Contaminants of Dispersion on Groundwater by Landfill Leachate

S.Senthamilselvi, Nirmala P. Ratchagar

Abstract: ***The major potential impacts of landfill leachate on the environment are groundwater and surface water contamination.To date, the percolation of bacteria and viruses by landfill leachate into the groundwater table poses a potential ris k to public health and the environment and potential risks.This p aper deals with the study of leachate dispersion of contaminants using generalized dispersion techniques for solvent transport in p orous media.The porous layer interface, the slip boundary condit ions of the beavers joseph bj are used, the governing equations a re analytically solved and the expressions for speed and dispersio n are obtained and graphically presented***

Keywords : Solid waste, permeability, generalized-dispersion technique.

I. INTRODUCTION

Groundwater contamination is a problem which affects every individual (Mritunjay et al.,2008). Groundwater flow and transport analysis have been an important research topic in the last three decades. The transport of dissolved contaminants or suspended contaminants (bacteria and virus) by flowing water is of great significance to study the relation between environmental protection and resource utilization (Zhang Qiau-fei et al., 2008).

Municipal solid waste MSW has become one of the main fac tors that adversely affect the environment as the MSW issues increase the incidence of groundwater contamination. Hence the study of groundwater contamination resulting from MSW landfill leachate has become a focused issue nowadays (Xiaoli Liu et al., (2006).

Most of the contaminants occur in nature as either point sources or distributed sources. Example of point source contamination are municipal waste sites (landfill), industrial discharges, leaks and spills etc. Distributed sources occur as a result of effluent from leaking sewers and septic tanks, oil and chemical pipelines.

Landfilling has long been the major disposal method for both domestic and industrial wastes.

Bacteria and virus from sewage sludges , waste water, septic tanks and other sources can be transported from groundwater to drinking water wells. During this transport, bacteria and virus can be either irreversibly or reversibly stored on surface material (Martin Reinhard,1984). Valsamy and Nirmala P.Ratchagar(2012) developed a mathematical model to study the unsteady transport of bacteria and virus in groundwater.

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The main objective of this paper is to study the dispersion of contaminants following the generalized dispesion model of Gill and Sankarasubramanian (1970). The number density of the contaminated particle is constant. The fluid is assumed to be viscous, incompressible and contaminated (fine and coarse). The generalized dispersion theory developed can be extended to consider the dispersion phenomena for a wide variety of flows which are too complex to solve analytically (Rudraiah et al.(1986), Nagarani et al.(2006), Mallika and Rudraiah (2011) and Meena Priya and Nirmala P.Ratchagar (2011)).

II. MATHEMATICAL FORMULATION

The continuity and momentum equation of the motion of unsteady , viscous, incompressible fluid with uniform distribution of contaminated particles are given by :

For fluid phase ,

$$\nabla.u = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} + (u.\nabla)u = -\frac{\nabla.p}{\rho} + \nu\nabla^2 u + \frac{KN}{\rho}(v-u) - \frac{\nu}{k}u \qquad (2)$$

For contaminated phase

$$\nabla.v = 0 \qquad (3)$$

$$\frac{\partial u}{\partial t} + (v.\nabla)v = \frac{K}{m}(u-v) \qquad (4)$$

where,

u = velocity of the fluid phase (LT^{-1}),

v = velocity of contaminated phase (LT^{-1}), ,

ρ = density of the fluid (ML^{-3}),

p = pressure of the fluid $(ML^{-1}T^{-2})$,

N = number density of contaminated particle (M^{-3}),

ν = kinematic viscosity (L^2T^{-1}),

K= $6\pi a\mu$ = Stoke's resistance (drag coefficient), dimensionless,

a = spherical radius of the contaminated particle (L^2),

m= mass of the contaminated particle (M),

μ = coefficient of viscosity of fluid particle $(ML^{-1}T^{-1})$,

k = permeability of porous medium (L^2),

t = time (T).

