

Detection of Multi Fuzzy Semipreclosed Sets in Topological Space



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ABSTRACT: In this paper, we study some of the properties of interval valued intuitionistic multi fuzzy generalized semi pre-closed sets. Also we have provided the relation between interval valued intuitionistic multi fuzzy generalized semipre closed sets with other interval valued intuitionistic multi fuzzy sets.

KEYWORDS: Fuzzy subset, interval valued intuitionistic multi fuzzy subset, interval valued intuitionistic multi fuzzy topological space, interval valued intuitionistic multi fuzzy interior, interval valued intuitionistic multi fuzzy closure.

I. INTRODUCTION:

In 1965, Zadeh [16] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Some interesting theorems and results on interval valued intuitionistic multi fuzzy generalized semipreclosed sets are provided in this paper.

II. PRELIMINARIES:

1.1 Definition[16]: Let $Z (\neq \emptyset)$ be a set. A **fuzzy subset** F of Z is a function $F: Z \rightarrow [0, 1]$.

1.2 Definition[16]: A **multi fuzzy subset** M of a set Z is defined as an object of the form $M = \{ \langle z, M_1(z), M_2(z), M_3(z), \dots, M_n(z) \rangle / z \in Z \}$, where $M_i: Z \rightarrow [0, 1]$ for all i . It is denoted as $M = \langle M_1, M_2, M_3, \dots, M_n \rangle$.

1.3 Definition[16]: Let $Z (\neq \emptyset)$ be a set. A **interval valued fuzzy subset** I of Z is a function $I: Z \rightarrow D[0, 1]$, where $D[0, 1]$ denotes the family of all closed subintervals of $[0, 1]$.

1.4 Definition[16]: A **interval valued multi fuzzy subset** A of a set Z is defined as an object of the form $A = \{ \langle z, A_1(z), A_2(z), A_3(z), \dots, A_n(z) \rangle / z \in Z \}$, where $A_i: Z \rightarrow D[0, 1]$ for all i , where $D[0, 1]$ denotes the family of all closed subintervals of $[0, 1]$. It is denoted as $A = \langle A_1, A_2, A_3, \dots, A_n \rangle$.

1.5 Definition[1]: An **intuitionistic fuzzy subset (IFS)** A of a set Z is defined as an object of the form $A = \{ \langle z, \lambda_A(z), \delta_A(z) \rangle / z \in Z \}$, where $\lambda_A: Z \rightarrow [0, 1]$ and $\delta_A: Z \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element z in Z respectively and for every z in Z satisfying $0 \leq \lambda_A(z) + \delta_A(z) \leq 1$.

1.6 Example: Let $Z = \{ a, b, c \}$ be a set. Then $A = \{ \langle a, 0.521, 0.342 \rangle, \langle b, 0.145, 0.713 \rangle, \langle c, 0.256, 0.341 \rangle \}$ is an intuitionistic fuzzy subset of Z .

1.7 Definition[1]: A **intuitionistic multi fuzzy subset (IMFS)** A of a set Z is defined as an object of the form $A = \{ \langle z, \lambda_A(z), \delta_A(z) \rangle / z \in Z \}$, where $\lambda_A(z) = (\lambda_{A1}(z), \lambda_{A2}(z), \dots, \lambda_{An}(z))$, $\lambda_{Ai}: Z \rightarrow [0, 1]$ for all i and $\delta_A(z) = (\delta_{A1}(z), \delta_{A2}(z), \dots, \delta_{An}(z))$, $\delta_{Ai}: Z \rightarrow [0, 1]$ for all i , define the degree of membership and the degree of non-membership of the element z in Z respectively and for every z in Z satisfying $0 \leq \lambda_{Ai}(z) + \delta_{Ai}(z) \leq 1$ for all i .

1.8 Definition: A **interval valued intuitionistic multi fuzzy subset (IVIMFS)** M of a set Z is defined as an object of the form $M = \{ \langle z, \lambda_M(z), \delta_M(z) \rangle / z \in Z \}$, where $\lambda_M(z) = (\lambda_{M1}(z), \lambda_{M2}(z), \dots, \lambda_{Mn}(z))$, $\lambda_{Mi}: Z \rightarrow D[0, 1]$ for all i and $\delta_M(z) = (\delta_{M1}(z), \delta_{M2}(z), \dots, \delta_{Mn}(z))$, $\delta_{Mi}: Z \rightarrow D[0, 1]$ for all i , define the degree of membership and the degree of non-membership of the element z in Z respectively and for every z in Z satisfying $0 \leq \sup \lambda_{Mi}(z) + \sup \delta_{Mi}(z) \leq 1$ for all i , where $D[0, 1]$ denotes the family of all closed subintervals of $[0, 1]$.

