Remarks on $\gamma_{\hat{\Omega}}^{\star}$-open sets and minimal $\gamma_{\hat{\Omega}}^{\star}$-open sets

$\gamma_{\hat{\Omega}}^{\star}$

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Abstract**: *Aim of this paper is to define $\gamma_{\hat{\Omega}}^{\star}$-open sets in a topological space and obtain their basic properties. Also, we define minimal $\gamma_{\hat{\Omega}}^{\star}$-open sets in a space and study the impact of two minimal $\gamma_{\hat{\Omega}}^{\star}$-open sets in a space with $\hat{\Omega}$-regular operation. However, the roll of minimal $\gamma_{\hat{\Omega}}^{\star}$-open sets in $\gamma_{\hat{\Omega}}^{\star}$-locally finite space has been discussed.***

Keywords* : *$\gamma_{\hat{\Omega}}^{\star}$-open set, $\gamma_{\hat{\Omega}}^{\star}$-Interior, $\gamma_{\hat{\Omega}}^{\star}$-Closure, minimal $\gamma_{\hat{\Omega}}^{\star}$-open set, $\gamma_{\hat{\Omega}}^{\star}$-locally finite space.

I. INTRODUCTION

In a topological space, the notion of minimal open sets had been introduced by Nakaoka and Oda[4] in 2001. In 2012, Lellis Thivagar et al.[1] introduced the class of $\hat{\Omega}$-closed sets in a space which is independent of closed sets. Recently, [3] operation on the class of $\hat{\Omega}$-open sets have been introduced and studied. In this paper, we introduce the class of $\gamma_{\hat{\Omega}}^{\star}$-open sets and investigate their basic properties in terms of it's closure. However, Nakaoka's idea of minimal open sets has been extended to $\gamma_{\hat{\Omega}}^{\star}$-open sets and some of its elementary properties have been derived. Moreover, the behaviour of minimal $\gamma_{\hat{\Omega}}^{\star}$-open sets in a $\gamma_{\hat{\Omega}}^{\star}$-locally finite space has been investigated.

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II. PRELIMINARIES

Some definitions and results that are used in this paper have been given in this section. Always X or (X,τ) denotes a topological space on which no separation axioms assumed, unless otherwise stated. For any subset A of X, the closure (res.interior) of A is denoted by $cl(A)(res.int(A))$.

Definition 2.1 *[2] A subset A of a topological space (X,τ) is called a* **semi-open** *set if $A \subseteq cl(int(A))$. $SO(X)$ denotes the set of all semi-open sets in (X,τ).It's complement is known as* **semi-closed** *set on X.*

Definition 2.2 *([1], Definition 3.1) Let (X,τ) be a topological space. A is said to be* **$\hat{\Omega}$-closed set** *if $\delta cl(A) \subseteq U$ when $A \subseteq U$, where U is a semi-open subset of X. The complement of $\hat{\Omega}$-closed set is an $\hat{\Omega}$-open set. The family of all $\hat{\Omega}$-closed sets in a space (X,τ) is denoted by $\tau_{\hat{\Omega}}$. Also $\hat{\Omega}O(X,\tau)$ or $\hat{\Omega}O(X)$ (resp. $\hat{\Omega}C(X,\tau)$ or $\hat{\Omega}C(X)$) denotes the set of all $\hat{\Omega}$-open sets (resp. $\hat{\Omega}$-closed sets) on the space X.*

Definition 2.3 *([3], Definition 3.1) A function $\gamma : \hat{\Omega}O(X,\tau) \rightarrow P(X)$ is called an* **operation on $\hat{\Omega}O(X,\tau)$**, *if $U \subseteq \gamma(U)$ for every set $U \in \hat{\Omega}O(X,\tau)$. For any operation $\gamma, \gamma(X) = X$, and $\gamma(\emptyset) = \emptyset$.*