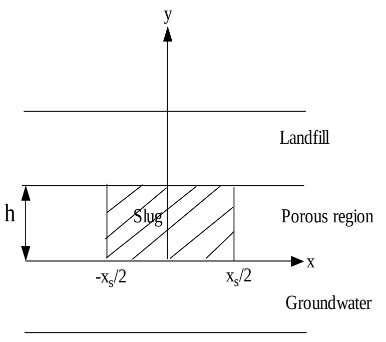

Figure1: Physical configuration

In this paper, we have assumed that the flow is unidirectional and parallel to a plates due to constant pressure gradient in that direction. Hence the momentum equation for fluid phase and contaminated phase in equation (2) and (3) takes the form

$$\frac{\partial u}{\partial t}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{KN}{\rho}(v-u)-\frac{\nu}{k}u \qquad (5)$$

$$\frac{\partial v}{\partial t}=\frac{K}{m}(u-v) \qquad (6)$$

Equation (5) and (6) are solved subject to the initial and boundary conditions;

u=0, v=0 at t=0

$u=u_a+\varepsilon e^{nt}u_b$ at y=0 and

$$\frac{du}{dy}=\frac{\alpha}{\sqrt{k}}(u_\beta-u_\beta{}^m,\ v=0,\ \phi=1+\varepsilon e^{nt+\lambda x}\ at\ y=h$$

We make these equations dimensionless using

$$u^*=\frac{uL}{\nu} \qquad v^*=\frac{vL}{\nu} \qquad t^*=\frac{t\nu^2}{L} \quad ;$$

$$x^*=\frac{x}{L};\ y^*=\frac{y}{L};\ p^*=\frac{p}{\rho\nu^2} \qquad (7)$$

Substituting equation (7) into equation (5) and (6) and for simplicity neglecting the asterisks we get,

$$\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}+S(v-u)-Xu \qquad (8)$$

$$\frac{\partial v}{\partial t}=\frac{1}{G}(u-v) \qquad (9)$$

where

$$S=\frac{KNL^2}{\nu\rho}$$

$$X=\frac{\rho L^2}{k},$$

$G=\dfrac{m\nu}{KL^2}$, particle mass parameter

Since the applied pressure gradient is constant for t> 0, then

$$-\frac{\partial p}{\partial x}=c_0 \qquad (10)$$

Hence the equation (8) and (9) becomes

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+S(v-u)-Xu+c_0 \qquad (11)$$

$$\frac{\partial v}{\partial t}=\frac{1}{G}(u-v) \qquad (12)$$

The initial and boundary condition becomes,

$u=0, v=0\ \ at\ \ t=0$

$u=u_a+\varepsilon e^{nt}u_b$ at y=0 and

$\dfrac{du}{dy}=\dfrac{\alpha}{\sqrt{k}}(u_\beta-u_\beta{}^m),\ at\ y=1$ for $t>0$

where, a, b and n are constants and ε is the perturbation parameter(less than unity).

III. METHOD OF SOLUTION

Velocity Equations (11) and (12) are partial differential equations which can be solved analytically by employing perturbation technique. This can be done by representing the velocity as

$$u(y,t)=u_0(y)+\varepsilon e^{n_1 t}u_1(y)+O(\varepsilon^2) \qquad (13)$$

$$v(y,t)=v_0(y)+\varepsilon e^{n_1 t}v_1(y)+O(\varepsilon^2) \qquad (14)$$

Substituting equation (13) into equations (11) and (12), neglecting the higher order of (ε^2) and solving subject to the boundary conditions,

$$\left.\begin{aligned}u_0=0,u_1=0 at y=0\\ u_0=a_1,u_1=b_1 at y=1\end{aligned}\right\} \quad (15)$$

