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Remarks on $\gamma^{\star}_{\hat{\Omega}}$-open sets and minimal $\gamma^{\star}_{\hat{\Omega}}$-open sets

$\gamma^{\star}_{\hat{\Omega}}$

K. Poorani, S. M. Meenarani, M. Anbuchelvi

Abstract*: *Aim of this paper is to define $\gamma^{\star}_{\hat{\Omega}}$-open sets in a topological space and obtain their basic properties. Also, we define minimal $\gamma^{\star}_{\hat{\Omega}}$-open sets in a space and study the impact of two minimal $\gamma^{\star}_{\hat{\Omega}}$-open sets in a space with $\hat{\Omega}$-regular operation. However, the roll of minimal $\gamma^{\star}_{\hat{\Omega}}$-open sets in $\gamma^{\star}_{\hat{\Omega}}$-locally finite space has been discussed.

Keywords* : *$\gamma^{\star}_{\hat{\Omega}}$-open set, $\gamma^{\star}_{\hat{\Omega}}$-Interior, $\gamma^{\star}_{\hat{\Omega}}$-Closure, minimal $\gamma^{\star}_{\hat{\Omega}}$-open set, $\gamma^{\star}_{\hat{\Omega}}$-locally finite space.

I. INTRODUCTION

In a topological space, the notion of minimal open sets had been introduced by Nakaoka and Oda[4] in 2001. In 2012, Lellis Thivagar et al.[1] introduced the class of $\hat{\Omega}$-closed sets in a space which is independent of closed sets. Recently, [3] operation on the class of $\hat{\Omega}$-open sets have been introduced and studied. In this paper, we introduce the class of $\gamma^{\star}_{\hat{\Omega}}$-open sets and investigate their basic properties in terms of it's closure. However, Nakaoka's idea of minimal open sets has been extended to $\gamma^{\star}_{\hat{\Omega}}$-open sets and some of its elementary properties have been derived. Moreover, the behaviour of minimal $\gamma^{\star}_{\hat{\Omega}}$-open sets in a $\gamma^{\star}_{\hat{\Omega}}$-locally finite space has been investigated.

II. PRELIMINARIES

Some definitions and results that are used in this paper have been given in this section. Always X or (X,τ) denotes a topological space on which no separation axioms assumed, unless otherwise stated. For any subset A of X, the closure (res.interior) of A is denoted by $cl(A)(res.int(A))$.

Definition 2.1 *[2] A subset A of a topological space (X,τ) is called a* **semi-open** *set if $A \subseteq cl(int(A))$. $SO(X)$ denotes the set of all semi-open sets in (X,τ). It's complement is known as* **semi-closed** *set on X.*

Definition 2.2 *([1], Definition 3.1) Let (X,τ) be a topological space. A is said to be* **$\hat{\Omega}$-closed set** *if $\delta cl(A) \subseteq U$ when $A \subseteq U$, where U is a semi-open subset of X. The complement of $\hat{\Omega}$-closed set is an $\hat{\Omega}$-open set. The family of all $\hat{\Omega}$-closed sets in a space (X,τ) is denoted by $\tau_{\hat{\Omega}}$. Also $\hat{\Omega}O(X,\tau)$ or $\hat{\Omega}O(X)$ (resp. $\hat{\Omega}C(X,\tau)$ or $\hat{\Omega}C(X)$) denotes the set of all $\hat{\Omega}$-open sets (resp. $\hat{\Omega}$-closed sets) on the space X.*

Definition 2.3 *([3], Definition 3.1) A function $\gamma : \hat{\Omega}O(X,\tau) \to P(X)$ is called an* **operation on $\hat{\Omega}O(X,\tau)$**, if $U \subseteq \gamma(U)$ for every set $U \in \hat{\Omega}O(X,\tau)$. For any operation $\gamma, \gamma(X) = X$, and $\gamma(\emptyset) = \emptyset$.

