefore U∈ $IVIMFSPC(X)$. Hence $IVIMFSPO(X) \subseteq IVIMFSPC(X)$. Let A∈ $IVIMFSPC(X)$. Then A^C∈ $IVIMFSPO(X) \subseteq IVIMFSPC(X)$. That is A^C∈ $IVIMFSPC(X)$. Therefore A∈ $IVIMFSPO(X)$. Hence $IVIMFSPC(X) \subseteq IVIMFSPO(X)$. Thus $IVIMFSPO(X) = IVIMFSPC(X)$.

Sufficiency: Suppose that $IVIMFSPO(X) = IVIMFSPC(X)$. Let A $\subseteq U$ and U be an IVIMFOS in $(X, \Im)$. Then U∈ $IVIMFSPO(X)$ and $ivimfspcl(A) \subseteq ivimfspcl(U) = U$, since U∈ IVIMFSPC(Z), by hypothesis. Therefore A is an IVIMFGSPCS in Z.

2.21 Theorem: If A is an IVIMFOS and an IVIMFGSPCS in $(X, \Im)$, then A is an IVIMFSPCS in $(X, \Im)$.

Proof: Since A$\subseteq A$ and A is an IVIMFOS in $(X, \Im)$, by hypothesis, $ivimfspcl(A) \subseteq A$.

But $A \subseteq ivimfspcl(A)$.

Therefore $ivimfspcl(A) = A$. Hence A is an IVIMFSPCS in $(X, \Im)$.

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