gives the velocity of fluid as

$$u=\left[\frac{1}{\sqrt{X}(e^{\sqrt{X}}+e^{-\sqrt{X}})}\left(\frac{c_0}{X}\sqrt{X}e^{\sqrt{X}}-\frac{\alpha}{\sqrt{k}}(u_\beta-u_\beta{}^m)\right)-\frac{c_0}{X}\right]e^{\sqrt{X}y}$$
$$-\left[\frac{1}{\sqrt{X}(e^{\sqrt{X}}+e^{-\sqrt{X}})}\left(\frac{c_0}{X}\sqrt{X}e^{\sqrt{X}}-\frac{\alpha}{\sqrt{k}}(u_\beta-u_\beta{}^m)\right)-\frac{c_0}{X}\right]e^{-\sqrt{X}y}$$
$$+\frac{c_0}{X}+\varepsilon e^{nt}A_2e^{\sqrt{\frac{S}{1+Gn}-S-X-ny}} \qquad (16)$$

and velocity of contaminants as

$$v=A_1e^{\sqrt{X}y}+B_1e^{-\sqrt{X}y}$$
$$+\frac{c_0}{X}+\frac{\varepsilon e^{nt}}{1+Gn}\left(A_2e^{\sqrt{\frac{S}{1+Gn}-S-X-ny}}\right.$$
$$\left.+B_2e^{-\sqrt{\frac{S}{1+Gn}-S-X-ny}}\right) \qquad (17)$$

where,

$$A_1=\left[\frac{1}{\sqrt{X}(e^{\sqrt{X}}+e^{-\sqrt{X}})}\left(\frac{c_0}{X}\sqrt{X}e^{\sqrt{X}}-\frac{\alpha}{\sqrt{k}}(u_\beta-u_\beta{}^m)\right)-\frac{c_0}{X}\right]$$

$$B_1=\left[\frac{1}{\sqrt{X}(e^{\sqrt{X}}+e^{-\sqrt{X}})}\left(\frac{c_0}{X}\sqrt{X}e^{\sqrt{X}}-\frac{\alpha}{\sqrt{k}}(u_\beta-u_\beta{}^m)\right)\right]$$

$$A_2=a-\frac{e^{\sqrt{\frac{S}{1+Gn}-S-X-na}}}{e^{\sqrt{\frac{S}{1+Gn}-S-X-na}}+e^{-\sqrt{\frac{S}{1+Gn}-S-X-na}}}$$

$$B_2=\frac{e^{\sqrt{\frac{S}{1+Gn}-S-X-na}}}{e^{\sqrt{\frac{S}{1+Gn}-S-X-na}}+e^{-\sqrt{\frac{S}{1+Gn}-S-X-na}}}$$

Average velocity is given by,

$$\bar{u}=\frac{e^{\sqrt{X}-1}}{X}\left(\frac{c_0}{X}\sqrt{X}e^{\sqrt{X}}-\frac{\alpha}{\sqrt{k}}(u_\beta-u_\beta{}^m)\right)+\frac{e^{-\sqrt{X}-1}}{X}\left(\frac{c_0}{X}\sqrt{X}e^{\sqrt{X}}-\frac{\alpha}{\sqrt{k}}(u_\beta-u_\beta{}^m)\right)+\varepsilon\left(\left(a-\frac{e^{\sqrt{m_1}}}{e^{\sqrt{m_1}}+e^{-\sqrt{m_1}}}\right)+\left(\frac{e^{\sqrt{m_1}}-1}{\sqrt{m_1}}\right)\right)-\varepsilon\left(\left(\frac{e^{\sqrt{m_1}}a}{e^{\sqrt{m_1}}+e^{-\sqrt{m_1}}}\right)\left(\frac{e^{-\sqrt{m_1}-1}}{\sqrt{m_1}}\right)\right)+\frac{c_0}{X}$$

Dispersion coefficient

The concentration of contaminants (fine and coarse) in the groundwater which diffuse in a fully developed flow, is given by

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \qquad (18)$$

with the initial and boundary conditions,

(i) $C(0,x,y)=\begin{cases}C_0, |x|\le\frac{x_s}{2}\\ 0,\ |x|>\frac{x_s}{2}\end{cases}$

(ii) $\frac{\partial C}{\partial y}(t,x,0)=\frac{\partial C}{\partial y}(t,x,h)=0$

(iii) $C(t,\infty,y)=\frac{\partial C}{\partial x}(t,\infty,y)=0 \qquad (19)$

where, C_0 is the concentration of the initial slug input of length x_s.