Definition 2.4 *([3], Definition 3.3) A non-empty set A of X is called* **$\gamma_{\hat{\Omega}}$-open set** *if for each $x \in A$, there exists an $\hat{\Omega}$ -open set U such that $x \in U$ and $\gamma(U) \subseteq A$. The complement of $\gamma_{\hat{\Omega}}$-open set is a $\gamma_{\hat{\Omega}}$-closed set. The set of all $\gamma_{\hat{\Omega}}$-open subsets of a topological space (X,τ) is denoted by $\tau_{\gamma_{\hat{\Omega}}}$.*

Definition 2.5 *([3], Definition 3.15) Let (X,τ) be a topological space. An operation γ is said to be* **$\hat{\Omega}$-regular** *if for every pair of sets $U,V \in \hat{\Omega}O(X,x)$, there exists an $\hat{\Omega}$-open set W containing x such that $\gamma(W) \subseteq \gamma(U) \cap \gamma(V)$.*

Remark 2.6 *([1], Remark 5.2) From the definition and Theorem 4.16, arbitrary intersection of an $\hat{\Omega}$-closed sets in a topological space (X,τ) is an $\hat{\Omega}$-closed set in (X,τ), $\hat{\Omega}cl(A)$ is the smallest $\hat{\Omega}$-closed set containing A.*

Theorem 2.7 *([3], Theorem 3.5) Arbitrary union of $\gamma_{\hat{\Omega}}$-open sets is a $\gamma_{\hat{\Omega}}$-open set in a topological space.*

Proposition 2.8 *([3], Proposition 3.8) Every $\gamma_{\hat{\Omega}}$-open set is an $\hat{\Omega}$-open in a space X.*

Proposition 2.9 *([3], Proposition 3.17) Intersection*

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* Correspondence Author
K.Poorani*, Research Scholar, Department of Mathematics, V.V.Vanniaperumal college for women,Virudhunagar. Email: kannapadaas@gmail.com
S.M.Meenarani, Associate Professor, Department of Mathematics, V.V.Vanniaperumal college for women,Virudhunagar. Email: smmeenarani@gmail.com
M.Anbuchelvi, Associate Professor, Department of Mathematics, V.V.Vanniaperumal college for women,Virudhunagar. Email: rsanbuchelvi@gmail.com

of any two $\gamma_{\hat{\Omega}}$-open sets is a $\gamma_{\hat{\Omega}}$-open in a space with an $\hat{\Omega}$-regular operation on $\hat{\Omega}O(X,\tau)$.

III. $\gamma^{\star}_{\hat{\Omega}}$--OPEN SETS

Definition 3.1 *A subset A of a space (X,τ) is said to be a* $\boldsymbol{\gamma^{\star}_{\hat{\Omega}}}$***-open*** *subset of X, if the following two axioms hold:*

i) A is a $\gamma_{\hat{\Omega}}$-open subset of X.

ii) For any $x \in A$, there exists an $\hat{\Omega}$-closed set F containing x such that $F \subseteq A$.

The complement of a $\gamma^{\star}_{\hat{\Omega}}$-open set is a $\gamma^{\star}_{\hat{\Omega}}$-closed set in X. However, $\gamma^{\star}_{\hat{\Omega}}O(X)$ (resp. $\gamma^{\star}_{\hat{\Omega}}C(X)$) denotes the set of all $\gamma^{\star}_{\hat{\Omega}}$-open sets (resp. $\gamma^{\star}_{\hat{\Omega}}$-closed sets) on X and $\gamma^{\star}_{\hat{\Omega}}O(X,x)$ denotes the set of all $\gamma^{\star}_{\hat{\Omega}}$-open sets containing x.

Example3.2 Consider a space with $X = \{a,b,c,d\}$ *and* $\tau = \{\emptyset,\{a\},\{c\},\{a,b\},\{a,c\},\{a,b,c\},\{a,c,d\},X\}$, Define $\gamma{:}\hat{\Omega}O(X,\tau) \to P(X)$ is defined by $\gamma(\emptyset) = \emptyset, \gamma(\{a\}) = \{a\}, \gamma(\{c\}) = \{c\}, \gamma(\{a,b\}) = \{a,b,c\}, \gamma(\{a,c\}) = \{a,c\}, \gamma(\{a,b,c\}) = X, \gamma(X) = X$ .Here, γ is an operation on $\hat{\Omega}O(X,\tau)$. $\tau_{\gamma_{\hat{\Omega}}} = \{\emptyset,\{a\},\{c\},\{a,c\},\{a,b,c\},X\}$, $\hat{\Omega}C(X) = \{\emptyset,\{d\},\{b,d\},\{c,d\},\{a,b,d\},\{b,c,d\},X\}$. Here, $\emptyset$ and X are the only $\gamma^{\star}_{\hat{\Omega}}$-open subsets of X.