1.9 Definition: Let M and N be any two interval valued intuitionistic multi fuzzy subsets of a set Z . We define the following relations and operations:

(i) $M \subseteq N$ iff $\lambda_M(z) \leq \lambda_N(z)$ and $\delta_M(z) \geq \delta_N(z)$, for all z in Z .

(ii) $M = N$ iff $\lambda_M(z) = \lambda_N(z)$ and $\delta_M(z) = \delta_N(z)$, for all z in Z .

(iii) $M^c = \{ \langle z, \delta_M(z), \lambda_M(z) \rangle / z \in Z \}$.

(iv) $M \cap N = \{ \langle z, \min\{\lambda_M(z), \lambda_N(z)\}, \max\{\delta_M(z), \delta_N(z)\} \rangle / z \in Z \}$.

(v) $M \cup N = \{ \langle z, \max\{\lambda_M(z), \lambda_N(z)\}, \min\{\delta_M(z), \delta_N(z)\} \rangle / z \in Z \}$.

1.10 Definition: Let Z be a set and \mathfrak{F} be a family of interval valued intuitionistic multi fuzzy subsets of Z . The family \mathfrak{F} is called an interval valued intuitionistic multi fuzzy topology (IVIMFT) on Z iff \mathfrak{F} satisfies the following axioms

(i) $0_Z, 1_Z \in \mathfrak{F}$,

(ii) If $\{ M_i; i \in I \} \subseteq \mathfrak{F}$, then $\bigcup_{i \in I} M_i \in \mathfrak{F}$,

(iii) If $M_1, M_2, M_3, \dots, M_n \in \mathfrak{F}$, then $\bigcap_{i=1}^{i=n} M_i \in \mathfrak{F}$. The pair

(Z, \mathfrak{F}) is called an interval valued intuitionistic multi fuzzy topological space (IVIMFTS). The members of \mathfrak{F} are called interval valued intuitionistic multi fuzzy open sets (IVIMFOSs) in Z . An interval valued intuitionistic multi fuzzy set M in Z is said to be interval valued intuitionistic multi fuzzy closed set (IVIMFCS) in Z iff if M^c is an interval valued intuitionistic multi fuzzy open set in Z .

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1.11 Definition: Let (Z, \mathfrak{S}) be an IVIMFTS and M be an IVIMFS in Z . Then the interval valued intuitionistic multi fuzzy interior and interval valued intuitionistic multi fuzzy closure are defined by

$$ivimfint(M) = \bigcup \{G : G \text{ is an IVIMFOS in } Z \text{ and } G \subseteq M\}$$

$$ivimfcl(M) = \{K : K \text{ is an IVIMFCS in } Z \text{ and } M \subseteq K\}$$

For any IVIMFS A in (Z, \mathfrak{S}) , we have

$$ivimfcl(M^c) = (ivimfint(M))^c \quad \text{and} \quad ivimfint(M^c) = (ivimfcl(M))^c$$

1.12 Definition: An IVIMFS M of an IVIMFTS (Z, \mathfrak{S}) is said to be an

(i) interval valued intuitionistic multi fuzzy regular closed set (IVIMFRCS) if $M = ivimfcl(ivimfint(M))$

(ii) interval valued intuitionistic multi fuzzy semiclosed set (IVIMFSCS) if $ivimfint(ivimfcl(M)) \subseteq M$

(iii) interval valued intuitionistic multi fuzzy preclosed set (IVIMFPCS) if $ivimfcl(ivimfint(M)) \subseteq M$

(iv) interval valued intuitionistic multi fuzzy α closed set (IVIMF α CS) if $ivimfcl(ivimfint(ivimfcl(M))) \subseteq M$

(v) interval valued intuitionistic multi fuzzy β closed set (IVIMF β CS) if $ivimfint(ivimfcl(ivimfint(M))) \subseteq M$

1.13 Definition: An IVIMFS M of an IVIMFTS (Z, \mathfrak{S}) is said to be an

(i) interval valued intuitionistic multi fuzzy generalized closed set (IVIMFGCS) if $ivimfcl(M) = U$ whenever $M \subseteq U$ and U is an IVIMFOS

(ii) interval valued intuitionistic multi fuzzy regular generalized closed set (IVIMFRGCS) if $ivimfcl(M) \subseteq U$, whenever $M \subseteq U$ and U is an IVIMFROS.