Introducing non-dimensionless variables,

$$\theta=\frac{C}{C_0};\quad X=\frac{Dx}{h^2\bar{u}};\quad X_s=\frac{Dx_s}{h^2\bar{u}};\quad Y=\frac{y}{h};$$

$$\tau=\frac{Dt}{h^2};\quad u^*=\frac{u}{\bar{u}};\quad Pe=\frac{h\bar{u}}{D};$$

The equation (17)becomes,

$$\frac{\partial\theta}{\partial\tau}+U\frac{\partial\theta}{\partial X_1}=\frac{1}{Pe^2}\frac{\partial^2\theta}{\partial X_1{}^2}+\frac{\partial^2\theta}{\partial Y^2} \qquad (20)$$

where, $\frac{1}{Pe^2}=\frac{D^2}{h^2\bar{u}^2}$ and $U=\frac{u-\bar{u}}{\bar{u}}$

Axial coordinate moving with the average velocity of flow is defined as $x_1=x-\bar{u}t$ which in dimensionless form is by

$X_1=X-\tau$, where, $X_1=\frac{Dx_1}{h^2\bar{u}}$

The non-dimensional initial and boundary conditions of (18) takes the form

(i) $\theta(0,x,y)=\begin{cases}1,|X_1|\le\frac{X_s}{2}\\ 0,\ |X_1|>\frac{X_s}{2}\end{cases}$

(ii) $\frac{\partial\theta}{\partial y}(\tau,X_1,0)=\frac{\partial\theta}{\partial Y}(\tau,X_1,1)=0$

(iii) $\theta(\tau,\infty,y)=\frac{\partial\theta}{\partial X_1}(\tau,\infty,Y)=0 \qquad (21)$

Following Gill and Sankarasubramanian (1970), the solution to equation (19) can be written as a series expansion in the form

$$\theta(\tau,X_1,Y)=\theta_m(\tau,X_1)+\sum_{k=1}^{\infty}f_k(\tau,Y)\frac{\partial^k\theta_m}{\partial X_1{}^k} \qquad (22)$$

where, θ_m is the dimensionless cross sectional average concentration, given by

$$\theta_m(\tau,X_1)=\int_0^1\theta(\tau,X_1,Y)dY \qquad (23)$$

Integrating equation (19) with respect to Y in [0,1] and substituting for θ_m we get ,

$$\frac{\partial\theta_m}{\partial\tau}=\frac{1}{Pe^2}\frac{\partial^2\theta_m}{\partial X_1^2}-\frac{\partial}{\partial X_1}\int_0^1 U\theta\, dY$$

(24)

The generalized dispersion model with time dependent dispersion coefficient can be written as

$$\frac{\partial\theta_m}{\partial\tau}=K_1\frac{\partial\theta_m}{\partial X_1}+K_2\frac{\partial^2\theta_m}{\partial X_1^2}+K_3\frac{\partial^3\theta_m}{\partial X_1^3}+\dots \qquad (25)$$

Introducing equations (24) in (23) and making use of the boundary condition (ii) of (18) gives

$$K_1 \frac{\partial \theta_m}{\partial X_1} + K_2 \frac{\partial^2 \theta_m}{\partial X_1^2} + K_3 \frac{\partial^3 \theta_m}{\partial X_1^3} + \dots$$

$$= \frac{1}{Pe^2} \frac{\partial^2 \theta_m}{\partial X_1^2}$$

$$- \frac{\partial}{\partial X_1} \int_0^1 U \left(\begin{array}{l} \theta_m(\tau, x_1) + f_1(\tau, y) \dfrac{\partial \theta_m}{\partial x_1} \\ + f_2(\tau, y) \dfrac{\partial^2 \theta_m}{\partial x_1^2} + \dots\dots \end{array} \right) dY \qquad (26)$$

