

Distribution of Arrivals

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Abstract: Many different distributions have been discussed by a number of authors. It has been observed that there is still more scope for density functions on arrivals which take significant role in lifetime statistical analysis. This paper proposes one probability density function for selected random variable.

Key Words: Random variable, continuous probability distribution, arrival rate, density function, arrivals, integrability.

I. INTRODUCTION

We have seen the random variables “Time for the first arrival”, “time for the rth arrival”, etc. Now the random variable of interest is instead of asking “In fixed time interval, how many arrivals take place?”, “We ask in a particular interval how likely there are successive arrivals”. This random variable is continuous if it holds up a p.d.f. f(x) which is continuous over the time axis with f(x) greater than

or equal to zero and $\int_{-\infty}^{\infty} f(x)dx = 1$

For $n = 0$ } Cases are absurd since we consider the two
 $n = 1$ }

consecutive arrivals in the time interval.

We consider $n = 2$.

Case - 1: $n = 2$

Here we propose

$$f(x) = \lambda [2P(2,t) - P(1,t)] \text{ ----- (1)}$$

$$f(x) = \lambda \left[2 \frac{e^{-\lambda t} (\lambda t)^2}{2!} - e^{-\lambda t} \lambda t \right]$$

Here f(x) is integrable and in general probability of arriving one customer in particular interval is greater than probability of arriving two customers in same time interval.

In equation (1) twice the first term is more than second term. Therefore equation (1) will give absolutely a positive number.

The arrival rate λ is positive quantity.

$$\therefore f(x) \geq 0 \forall x$$

$$\text{And } \int_{-\infty}^{\infty} f(x)dx = \int_0^{\infty} f(x)dx$$

$$\text{Now } \int_0^{\infty} \lambda [2P(2,t) - P(1,t)] dt$$

$$\int_0^{\infty} \lambda \left[2 \frac{(\lambda t)^2 e^{-\lambda t}}{2!} - \frac{e^{-\lambda t} (\lambda t)^1}{1!} \right] dt$$

$$\int_0^{\infty} \left[2 \frac{(\lambda t)^2 e^{-\lambda t}}{2!} \lambda - \frac{e^{-\lambda t} (\lambda t)^1}{1!} \lambda \right] dt$$

$$= 1$$

$$\therefore \int_{-\infty}^{\infty} f(x)dx = \int_0^{\infty} f(x)dx = 1 \text{ for } n = 2$$

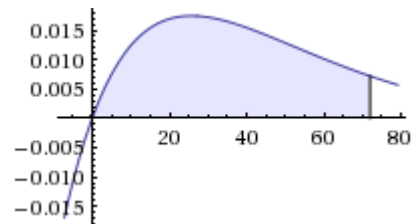


Figure - 1

Case - 2: When n value is equals to three:

$$f(x) = \lambda [2P(3,t) - P(2,t)] \text{ ----- (2)}$$

Here f(x) is integrable and in general probability of arriving two customers in particular interval is greater than probability of arriving three customers in that time interval. Also twice the P (3, t) is more than second term in equation (2)

Therefore equation (2) is a positive quantity.

The arrival rate λ is a positive value.

$$\therefore f(x) \geq 0 \forall x$$

$$\therefore \lambda [2P(3,x) - P(2,x)] \geq 0$$

$$\int_0^{\infty} \lambda [2P(3,t) - P(2,t)] dt$$

$$\int_0^{\infty} \lambda \left[2 \frac{(\lambda t)^3 e^{-\lambda t}}{3!} - \frac{e^{-\lambda t} (\lambda t)^2}{2!} \right] dt$$

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$$\int_0^{\infty} \left[2 \frac{(\lambda t)^3 e^{-\lambda t}}{3!} \lambda - \frac{e^{-\lambda t} (\lambda t)^2}{2!} \lambda \right] dt$$

= 1

$f(x) = \lambda [2P(3, x) - P(2, x)]$ is p.d.f. function when $n=3$.

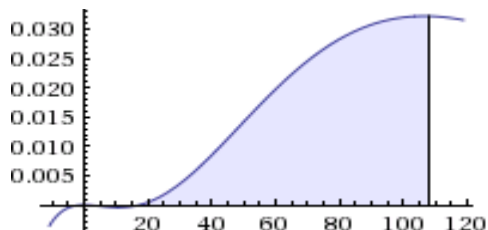


Figure - 2

Case - 3: $n = 4$

$$f(x) = \lambda [2P(4, x) - P(3, x)] \text{-----(3)}$$

$$= \lambda \left[2 \frac{e^{-\lambda x} (\lambda x)^4}{4!} - \frac{e^{-\lambda x} (\lambda x)^3}{3!} \right]$$

Here $f(x)$ is integrable and since the probability of arriving four persons to system $P(4,t)$ is less than probability of arriving 3 persons $P(3, t)$ to that system in the same time 't'. Also twice $P(4,t)$ is more than second term in equation (3).

Therefore equation (3) is absolutely positive value.

The arrival rate λ is positive quantity.

$$\therefore \lambda [2P(4, t) - P(3, t)] \geq 0$$

$$\int_0^{\infty} \lambda [2P(4, t) - P(3, t)] dt$$

$$\int_0^{\infty} \left[2 \frac{(\lambda t)^4 e^{-\lambda t}}{4!} \lambda - \frac{e^{-\lambda t} (\lambda t)^3}{3!} \lambda \right] dt$$

2(1) - 1 = 1

$f(x) = \lambda [2P(4, x) - P(3, x)]$ is p.d.f. function when $n = 4$.

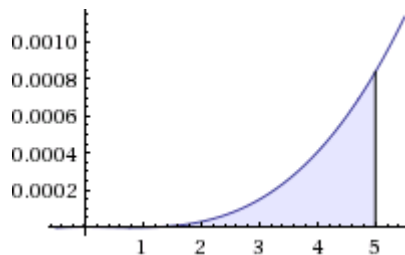


Figure – 3

When it comes to the finite interval, we need to multiply the normalizer to get the area under the curve equals to 1.

In general, we define

$$f(x) = \begin{cases} \lambda [2\mathfrak{T}(P(n, x) - P(n-1, x))] & \text{when } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

With \mathfrak{T} is the normalizing constant.

II. CONCLUSION

In this paper we specified for the random variable for the successive arrivals, the probability density function,

$$f(x) = \begin{cases} \lambda [2\mathfrak{T}(P(n, x) - P(n-1, x))] & \text{when } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

FUTURE WORK

We can find CDF and all statistical properties of the above distribution.

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