1.14 Definition: An IVIMFS M of an IVIMFTS (Z, \mathfrak{S}) is said to be an

(i) interval valued intuitionistic multi fuzzy semipreclosed set (IVIMFSPCS) if there exists an IVIMFPCS N such that $ivimfint(N) \subseteq M \subseteq N$.

(ii) interval valued intuitionistic multi fuzzy semipreopen set (IVIMFSPOS) if there exists an IVIMFPOS N such that $N \subseteq M \subseteq ivimfcl(N)$

1.15 Definition: Let A be an IVIMFS in an IVIMFTS (Z, \mathfrak{S}) . Then the interval valued intuitionistic multi fuzzy semipre interior of M ($ivimfspint(M)$) and the interval valued intuitionistic multi fuzzy semipre closure of M ($ivimfspcl(M)$) are defined by $ivimfspint(M) = \bigcup \{G : G \text{ is an IVIMFSPOS in } Z \text{ and } G \subseteq M\}$

$$ivimfspcl(M) = \bigcap \{K : K \text{ is an IVIMFSPCS in } Z \text{ and } M \subseteq K\}$$

For any IVIMFS M in (Z, \mathfrak{S}) , we have $ivimfspcl(M^c) = (ivimfspint(M))^c$ and $ivimfspint(M^c) = (ivimfspcl(M))^c$.

1.16 Definition: An IVIMFS M in IVIMFTS (Z, \mathfrak{S}) is said to be an interval valued intuitionistic multi fuzzy generalized semipreclosed set (IVIMFGSPCS) if $ivimfspcl(M) \subseteq U$ whenever $M \subseteq U$ and U is an IVIMFOS in (Z, \mathfrak{S}) .

1.17 Example: Let $X = \{a, b\}$ and $G = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] \rangle \}$. Then $\tau = \{0_Z, G, 1_Z\}$ is an IVIMFTS on Z .

Let $M = \{ \langle a, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] \rangle, \langle b, [0.2, 0.2], [0.25, 0.25], [0.3, 0.3], [0.7, 0.7], [0.65, 0.65], [0.6, 0.6] \rangle \}$ is an IVIMFGSPCS in (Z, \mathfrak{S}) .

III. EXPERIMENTS AND RESULTS DESCRIPTION

2.1 Theorem: Every IVIMFCS in (Z, \mathfrak{S}) is an IVIMFGSPCS in (Z, \mathfrak{S}) .

Proof: Let A be an IVIMFCS in (Z, \mathfrak{S}) . Assume that $A \subseteq U$ and U is an IVIMFOS in (Z, \mathfrak{S}) . Then $ivimfspcl(A) \subseteq ivimfcl(A) = A \subseteq U$, by hypothesis. Hence A is an IVIMFGSPCS in (Z, \mathfrak{S}) .

2.2 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{a, b\}$ and $G = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] \rangle \}$. Then $\tau = \{0_Z, G, 1_Z\}$ is an IVIMFTS on Z . Let $A = \{ \langle a, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] \rangle, \langle b, [0.2, 0.2], [0.25, 0.25], [0.3, 0.3], [0.7, 0.7], [0.65, 0.65], [0.6, 0.6] \rangle \}$ be an IVIMFS in Z . Then A is an IVIMFGSPCS but not an IVIMFCS in Z .

2.3 Theorem: Every IVIMFRCS in (Z, \mathfrak{S}) is an IVIMFGSPCS in (Z, \mathfrak{S}) .

Proof: Since every IVIMFRCS is an IVIMFCS, the proof is obvious from Theorem 2.1.