Comparing the coefficient of $\dfrac{\partial^k \theta_m}{\partial X_1^k}$ $(k = 1,2,3,\dots\dots)$, we get

$$K_1 = -\int_0^1 U dY \qquad (27)$$

$$K_2(\tau) = \frac{1}{Pe^2} - \int_0^1 U f_1(\tau, y) dY \qquad (28)$$

$$K_3(\tau) = -\int_0^1 U f_2(\tau, y) dY, \dots \qquad (29)$$

Substituting equation (21) in (19)

$$\frac{\partial(\theta_m(\tau, X_1))}{\partial \tau} + f_1(\tau, y) \frac{\partial \theta_m}{\partial X_1}(\tau, X_1)$$

$$+ f_2(\tau, y) \frac{\partial^2 \theta_m}{\partial X_1^2}(\tau, X_1) + \dots)$$

$$+ U \frac{\partial(\theta_m(\tau, X_1))}{\partial \tau} + f_1(\tau, y) \frac{\partial \theta_m}{\partial X_1}(\tau, X_1)$$

$$+ f_2(\tau, y) \frac{\partial^2 \theta_m}{\partial X_1^2}(\tau, X_1) + \dots)$$

$$= \frac{1}{Pe^2} \frac{\partial^2(\theta_m(\tau, X_1))}{\partial X_1} + f_1(\tau, y) \frac{\partial \theta_m}{\partial X_1}(\tau, X_1)$$

$$+ f_2(\tau, y) \frac{\partial^2 \theta_m}{\partial X_1^2}(\tau, X_1) + \dots)$$

$$+ \frac{\partial^2(\theta_m(\tau, X_1))}{\partial Y^2} + f_1(\tau, y) \frac{\partial \theta_m}{\partial X_1}(\tau, X_1)$$

$$+ f_2(\tau, y) \frac{\partial^2 \theta_m}{\partial X_1^2}(\tau, X_1) + \dots)$$

(30)

Following Gill and Sankarasubramanian (1970) and noting

$$\frac{\partial^{k+1} \theta_m}{\partial \tau \partial X_1^k} = \sum_{i=1}^{\infty} K_i(\tau) \frac{\partial^{k+i} \theta_m}{\partial X_1^{k+i}} \qquad for\ k = 1,2,3,\dots\dots$$

we get,

$$\left[\frac{\partial f_1}{\partial \tau} - \frac{\partial^2 f_1}{\partial Y^2} + U + k_1(\tau) \right] \frac{\partial \theta_m}{\partial X_1}$$

$$+ \left[\frac{\partial f_2}{\partial \tau} - \frac{\partial^2 f_2}{\partial Y^2} + Uf + k_1(\tau) f_1 + k_2(\tau) - \frac{1}{Pe^2} \right] \frac{\partial^2 \theta_m}{\partial X_1^2}$$

$$+ \sum_{k=1}^{\infty} \left[\begin{array}{l} \dfrac{\partial f_{k+2}}{\partial \tau} - \dfrac{\partial^2 f_{k+2}}{\partial Y^2} + U f_{k+1} \\ + k_1(\tau) f_{k+1} + \left(k_2(\tau) - \dfrac{1}{Pe^2} \right) f_k + \displaystyle\sum_{i=3}^{k+2} k_i f_{k+2-i} \end{array} \right] \frac{\partial^{k+2} \theta_m}{\partial X_1^{k+2}}$$