Definition 2.4 *([3], Definition 3.3) A non-empty set A of X is called* **$\gamma_{\hat{\Omega}}$-open set** *if for each $x \in A$, there exists an $\hat{\Omega}$-open set U such that $x \in U$ and $\gamma(U) \subseteq A$. The complement of $\gamma_{\hat{\Omega}}$-open set is a $\gamma_{\hat{\Omega}}$-closed set. The set of all $\gamma_{\hat{\Omega}}$-open subsets of a topological space (X,τ) is denoted by $\tau_{\gamma_{\hat{\Omega}}}$.*

Definition 2.5 *([3], Definition 3.15) Let (X,τ) be a topological space. An operation γ is said to be* **$\hat{\Omega}$-regular** *if for every pair of sets $U,V \in \hat{\Omega}O(X,x)$, there exists an $\hat{\Omega}$-open set W containing x such that $\gamma(W) \subseteq \gamma(U) \cap \gamma(V)$.*

Remark 2.6 *([1], Remark 5.2) From the definition and Theorem 4.16, arbitrary intersection of an $\hat{\Omega}$-closed sets in a topological space (X,τ) is an $\hat{\Omega}$-closed set in (X,τ), $\hat{\Omega}cl(A)$ is the smallest $\hat{\Omega}$-closed set containing A.*

Theorem 2.7 *([3], Theorem 3.5) Arbitrary union of $\gamma_{\hat{\Omega}}$-open sets is a $\gamma_{\hat{\Omega}}$-open set in a topological space.*

Proposition 2.8 *([3], Proposition 3.8) Every $\gamma_{\hat{\Omega}}$-open set is an $\hat{\Omega}$-open in a space X.*

Proposition 2.9 *([3], Proposition 3.17) Intersection of any two $\gamma_{\hat{\Omega}}$-open sets is a $\gamma_{\hat{\Omega}}$-open in a space with an $\hat{\Omega}$-regular operation on $\hat{\Omega}O(X,\tau)$.*

III. $\gamma^{\star}_{\hat{\Omega}}$--OPEN SETS

Definition 3.1 *A subset A of a space (X,τ) is said to be a* $\boldsymbol{\gamma^{\star}_{\hat{\Omega}}}$***-open*** *subset of X, if the following two axioms hold:*

i) A is a $\gamma_{\hat{\Omega}}$-open subset of X.

ii) For any $x \in A$, there exists an $\hat{\Omega}$-closed set F containing x such that $F \subseteq A$.

The complement of a $\gamma^{\star}_{\hat{\Omega}}$-open set is a $\gamma^{\star}_{\hat{\Omega}}$-closed set in X. However, $\gamma^{\star}_{\hat{\Omega}}O(X)$ (resp. $\gamma^{\star}_{\hat{\Omega}}C(X)$) denotes the set of all $\gamma^{\star}_{\hat{\Omega}}$-open sets (resp. $\gamma^{\star}_{\hat{\Omega}}$-closed sets) on X and $\gamma^{\star}_{\hat{\Omega}}O(X,x)$ denotes the set of all $\gamma^{\star}_{\hat{\Omega}}$-open sets containing x.

Example3.2 Consider a space with $X = \{a,b,c,d\}$ *and* $\tau = \{\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},\{a,b,c\},\{a,c,d\},X\}$, Define $\gamma{:}\hat{\Omega}O(X,\tau)\rightarrow P(X)$ is defined by $\gamma(\emptyset) = \emptyset, \gamma(\{a\}) = \{a\}, \gamma(\{c\}) = \{c\}, \gamma(\{a,b\}) = \{a,b,c\}, \gamma(\{a,c\}) = \{a,c\}, \gamma(\{a,b,c\}) = X, \gamma(X) = X$. Here, γ is an operation on $\hat{\Omega}O(X,\tau)$. $\tau_{\gamma_{\hat{\Omega}}} = \{\emptyset,\{a\},\{c\},\{a,c\},\{a,b,c\},X\}$, $\hat{\Omega}C(X) = \{\emptyset,\{d\},\{b,d\},\{c,d\},\{a,b,d\},\{b,c,d\},X\}$. Here, $\emptyset$ and X are the only $\gamma^{\star}_{\hat{\Omega}}$-open subsets of X.