2.4 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{a, b\}$ and $G = \{ \langle a, [0.4, 0.8], [0.45, 0.85], [0.5, 0.9], [0, 0], [0, 0], [0, 0] \rangle, \langle b, [0.3, 0.6], [0.35, 0.65], [0.4, 0.7], [0, 0], [0, 0], [0, 0] \rangle \}$. Then $\tau = \{0_Z, G, 1_Z\}$ is an IVIMFTS on Z . Let $A = \{ \langle a, [0.3, 0.6], [0.35, 0.65], [0.4, 0.7], [0, 0], [0, 0], [0, 0] \rangle, \langle b, [0.2, 0.4], [0.25, 0.45], [0.3, 0.5], [0, 0], [0, 0], [0, 0] \rangle \}$ be an IVIMFS in Z . Then A is an IVIMFGSPCS but not an IVIMFRCS in Z .

2.5 Theorem: Every IVIMFGCS in (X, \mathfrak{S}) is an IVIMFGSPCS in (Z, \mathfrak{S}) .

Proof: Let A be an IVIMFGCS in (Z, \mathfrak{S}) . Then assume that $A \subseteq U$ and U is an IVIMFOS in (Z, \mathfrak{S}) . Since $ivimfspcl(A) \subseteq ivimfcl(A)$ and $ivimfcl(A) \subseteq U$, by hypothesis, A is an IVIMFGSPCS in Z .

2.6 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{a, b\}$ and $G = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] \rangle \}$. Then $\tau = \{0_Z, G, 1_Z\}$ is an IVIMFTS on Z .

Let $A = \{ \langle a, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.6, 0.6], [0.55, 0.55], [0.5, 0.5] \rangle, \langle b, [0.2, 0.2], [0.25, 0.25], [0.3, 0.3], [0.7, 0.7], [0.65, 0.65], [0.6, 0.6] \rangle \}$ be an IVIMFS in Z . Then A is an IVIMFGSPCS but not an IVIMFGCS in Z .

2.7 Theorem: Every IVIMFSPCS in (Z, \mathfrak{S}) is an IVIMFGSPCS in (Z, \mathfrak{S}) .

Proof: Let A be an IVIMFSPCS in Z . Assume that $A \subseteq U$ and U is an IVIMFOS in (Z, \mathfrak{S}) . Then since $ivimfspcl(A) = A$, we have $ivimfspcl(A) \subseteq U$. Hence A is an IVIMFGSPCS in (Z, \mathfrak{S}) .

2.8 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{a, b\}$ and $G = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle \}$. Then $\tau = \{0_Z, G, 1_Z\}$ is an IVIMFOS on Z . Let $A = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] \rangle \}$ be an IVIMFS in Z . Then A is an IVIMFGSPCS but not an IVIMFSPCS in Z .

2.9 Theorem: Every IVIMF α CS in (Z, \mathfrak{S}) is an IVIMFGSPCS in (Z, \mathfrak{S}) .

Proof: Since every IVIMF α CS is an IVIMFSPCS, the proof is obvious from Theorem 2.7.

2.10 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{a, b\}$ and $G = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle \}$. Then $\tau = \{0_Z, G, 1_Z\}$ is an IVIMFOS on Z . Let $A = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] \rangle \}$ be an IVIMFS in Z . Then A is an IVIMFGSPCS but not an IVIMF α CS in (Z, \mathfrak{S}) .

2.11 Theorem: Every IVIMF β CS in (Z, \mathfrak{S}) is an IVIMFGSPCS in (Z, \mathfrak{S}) .

Proof: Let A be an IVIMF β CS in Z . Assume that $A \subseteq U$, U is an IVIMFOS in (Z, \mathfrak{S}) .

Since $ivimf\beta cl(A) = A$, we have $ivimf\beta cl(A) \subseteq U$. Hence A is an IVIMFGSPCS in (Z, \mathfrak{S}) .

2.12 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{a, b\}$ and $G = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle \}$. Then $\tau = \{0_Z, G, 1_Z\}$ is an IVIMFOS on Z . Let $A = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] \rangle \}$ be an IVIMFS in Z . Then A is an IVIMFGSPCS but not an IVIMF β CS in Z .

2.13 Theorem: Every IVIMFSCS in (Z, \mathfrak{S}) is an IVIMFGSPCS in (Z, \mathfrak{S}) .

Proof: Let A be an IVIMFSCS in (Z, \mathfrak{S}) . Since every IVIMFSCS is an IVIMFSPCS and by theorem 2.7, we have A is an IVIMFGSPCS in (Z, \mathfrak{S}) .

2.14 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{a, b\}$ and $G = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle \}$. Then $\tau = \{0_Z, G, 1_Z\}$ is an IVIMFOS on Z . Let $A = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] \rangle \}$ be an IVIMFS in Z . Then A is an IVIMFGSPCS but not an IVIMFSCS in Z .