Proposition 3.3 *In a space (X,τ), the following statements hold.*

i) Every $\gamma^{\star}_{\hat{\Omega}}$-open subset is a $\gamma_{\hat{\Omega}}$-open set.

ii) Every $\gamma^{\star}_{\hat{\Omega}}$-open set is an $\hat{\Omega}$-open set.

Proof. *i)* It follows straightly from the definition.
ii) It follows by combining *i)* and By Proposition 2.8

Remark 3.4 *From the following example, it is seen that there are some $\gamma_{\hat{\Omega}}$-open subsets which fail to be the $\gamma^{\star}_{\hat{\Omega}}$-open sets in a space (X,τ).*

Example 3.5 *Let* $X = \{a,b,c\}, \tau = \{\emptyset,\{a\},\{a,b\},X\}$; $\hat{\Omega}O(X) = \{\emptyset,\{a\},X\}$; $\hat{\Omega}C(X) = \{\emptyset,\{b,c\},X\}$.

Define an operation $\gamma{:}\hat{\Omega}O(X) \to P(X)$ as follows:

$$\gamma(W) = \begin{cases} W & \text{if } W = \{a\} \text{ or } \emptyset \\ X & \text{Otherwise} \end{cases}$$

Here , $\tau_{\gamma_{\hat{\Omega}}} = \{\emptyset,\{a\},X\}$; $\gamma^{\star}_{\hat{\Omega}}O(X) = \{\emptyset,X\}$.

Now $\{a\} \in \tau_{\gamma_{\hat{\Omega}}}$ *but* $\{a\} \notin \gamma^{\star}_{\hat{\Omega}}O(X)$.

Theorem 3.6. *Arbitrary union of $\gamma^{\star}_{\hat{\Omega}}$-open sets in a space (X,τ) is a $\gamma^{\star}_{\hat{\Omega}}$-open set.*

Proof. Let $\{U_i / i \in I\}$ be any collection of $\gamma^{\star}_{\hat{\Omega}}$-open sets in a space X and let $U = \bigcup_{i \in I} U_i$. By Theorem 2.7, U is a $\gamma_{\hat{\Omega}}$-open subset of X. Let $x \in U$ be arbitrary. Then $x \in U_i$ for some $i \in I$. Since U_i is a $\gamma^{\star}_{\hat{\Omega}}$-open subset of X, there exist an $\hat{\Omega}$-closed set F containing x such that $F \subseteq U_i \subseteq U$. Therefore, U is a $\gamma^{\star}_{\hat{\Omega}}$-open subset of X.

Remark 3.7. *The intersection of any two $\gamma^{\star}_{\hat{\Omega}}$-open sets is not always a $\gamma^{\star}_{\hat{\Omega}}$-open set as seen from the following example.*

Example 3.8. *Let $X = \{a,b,c\}$ and $\tau = P(X)$. Define an operation $\gamma{:}\hat{\Omega}O(X) \to P(X)$ as follows:*

$$\gamma(W) = \begin{cases} W & \text{if } W = \{a,b\} \text{ or } \{a,c\} \text{ or } \emptyset \\ X & \text{Otherwise} \end{cases}$$

$\gamma^{\star}_{\hat{\Omega}}O(X) = \{\emptyset,\{a,b\},\{a,c\},X\}$. *Here, the two sets $\{a,b\}$ and $\{a,c\}$ are $\gamma^{\star}_{\hat{\Omega}}$-open sets but $\{a\}$ is not a $\gamma^{\star}_{\hat{\Omega}}$-open in X.*

Proposition 3.9. *If U and V are $\gamma^{\star}_{\hat{\Omega}}$-open sets in a space X, then $U \cap V$ is a $\gamma^{\star}_{\hat{\Omega}}$-open set provided γ is an $\hat{\Omega}$-regular operation.*

Proof. Let U and V be any two $\gamma^{\star}_{\hat{\Omega}}$-open subsets of X. By Proposition 2.9, $U \cap V$ is a $\gamma_{\hat{\Omega}}$-open subset of X. Let $x \in U \cap V$ be arbitrary. Since U and V are $\gamma^{\star}_{\hat{\Omega}}$-open subsets of X, there exists two $\hat{\Omega}$-closed sets F_1 and F_2 containing x such that $F_1 \subseteq U$ and $F_2 \subseteq V$. By choosing F as $F_1 \cap F_2$, F is an $\hat{\Omega}$-closed set containing x such that $F \subseteq U \cap V$. Therefore, $U \cap V$ is a $\gamma^{\star}_{\hat{\Omega}}$-open subset of X.