Proposition 3.3 *In a space (X,τ), the following statements hold.*

i) Every $\gamma^{\star}_{\hat{\Omega}}$-open subset is a $\gamma_{\hat{\Omega}}$-open set.

ii) Every $\gamma^{\star}_{\hat{\Omega}}$-open set is an $\hat{\Omega}$-open set.

Proof. *i)* It follows straightly from the definition.

ii) It follows by combining *i)* and By Proposition 2.8

Remark 3.4 *From the following example, it is seen that there are some $\gamma_{\hat{\Omega}}$-open subsets which fail to be the $\gamma^{\star}_{\hat{\Omega}}$-open sets in a space (X,τ).*

Example 3.5 *Let* $X = \{a,b,c\}, \tau = \{\emptyset,\{a\},\{a,b\},X\}$; $\hat{\Omega}O(X) = \{\emptyset,\{a\},X\}$; $\hat{\Omega}C(X) = \{\emptyset,\{b,c\},X\}$.

Define an operation $\gamma{:}\hat{\Omega}O(X)\rightarrow P(X)$ as follows:

$$\gamma(W) = \begin{cases} W & \text{if } W = \{a\} \text{ or } \emptyset \\ X & \text{Otherwise} \end{cases}$$

Here, $\tau_{\gamma_{\hat{\Omega}}} = \{\emptyset,\{a\},X\}$; $\gamma^{\star}_{\hat{\Omega}}O(X) = \{\emptyset,X\}$.

Now $\{a\} \in \tau_{\gamma_{\hat{\Omega}}}$ *but* $\{a\} \notin \gamma^{\star}_{\hat{\Omega}}O(X)$.

Theorem 3.6. *Arbitrary union of $\gamma^{\star}_{\hat{\Omega}}$-open sets in a space (X,τ) is a $\gamma^{\star}_{\hat{\Omega}}$-open set.*

Proof. Let $\{U_i / i \in I\}$ be any collection of $\gamma^{\star}_{\hat{\Omega}}$-open sets in a space X and let $U = \bigcup_{i \in I} U_i$. By Theorem 2.7, U is a $\gamma_{\hat{\Omega}}$-open subset of X. Let $x \in U$ be arbitrary. Then $x \in U_i$ for some $i \in I$. Since U_i is a $\gamma^{\star}_{\hat{\Omega}}$-open subset of X, there exist an $\hat{\Omega}$-closed set F containing x such that $F \subseteq U_i \subseteq U$. Therefore, U is a $\gamma^{\star}_{\hat{\Omega}}$-open subset of X.

Remark 3.7. *The intersection of any two $\gamma^{\star}_{\hat{\Omega}}$-open sets is not always a $\gamma^{\star}_{\hat{\Omega}}$-open set as seen from the following example.*

Example 3.8. *Let $X = \{a,b,c\}$ and $\tau = P(X)$. Define an operation $\gamma{:}\hat{\Omega}O(X)\rightarrow P(X)$ as follows:*

$$\gamma(W) = \begin{cases} W & \text{if } W = \{a,b\} \text{ or } \{a,c\} \text{ or } \emptyset \\ X & \text{Otherwise} \end{cases}$$

$\gamma^{\star}_{\hat{\Omega}}O(X) = \{\emptyset,\{a,b\},\{a,c\},X\}$. *Here, the two sets $\{a,b\}$ and $\{a,c\}$ are $\gamma^{\star}_{\hat{\Omega}}$-open sets but $\{a\}$ is not a $\gamma^{\star}_{\hat{\Omega}}$-open in X.*

Proposition 3.9. *If U and V are $\gamma^{\star}_{\hat{\Omega}}$-open sets in a space X, then $U \cap V$ is a $\gamma^{\star}_{\hat{\Omega}}$-open set provided γ is an $\hat{\Omega}$-regular operation.*

Proof. Let U and V be any two $\gamma^{\star}_{\hat{\Omega}}$-open subsets of X. By Proposition 2.9, $U \cap V$ is a $\gamma_{\hat{\Omega}}$-open subset of X. Let $x \in U \cap V$ be arbitrary. Since U and V are $\gamma^{\star}_{\hat{\Omega}}$-open subsets of X, there exists two $\hat{\Omega}$-closed sets F_1 and F_2 containing x such that $F_1 \subseteq U$ and $F_2 \subseteq V$. By choosing F as $F_1 \cap F_2$, F is an $\hat{\Omega}$-closed set containing x such that $F \subseteq U \cap V$. Therefore, $U \cap V$ is a $\gamma^{\star}_{\hat{\Omega}}$-open subset of X.