2.15 Theorem: Every IVIMFPCS in (Z, \mathfrak{S}) is an IVIMFGSPCS in (Z, \mathfrak{S}) .

Proof: Since every IVIMFPCS is an IVIMFSPCS, the proof is obvious from Theorem 2.7.

2.16 Remark: The converse of above theorem is not necessary true

Proof: Consider the example, let $X = \{a, b\}$ and $G = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle \}$. Then $\tau = \{0_Z, G, 1_Z\}$ is an IVIMFOS on Z . Let $A = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] \rangle \}$ be an IVIMFS in Z . Then A is an IVIMFGSPCS but not an IVIMFPCS in Z .

2.17 Remark: The union of any two IVIMFGSPCS in (Z, \mathfrak{S}) is not an IVIMFGSPCS in (Z, \mathfrak{S}) .

Proof: Consider the example, let $X = \{a, b\}$ and $A_1 = \{ \langle a, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] \rangle, \langle b, [0.8, 0.8], [0.85, 0.85], [0.9, 0.9], [0.2, 0.2], [0.15, 0.15], [0.1, 0.1] \rangle \}$ and

$A_2 = \{ \langle a, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle, \langle b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] \rangle \}$. Then $\tau = \{0_Z, A_1, A_2, 1_Z\}$ is an IVIMFOS on Z . Let $A = \{ \langle a, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle, \langle b, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] \rangle \}$ and

$B = \{ \langle a, [0.4, 0.4], [0.45, 0.45], [0.5, 0.5], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle, \langle b, [0.8, 0.8], [0.85, 0.85], [0.9, 0.9], [0.2, 0.2], [0.15, 0.15], [0.1, 0.1] \rangle \}$ be two IVIMFSs in Z . Then A and B are IVIMFGSPCS but

$A \cup B$ is not an IVIMFGSPCS in Z , since $A \cup B = \{ \langle a, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle, \langle b, [0.8, 0.8], [0.85, 0.85], [0.9, 0.9], [0.2, 0.2], [0.15, 0.15], [0.1, 0.1] \rangle \} \subseteq A_1$ but $ivimfspcl(A \cup B) = 1_Z \notin A_1$.

2.18 Remark: The intersection of two IVIMFGSPCS in (Z, \mathfrak{S}) is not an IVIMFGSPCS in (Z, \mathfrak{S}) .

Proof: Consider the example, let $X = \{a, b\}$ and $G = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle \}$.



$[0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle \}$. Then $\tau = \{0_Z, G, 1_Z\}$ is an IVIMFSTS on Z . Let $A = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.7, 0.7], [0.75, 0.75], [0.8, 0.8], [0.3, 0.3], [0.25, 0.25], [0.2, 0.2] \rangle \}$ and $B = \{ \langle a, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle, \langle b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle \}$ be IVIMFSTS in Z . Then A and B are IVIMFGSPCS but $A \cap B$ is not an IVIMFGSPCS in Z , since $A \cap B = \{ \langle a, [0.5, 0.5], [0.55, 0.55], [0.6, 0.6], [0.5, 0.5], [0.45, 0.45], [0.4, 0.4] \rangle, \langle b, [0.6, 0.6], [0.65, 0.65], [0.7, 0.7], [0.4, 0.4], [0.35, 0.35], [0.3, 0.3] \rangle \} \subseteq G$ but $ivimfspcl(A \cap B) = 1_Z \notin G$.

2.19 Theorem: Let (Z, \mathfrak{T}) be an IVIMFSTS. Then for every $A \in IVIMFGSPC(Z)$ and for every $B \in IVIMFSTS(Z)$, $A \subseteq B \subseteq ivimfspcl(A)$ implies $B \in IVIMFGSPC(Z)$.

Proof: Let $B \subseteq U$ and U be an IVIMFOS in (Z, \mathfrak{T}) . Then since $A \subseteq B, A \subseteq U$. By hypothesis, $B \subseteq ivimfspcl(A)$. Therefore

$$ivimfspcl(B) \subseteq ivimfspcl(ivimfspcl(A)) = ivimfspcl(A) \subseteq U$$

, since A is an IVIMFGSPCS in (Z, \mathfrak{T}) . Hence $B \in IVIMFGSPC(Z)$.