$= 0 \qquad (31)$

with $f_0 = 1$. Equating the coefficients of $\dfrac{\partial^k \theta_m}{\partial X_1^k}$ $(k = 1,2,3,\dots\dots\dots)$ to zero, a set of differential equations are obtained: $\dfrac{\partial f_1}{\partial \tau} = \dfrac{\partial^2 f_1}{\partial Y^2} - U - K_1(\tau)$

(32)

$$\frac{\partial f_2}{\partial \tau} = \frac{\partial^2 f_2}{\partial Y^2} - U f_1 - K_1(\tau) f_1 - K_2(\tau) + \frac{1}{Pe^2} \qquad (33)$$

$$\frac{\partial f_{k+2}}{\partial \tau} = \frac{\partial^2 f_{k+2}}{\partial Y^2} - U f_{k+1} - K_1(\tau) f_{k+1}$$

$$- \left(K_2(\tau) - \frac{1}{Pe^2} \right) f_k - \sum_{i=3}^{k+2} K_i f_{k+2-i} \qquad (34)$$

To evaluate K_i's we should know f_k's. Therefore we have to solve equation (30) subject to the initial and boundary conditions,

(i) $f_k(0, y) = 0 \qquad (35)$

(ii) $\dfrac{\partial f_k}{\partial Y}(\tau, 0) = 0 \qquad (36)$

(iii) $\dfrac{\partial f_k}{\partial Y}(\tau, 1) = 0 \qquad (37)$

(iv) $\displaystyle\int_0^1 f_k(\tau, Y) dY = 0$, for k=1,2,3,…

(38)

From equation (26), we get K_1 as

$$K_1(\tau) = 0 \qquad (39)$$

Equation (27) implies

$$K_2(\tau) = \frac{1}{Pe^2} - \int_0^1 U f_1 dY$$

Let

$$f_1 = f_{10}(y) + f_{11}(\tau, y) \qquad (40)$$

where, $f_{10}(y)$ corresponds to an infinitely wide slug which is independent of τ satisfies

(i) $f_{11} = -f_{10}(Y) \quad at\ \ \tau = 0$

(ii) $\dfrac{\partial f_{11}}{\partial Y} = 0 \quad at\ \ Y = 0$

(iii) $\frac{\partial f_{11}}{\partial Y} = 0 \quad at \ \ Y = 1$

(iv) $\int_0^1 f_{11} dY = 0$ (41)

Substituting (39) in (31) gives $\frac{d^2 f_{10}(Y)}{dY^2} = U(Y)$ and $\frac{\partial f_{11}}{\partial \tau} = \frac{\partial^2 f_{11}}{\partial Y^2}$ is the well-known heat conduction equation which is solved by separation of variables.

Hence the solution of f_1 is given by

$$f_1 = \frac{1}{m_3}\left[\frac{1}{X\sqrt{X(e^{\sqrt{X}} + e^{-\sqrt{X}})}}\left(\frac{c_0}{X} X e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m) - \frac{c_0}{X}\right)\right] e^{\sqrt{X} y}$$

$$+ \left[\frac{1}{X\sqrt{X(e^{\sqrt{X}} + e^{-\sqrt{X}})}}\left(\frac{c_0}{X} X e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m) - \frac{c_0}{X}\right)\right] e^{-\sqrt{X} y}$$

$$- \frac{c_0 y^2}{2X} + \frac{A_2 e^{\sqrt{m_1} y}}{m_1} + \frac{B_2 e^{\sqrt{m_1} y}}{m_1}$$

$$+ c_1 y + c_2 + \sum_{m=1}^{\infty} A_m Cos(\lambda_m Y) e^{-\lambda_m^2 \tau}$$