$\gamma^{\star}_{\hat{\Omega}}$-Closure and $\gamma^{\star}_{\hat{\Omega}}$-Interior of a set have been defined in the usual manner as follows:

Definition 3.10. $\boldsymbol{\gamma^{\star}_{\hat{\Omega}}}$***-closure*** *of any subset A of a space (X,τ) is denoted by $\gamma^{\star}_{\hat{\Omega}}cl(A)$ and defined by*

$$\gamma^{\star}_{\hat{\Omega}}cl(A) = \bigcap\{F / F \in \gamma^{\star}_{\hat{\Omega}}C(X) \text{ such that } A \subseteq F\}.$$

Always $\gamma^{\star}_{\hat{\Omega}}cl(\emptyset) = \emptyset$; $\gamma^{\star}_{\hat{\Omega}}cl(X) = X$; $\gamma^{\star}_{\hat{\Omega}}cl(A)$ is a $\gamma^{\star}_{\hat{\Omega}}$-closed subset of X and $\gamma^{\star}_{\hat{\Omega}}(\gamma^{\star}_{\hat{\Omega}}cl(A)) = \gamma^{\star}_{\hat{\Omega}}cl(A)$.

Definition 3.11. $\boldsymbol{\gamma^{\star}_{\hat{\Omega}}}$***-interior*** *of any subset A of a space (X,τ) is denoted by $\gamma^{\star}_{\hat{\Omega}}int(A)$ and defined by*

$$\gamma^{\star}_{\hat{\Omega}}int(A) = \bigcup\{U / U \in \gamma^{\star}_{\hat{\Omega}}O(X) \text{ such that } U \subseteq A\}.$$

Always, $\gamma^{\star}_{\hat{\Omega}}int(\emptyset) = \emptyset$; $\gamma^{\star}_{\hat{\Omega}}int(X) = X$ and $\gamma^{\star}_{\hat{\Omega}}int(A)$ is a $\gamma^{\star}_{\hat{\Omega}}$-open subset of X.

Proposition 3.12. *If U and V are any two subsets of a space X, then the following statements are true.*

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i) $U \subseteq V \Rightarrow \gamma^{\star}_{\hat{\Omega}}int(U) \subseteq \gamma^{\star}_{\hat{\Omega}}int(V)$.

ii) U is $\gamma^{\star}_{\hat{\Omega}}$-open iff $U = \gamma^{\star}_{\hat{\Omega}}int(U)$.

iii) $U \subseteq V \Rightarrow \gamma^{\star}_{\hat{\Omega}}cl(U) \subseteq \gamma^{\star}_{\hat{\Omega}}cl(V)$.

iv) U is $\gamma^{\star}_{\hat{\Omega}}$-closed iff $U = \gamma^{\star}_{\hat{\Omega}}cl(U)$.

v) $X\backslash\gamma^{\star}_{\hat{\Omega}}int(U) = \gamma^{\star}_{\hat{\Omega}}cl(X\backslash U)$.

vi) $X\backslash\gamma^{\star}_{\hat{\Omega}}cl(U) = \gamma^{\star}_{\hat{\Omega}}int(X\backslash U)$.

Proof. It follows from the definition.

Proposition 3.13. *For any point x in a space X and for any subset A of X,* $x \in \gamma^{\star}_{\hat{\Omega}}cl(A)$ iff $U \cap A \neq \emptyset$ for any $U \in \gamma^{\star}_{\hat{\Omega}}O(X,x)$.