2.20 Theorem: Let (X, \mathfrak{T}) be an IVIMFSTS. Then every IVIMFOS in (X, \mathfrak{T}) is an IVIMFGSPCS in (X, \mathfrak{T}) if and only if $IVIMFSPC(Z) = IVIMFSPC(Z)$.

Proof: Necessity: Suppose that every IVIMFOS in (X, \mathfrak{T}) is an IVIMFGSPCS in (X, \mathfrak{T}) . Let $U \in IVIMFOS(X)$. Then $U \in IVIMFSPC(X)$ and by hypothesis, $ivimfspcl(U) \subseteq U \subseteq ivimfspcl(U)$. This implies $ivimfspcl(U) = U$. Therefore $U \in IVIMFSPC(X)$. Hence $IVIMFSPC(X) \subseteq IVIMFSPC(X)$. Let $A \in IVIMFSPC(X)$. Then $A^c \in IVIMFSPC(X) \subseteq IVIMFSPC(X)$. That is $A^c \in IVIMFSPC(X)$. Therefore $A \in IVIMFSPC(X)$. Hence $IVIMFSPC(X) \subseteq IVIMFSPC(X)$. Thus $IVIMFSPC(X) = IVIMFSPC(X)$.

Sufficiency: Suppose that $IVIMFSPC(X) = IVIMFSPC(X)$. Let $A \subseteq U$ and U be an IVIMFOS in (X, \mathfrak{T}) . Then $U \in IVIMFSPC(X)$ and $ivimfspcl(A) \subseteq ivimfspcl(U) = U$, since $U \in IVIMFSPC(Z)$, by hypothesis. Therefore A is an IVIMFGSPCS in Z .

2.21 Theorem: If A is an IVIMFOS and an IVIMFGSPCS in (X, \mathfrak{T}) , then A is an IVIMFSPCS in (X, \mathfrak{T}) .

Proof: Since $A \subseteq A$ and A is an IVIMFOS in (X, \mathfrak{T}) , by hypothesis, $ivimfspcl(A) \subseteq A$. But $A \subseteq ivimfspcl(A)$. Therefore $ivimfspcl(A) = A$. Hence A is an IVIMFSPCS in (X, \mathfrak{T}) .

REFERENCES

1. Funes, José Félix, et al. "Defect detection from multi-frequency limited data via topological sensitivity." Journal of Mathematical Imaging and Vision 55.1 (2016): 19-35.
2. Glegrave, Romain Lucas, Dieter Pelz, and Rudy Palm. "The efficiency of the linear classification rule in multi-group discriminant analysis." African Journal of Mathematics and Computer Science Research 3.1 (2010): 019-025.
3. Benz, Ursula C., et al. "Multi-resolution, object-oriented fuzzy analysis of remote sensing data for GIS-ready information." ISPRS Journal of photogrammetry and remote sensing 58.3-4 (2004): 239-258.
4. Westerman, Wayne. Hand tracking, finger identification, and chordic manipulation on a multi-touch surface. Diss. University of Delaware, 1999.
5. D'Orazio, Tiziana, and Cataldo Guaragnella. "A survey of automatic event detection in multi-camera third generation surveillance systems." International Journal of Pattern Recognition and Artificial Intelligence 29.01 (2015): 1555001.
6. Toosi, Adel Nadjaran, and Mohsen Kahani. "A new approach to intrusion detection based on an evolutionary soft computing model using neuro-fuzzy classifiers." Computer communications 30.10 (2007): 2201-2212.
7. Li, Zhixiong, et al. "Blind vibration component separation and nonlinear feature extraction applied to the nonstationary vibration signals for the gearbox multi-fault diagnosis." Measurement 46.1 (2013): 259-271.
8. Longbotham, Nathan, et al. "Multi-modal change detection, application to the detection of flooded areas: Outcome of the 2009–2010 data fusion contest." IEEE Journal of selected topics in applied earth observations and remote sensing 5.1 (2012): 331-342.
9. Lemos, Andre, Waldir Caminhas, and Fernando Gomide. "Adaptive fault detection and diagnosis using an evolving fuzzy classifier." Information Sciences 220 (2013): 64-85.
10. Markou, Markos, and Sameer Singh. "Novelty detection: a review—part 2: neural network based approaches." Signal processing 83.12 (2003): 2499-2521.