(42)

$$m_3 = \frac{e^{\sqrt{X}-1}}{\sqrt{X}}\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m)\right) + \frac{e^{-\sqrt{X}-1}}{\sqrt{X}}\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m)\right)$$
$$+ \varepsilon\left(\left(a - \frac{e^{\sqrt{m_1}}}{e^{\sqrt{m_1}} + e^{-\sqrt{m_1}}}\right) + \left(\frac{e^{\sqrt{m_1}} - 1}{\sqrt{m_1}}\right) - \left(\frac{e^{\sqrt{m_1}} a}{e^{\sqrt{m_1}} + e^{-\sqrt{m_1}}}\right)\left(\frac{e^{-\sqrt{m_1}} - 1}{\sqrt{m_1}}\right)\right) + \frac{c_0}{X}$$

$$c_1 = \left(-\frac{1}{m_3}\left(\frac{1}{X\sqrt{X}(e^{\sqrt{X}} + e^{-\sqrt{X}})}\right)\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m) - \frac{c_0}{X}\right)\right)$$
$$+ \left(\frac{1}{X\sqrt{X}(e^{\sqrt{X}} + e^{-\sqrt{X}})}\right)\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m)\right) + \frac{A_2}{\sqrt{m_1}} - \frac{B_2}{\sqrt{m_2}}$$

$$c_2 = \left(-\frac{1}{m_3}\left(\frac{1}{X\sqrt{X}(e^{\sqrt{X}} + e^{-\sqrt{X}})}\right)\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m)(e^{\sqrt{X}} + e^{-\sqrt{X}}) - \frac{c_0}{X}\right)\right)$$
$$+ \left(\frac{1}{X\sqrt{X}(e^{\sqrt{X}} + e^{-\sqrt{X}})}\right)\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m)\right) + \frac{A_2}{m_1\sqrt{m_1}(e^{\sqrt{m_1}-1})} - \frac{B_2}{m_2\sqrt{m_2}(e^{\sqrt{m_2}-1})}$$
$$+ \frac{c_0}{6X} + \frac{c}{2} - \frac{m_2}{6}$$

$$A_m = 2\left(\frac{1}{m_3}\left(\frac{1}{X\sqrt{X}(e^{\sqrt{X}} + e^{-\sqrt{X}})}\right)\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m) - \frac{c_0}{X}\right) e^{\sqrt{X}}\right)$$
$$- \frac{1}{m_3}\left(\frac{1}{X\sqrt{X}(e^{\sqrt{X}} + e^{-\sqrt{X}})}\right)\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m)\right) e^{-\sqrt{X}} + \frac{c_0}{X} + \frac{\sqrt{m_1} A_2 e^{-\sqrt{m_1}}}{m_1}$$
$$+ \frac{\sqrt{m_2} B_2 e^{-\sqrt{m_{21}}}}{m_2} - m_3 + c_1\left(\frac{-\cos(\lambda m)}{(\lambda m)^2}\right)\left(\frac{-1}{m_3}\left(\frac{1}{X\sqrt{X}(e^{\sqrt{X}} + e^{-\sqrt{X}})}\right)\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m) - \frac{c_0}{X}\right)\right)$$
$$- \left(\frac{1}{X\sqrt{X}(e^{\sqrt{X}} + e^{-\sqrt{X}})}\right)\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m)\right) + \frac{\sqrt{m_1} A_2}{m_1} - \frac{\sqrt{m_2} B_2}{m_2} - m_3 + c_1\left(\frac{-1}{(\lambda m)^2}\right)$$
$$\left(\frac{1}{m_3}\left(\frac{1}{X\sqrt{X}(e^{\sqrt{X}} + e^{-\sqrt{X}})}\right)\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m) - \frac{c_0}{X}\right)(e^{\sqrt{X}} - 1)\right) + \left(\frac{\cos(\lambda m)}{(\lambda m)^4}\right)$$
$$+ \left(\frac{1}{(e^{\sqrt{X}} + e^{-\sqrt{X}})}\right)\left(\frac{c_0}{X}\sqrt{X} e^{\sqrt{X}} - \frac{\alpha}{\sqrt{X}}(u_\beta - u_\beta{}^m)\right)$$

and $\lambda m = m\pi$

Therefore substituting f_1 in equation (27) gives the solution of K_2. Similarly $K_3(\tau)$, $K_4(\tau)$,...... are obtained and we found that $K_i(\tau), i > 2$ are negligibly small compared to $K_2(\tau)$.