Proof. Assume that $x \in \gamma^{\star}_{\hat{\Omega}}cl(A)$ and there exists a $\gamma^{\star}_{\hat{\Omega}}$-open set U containing x such that $U \cap A = \emptyset$. Now, $X\backslash U$ is a $\gamma^{\star}_{\hat{\Omega}}$ -closed set such that $A \subseteq X\backslash U$. That is, $X\backslash U \in \{F/F \in \gamma^{\star}_{\hat{\Omega}}C(X);\ A \subseteq F\}$. In particular, $\gamma^{\star}_{\hat{\Omega}}cl(A) \subseteq X\backslash U$, a contradiction to $x \in \gamma^{\star}_{\hat{\Omega}}cl(A)$.

Conversely,on contrary $x \notin \gamma^{\star}_{\hat{\Omega}}cl(A) = \bigcap\{F/F \in \gamma^{\star}_{\hat{\Omega}}C(X);\ A \subseteq F\}$. Then $x \notin F$ for some $\gamma^{\star}_{\hat{\Omega}}$ -closed set F such that $A \subseteq F$. If U is chosen as $X\backslash F$, then U is a $\gamma^{\star}_{\hat{\Omega}}$-open subset of X such that $U \cap A = \emptyset$, a contradiction to hypothesis.

IV. MINIMAL $\gamma^{\star}_{\hat{\Omega}}$–OPEN SETS AND $\gamma^{\star}_{\hat{\Omega}}$–LOCALLY FINITE SPACE

Definition 4.1. *A $\gamma^{\star}_{\hat{\Omega}}$- open subset U of a space X is said to be a* **minimal $\gamma^{\star}_{\hat{\Omega}}$-open set** *if $\emptyset$ and U are the only $\gamma^{\star}_{\hat{\Omega}}$-open subsets of U.*

Example 4.2 *Let $X = \{a,b,c\}$ and $\tau = P(X) = \hat{\Omega}O(X,\tau)$. Define an operation $\gamma{:}\hat{\Omega}O(X)\to P(X)$ as follows:*

$$\gamma(W) = \begin{cases} W & if\ W = \{a,b\}\ or\ \{a,c\}\ or\ \emptyset \\ X & Otherwise \end{cases}$$

$\gamma^{\star}_{\hat{\Omega}}O(X) = \{\emptyset,\{a,b\},\{a,c\},X\} = \gamma_{\hat{\Omega}}O(X)$. *Here $\{a,b\}$ and $\{a,c\}$ are minimal $\gamma^{\star}_{\hat{\Omega}}$-open sets in X.*

Proposition 4.3. *Let (X,τ) be any topological space. If U is a non-empty $\gamma^{\star}_{\hat{\Omega}}$ -open subset of X such that $\gamma^{\star}_{\hat{\Omega}}cl(U) = \gamma^{\star}_{\hat{\Omega}}cl(V)$ for any non-empty subset V of U, then U is a minimal $\gamma^{\star}_{\hat{\Omega}}$-open set.*

Proof. On contrary, U is assumed to be a non-minimal $\gamma^{\star}_{\hat{\Omega}}$-open subset of X such that $\gamma^{\star}_{\hat{\Omega}}cl(U) = \gamma^{\star}_{\hat{\Omega}}cl(V)$ for any non-empty subset V of U. By the definition of minimal open set, there exists a non-empty $\gamma^{\star}_{\hat{\Omega}}$-open set W such that $W \subsetneq U$. Choose $x \in U\backslash W$. Now W is a $\gamma^{\star}_{\hat{\Omega}}$*-open set such that $x \notin W$.Therefore,* $W \cap \gamma^{\star}_{\hat{\Omega}}cl(\{x\}) = \emptyset$ *or* $\gamma^{\star}_{\hat{\Omega}}cl(\{x\}) \subseteq X\backslash W$. *Now $\{x\}$ is a non-empty subset of U. By hypothesis,* $\gamma^{\star}_{\hat{\Omega}}cl(U) = \gamma^{\star}_{\hat{\Omega}}cl(\{x\}) \subseteq X\backslash W$. *However,* $W \subsetneq U \subseteq \gamma^{\star}_{\hat{\Omega}}cl(U) = \gamma^{\star}_{\hat{\Omega}}cl(\{x\}) \subseteq X\backslash W$ Which implies that $W \subseteq X\backslash W$, a contradiction.

Proposition 4.4. *Let γ be an $\hat{\Omega}$-regular operation in a space X. If U is a minimal $\gamma^{\star}_{\hat{\Omega}}$-open set and V is a $\gamma^{\star}_{\hat{\Omega}}$-open set then either $U \cap V = \emptyset$ or $U \subseteq V$.*

Proof. *Assume that $U \cap V = \emptyset$. By Proposition 3.9, $U \cap V$ is a $\gamma^{\star}_{\hat{\Omega}}$-open set. By the definition of minimal $\gamma^{\star}_{\hat{\Omega}}$-open sets, $U \cap V = V$. Thus $U \subseteq V$.*

Proposition 4.5. *Let γ be an $\hat{\Omega}$-regular operation in a space X. If U and V are any two minimal $\gamma^{\star}_{\hat{\Omega}}$-open sets, then either $U \cap V = \emptyset$ or $U = V$*

Proof. Assume that $U \cap V = \emptyset$. Apply Proposition 4.4 for both minimal $\gamma^{\star}_{\hat{\Omega}}$-open sets U and V. Then $U = V$.

Proposition 4.6. *Let γ be an $\hat{\Omega}$-regular operation in a space X and U be a minimal $\gamma^{\star}_{\hat{\Omega}}$-open set of X. Then for any $x \in U$, $U \subseteq V$ where V is any $\gamma^{\star}_{\hat{\Omega}}$-open set containing x.*

Proof. Assume the contrary that there exists $x \in U$ such that $U \nsubseteq V$ for any $\gamma^{\star}_{\hat{\Omega}}$-open subset V containing x. By Proposition 3.9, $U \cap V$ is a $\gamma^{\star}_{\hat{\Omega}}$ -open subset of X. As $x \in U \cap V, U \cap V \neq \emptyset$ such that $U \cap V \subseteq U$, a contradiction to U is a minimal $\gamma^{\star}_{\hat{\Omega}}$-open subset.

Proposition 4.7. *If U is a minimal $\gamma^{\star}_{\hat{\Omega}}$-open subset of a space X*, then

$$U = \bigcap_{x \in U}\{V/V \in \gamma^{\star}_{\hat{\Omega}}O(X,x)\}.$$

Proof. By Proposition 4.6, for any $x \in U$, $U \subseteq V$ for any $\gamma^{\star}_{\hat{\Omega}}$ -open subset V containing x. Then $U \subseteq \bigcap\{V/V \in \gamma^{\star}_{\hat{\Omega}}O(X,x)\}$. In particular, $\bigcap\{V/V \in \gamma^{\star}_{\hat{\Omega}}O(X,x)\} \subseteq U$ .Now $U = \bigcap\{V/V \in \gamma^{\star}_{\hat{\Omega}}O(X,x)\}$.

Proposition 4.8. *Let γ be an $\hat{\Omega}$-regular operation of a space X. If U is a minimal $\gamma^{\star}_{\hat{\Omega}}$-open subset of X, then for any $x \in X\backslash U$, and for any $\gamma^{\star}_{\hat{\Omega}}$-open set V containing x, either $V \cap U = \emptyset$ or $U \subseteq V$.*

Proof. It follows from Proposition 4.4.

Proposition 4.9. *Let γ be an $\hat{\Omega}$-regular operation in a space X. If U is a non-empty minimal $\gamma^{\star}_{\hat{\Omega}}$-open subset of X, then $U \subseteq \gamma^{\star}_{\hat{\Omega}}cl(V)$ for any non-empty subset V of U.*

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Proof. Let $x \in U$ be arbitrary and W be any $\gamma^{\star}_{\hat{\Omega}}$-open set containing x. By Proposition 4.4, either $U \cap W = \emptyset$ or $U \subseteq W$. Since $x \in U \cap W$, the only possibility is $U \subseteq W$. Then, $V = U \cap V \subseteq W \cap V$. As $V \neq \emptyset, W \cap V \neq \emptyset$. By Proposition 3.13, $x \in \gamma^{\star}_{\hat{\Omega}}cl(V)$.