The dispersion model (24) takes the form

$$\frac{\partial \theta_m}{\partial \tau} = K_2 \frac{\partial^2 \theta_m}{\partial X_1^2}$$

In a similar manner we apply generalized dispersion method to find the velocity and dispersion coefficient for the contaminant phase.

IV. RESULTS AND DISCUSSION

In this paper we have studied the dispersion of contaminants consisting a mixture of solid phase (fine and coarse) and fluid phase. Results of velocity and dispersion coefficient are obtained analytically and the numerical values have been computed using MATHEMATICA 8.0.

Figures 2 and 3 represents that the effect of particle mass parameter on the velocity profiles of contaminated phase and fluid phase at a given instant of time.It is clear that, the velocity of fluid is greater than the velocity of fine and coarse particle.

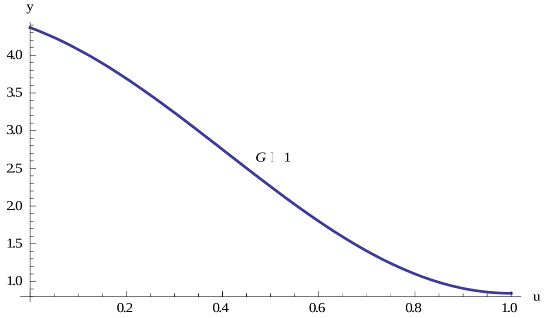

Fig. 2. Velocity profile for fluid phase

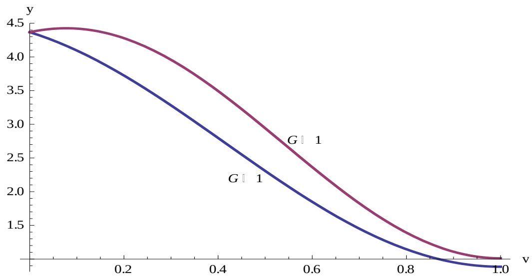

Fig. 3. Velocity profile for contaminant phase

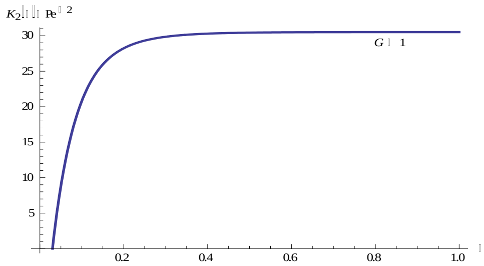

Fig. 4. Dispersion coefficient varying with dimensionless time for fluid phase

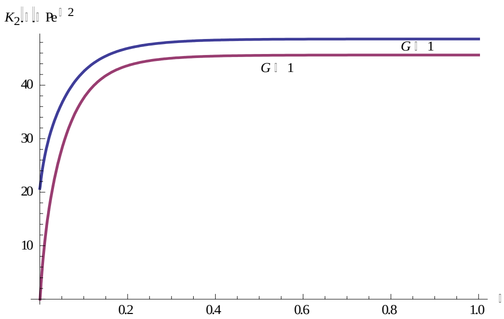

Fig.5. Dispersion coefficient varying with dimensionless time for contaminant phase

The time-dependent dispersion coefficient is evaluated using the generalized dispersion model which is valid for all time. The dominant dispersion coefficient is computed for different values of particle mass parameter and it is observed from the figure that the dispersion coefficient is greatest for coarse particle when compared with the fine particle and fluid phase.

V. CONCLUSION

Groundwater contamination by pathogenic bacteria and viruses has long been recognized as a serious hazard to human health . The release of leachate to the environment is one of the major environmental impacts related to disposal of waste. Disposed waste in landfills undergoes a series of 9 phases where the waster is decomposed. During the decomposition leachate is generated by excess rainwater infiltrating the waste.

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