Characterization of minimal $\gamma^{\star}_{\hat{\Omega}}$-open set in a space is given as follows.

Theorem 4.10. *Let γ be an $\hat{\Omega}$-regular operation in a space X and U be any non-empty $\gamma^{\star}_{\hat{\Omega}}$-open subset of X. Then the following statements are equivalent:*

i) U is a minimal $\gamma^{\star}_{\hat{\Omega}}$-open set.

ii) $U \subseteq \gamma^{\star}_{\hat{\Omega}}cl(V)$ for any non-empty subset V of U.

iii) $\gamma^{\star}_{\hat{\Omega}}cl(U) = \gamma^{\star}_{\hat{\Omega}}cl(V)$ for any non-empty subset V of U.

Proof. $i) \Rightarrow ii)$ It follows from Proposition 4.9.

$ii) \Rightarrow iii)$ Let V be any non-empty subset of U. By hypothesis $U \subseteq \gamma^{\star}_{\hat{\Omega}}cl(V)$ and hence $\gamma^{\star}_{\hat{\Omega}}cl(U) \subseteq \gamma^{\star}_{\hat{\Omega}}(\gamma^{\star}_{\hat{\Omega}}cl(V)) = \gamma^{\star}_{\hat{\Omega}}cl(V)$. On the other hand, $V \subseteq U \Rightarrow \gamma^{\star}_{\hat{\Omega}}cl(V) \subseteq \gamma^{\star}_{\hat{\Omega}}cl(U)$.

$iii) \Rightarrow i)$ On contrary, U is assumed to be a non-minimal $\gamma^{\star}_{\hat{\Omega}}$-open subset. Then, a non-empty $\gamma^{\star}_{\hat{\Omega}}$-open set V can be chosen such that $V \subsetneqq U$. Having chosen $x \in U \backslash V$, $V \cap \gamma^{\star}_{\hat{\Omega}}cl(\{x\}) = \emptyset$ and hence $\gamma^{\star}_{\hat{\Omega}}cl(\{x\}) \subseteq X \backslash V$. But by hypothesis, $\gamma^{\star}_{\hat{\Omega}}cl(\{x\}) = \gamma^{\star}_{\hat{\Omega}}cl(U)$ .It is concluded that $V \subseteq U \subseteq \gamma^{\star}_{\hat{\Omega}}cl(U) = \gamma^{\star}_{\hat{\Omega}}cl(\{x\}) \subseteq X \backslash V$, a contradiction. Therefore, U is a minimal $\gamma^{\star}_{\hat{\Omega}}$-open set.

Definition 4.11. *A space X is said to be* $\boldsymbol{\gamma^{\star}_{\hat{\Omega}}}$***-locally finite space****, if for every $x \in X$ there exists a finite $\gamma^{\star}_{\hat{\Omega}}$-open set U in X such that $x \in U$.Every finite topological space is always $\gamma^{\star}_{\hat{\Omega}}$-locally finite space.*

Proposition 4.12. *Every non-empty finite $\gamma^{\star}_{\hat{\Omega}}$-open set in a space X contains some non-empty finite minimal $\gamma^{\star}_{\hat{\Omega}}$-open set.*

Proof. Let V be any non-empty finite $\gamma^{\star}_{\hat{\Omega}}$-open set in X. If V is a minimal $\gamma^{\star}_{\hat{\Omega}}$-open set in X, then it meets our requirement. Suppose that V is not a minimal $\gamma^{\star}_{\hat{\Omega}}$-open set in X. Then, there exists a non-empty finite $\gamma^{\star}_{\hat{\Omega}}$-open set W_1 such that $W_1 \subsetneqq V$. If W_1 is a minimal $\gamma^{\star}_{\hat{\Omega}}$-open set, then the condition is satisfied. If not, by proceeding like this we would arrive at a sequence of non-empty finite $\gamma^{\star}_{\hat{\Omega}}$-open sets $W_1, W_2, \ldots W_n \ldots$ such that $\ldots \subsetneqq W_n \ldots \subsetneqq W_2 \subsetneqq W_1 \subsetneqq V$. Since V is finite, there exists $m \in \mathbb{N}$ such that W_m is a minimal $\gamma^{\star}_{\hat{\Omega}}$-open set such that $W_m \subseteq V$. Hence the result holds.

Proposition 4.13. *Let γ be an $\hat{\Omega}$-regular operation in a $\gamma^{\star}_{\hat{\Omega}}$-locally finite space X. Then every non-empty $\gamma^{\star}_{\hat{\Omega}}$-open set in a space X contains some non-empty finite minimal $\gamma^{\star}_{\hat{\Omega}}$-open set.*

Proof. Let V be any non-empty $\gamma^{\star}_{\hat{\Omega}}$-open subset of X. Then choose $x \in V$. By the definition of $\gamma^{\star}_{\hat{\Omega}}$-locally finite space, there exists a finite $\gamma^{\star}_{\hat{\Omega}}$-open set U such that $x \in U$. Since U is finite, $U \cap V$ is also finite. By Proposition 3.9, $U \cap V$ is a finite $\gamma^{\star}_{\hat{\Omega}}$-open set containing x. By Proposition 4.12, there exists non-empty finite minimal $\gamma^{\star}_{\hat{\Omega}}$-open set W such that $W \subseteq U \cap V \subseteq V$.

Proposition 4.14. *Let X be a topological space and γ be an $\hat{\Omega}$-regular operation on X. Then the following are true.*

i) Let V_α, $\alpha \in I$ be any $\gamma^{\star}_{\hat{\Omega}}$-open set and U be a non-empty finite $\gamma^{\star}_{\hat{\Omega}}$-open set in X. Then $U \cap \left(\bigcap_{\alpha \in I} V_\alpha\right)$ is a finite $\gamma^{\star}_{\hat{\Omega}}$-open set.

ii) Let V_α be a $\gamma^{\star}_{\hat{\Omega}}$-open set for any $\alpha \in I$ and V_β be a non-empty finite $\gamma^{\star}_{\hat{\Omega}}$-open set for any $\beta \in J$. Then $(\bigcup_{\beta \in J} V_\beta) \cap (\bigcap_{\alpha \in I} V_\alpha)$ is a $\gamma^{\star}_{\hat{\Omega}}$-open set.

Proof. *i)* Let V_α be a $\gamma^{\star}_{\hat{\Omega}}$-open subset for any $\alpha \in I$ and U be a non-empty finite $\gamma^{\star}_{\hat{\Omega}}$-open set in X. Since U is finite, $U \cap \left(\bigcap_{\alpha \in I} V_\alpha\right)$ is a finite set. Then, there exists a positive integer n such that $U \cap \left(\bigcap_{\alpha \in I} V_\alpha\right) = U \cap (\bigcap_{i=1}^{n} V_{\alpha i})$. By Proposition 3.9, $U \cap \left(\bigcap_{\alpha \in I} V_\alpha\right)$ is $\gamma^{\star}_{\hat{\Omega}}$-open set.

ii) Let U_α be a $\gamma^{\star}_{\hat{\Omega}}$-open set for any $\alpha \in I$ and V_β be a non-empty finite $\gamma^{\star}_{\hat{\Omega}}$-open set for any $\beta \in J$.By Theorem 3.6, $(\bigcup_{\beta \in J} V_\beta)$ is $\gamma^{\star}_{\hat{\Omega}}$-open set. By (i), $(\bigcup_{\beta \in J} V_\beta) \cap (\bigcap_{\alpha \in I} V_\alpha)$ is *$\gamma^{\star}_{\hat{\Omega}}$-open set in X.*

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