$\gamma^{\star}_{\hat{\Omega}}$-Closure and $\gamma^{\star}_{\hat{\Omega}}$-Interior of a set have been defined in the usual manner as follows:

Definition 3.10. $\boldsymbol{\gamma^{\star}_{\hat{\Omega}}}$***-closure*** *of any subset A of a space (X,τ) is denoted by $\gamma^{\star}_{\hat{\Omega}}cl(A)$ and defined by*

$$\gamma^{\star}_{\hat{\Omega}}cl(A) = \bigcap\{F / F \in \gamma^{\star}_{\hat{\Omega}}C(X) \text{ such that } A \subseteq F\}.$$

Always $\gamma^{\star}_{\hat{\Omega}}cl(\emptyset) = \emptyset$; $\gamma^{\star}_{\hat{\Omega}}cl(X) = X$; $\gamma^{\star}_{\hat{\Omega}}cl(A)$ is a $\gamma^{\star}_{\hat{\Omega}}$-closed subset of X and $\gamma^{\star}_{\hat{\Omega}}(\gamma^{\star}_{\hat{\Omega}}cl(A)) = \gamma^{\star}_{\hat{\Omega}}cl(A)$.

Definition 3.11. $\boldsymbol{\gamma^{\star}_{\hat{\Omega}}}$***-interior*** *of any subset A of a space (X,τ) is denoted by $\gamma^{\star}_{\hat{\Omega}}int(A)$ and defined by*

$$\gamma^{\star}_{\hat{\Omega}}int(A) = \bigcup\{U / U \in \gamma^{\star}_{\hat{\Omega}}O(X) \text{ such that } U \subseteq A\}.$$

Always, $\gamma^{\star}_{\hat{\Omega}}int(\emptyset) = \emptyset$; $\gamma^{\star}_{\hat{\Omega}}int(X) = X$ and $\gamma^{\star}_{\hat{\Omega}}int(A)$ is a $\gamma^{\star}_{\hat{\Omega}}$-open subset of X.

Proposition 3.12. *If U and V are any two subsets of a space X, then the following statements are true.*

i) $U \subseteq V \Rightarrow \gamma^{\star}_{\hat{\Omega}}int(U) \subseteq \gamma^{\star}_{\hat{\Omega}}int(V)$.

ii) U is $\gamma_{\hat{\Omega}}$-open iff $U = \gamma^{\star}_{\hat{\Omega}}int(U)$.

iii) $U \subseteq V \Rightarrow \gamma_{\hat{\Omega}}^{\star}cl(U) \subseteq \gamma_{\hat{\Omega}}^{\star}cl(V)$.

iv) U is $\gamma_{\hat{\Omega}}^{\star}$-closed iff $U = \gamma_{\hat{\Omega}}^{\star}cl(U)$.

v) $X \setminus \gamma_{\hat{\Omega}}^{\star}int(U) = \gamma_{\hat{\Omega}}^{\star}cl(X \setminus U)$.

vi) $X \setminus \gamma_{\hat{\Omega}}^{\star}cl(U) = \gamma_{\hat{\Omega}}^{\star}int(X \setminus U)$.

Proof. It follows from the definition.

Proposition 3.13. *For any point x in a space X and for any subset A of X, $x \in \gamma_{\hat{\Omega}}^{\star}cl(A)$* iff $U \cap A \neq \emptyset$ for any $U \in \gamma_{\hat{\Omega}}^{\star}O(X,x)$.

Proof. Assume that $x \in \gamma_{\hat{\Omega}}^{\star}cl(A)$ and there exists a $\gamma_{\hat{\Omega}}^{\star}$-open set U containing x such that $U \cap A = \emptyset$. Now, $X \setminus U$ is a $\gamma_{\hat{\Omega}}^{\star}$ -closed set such that $A \subseteq X \setminus U$. That is, $X \setminus U \in \{F / F \in \gamma_{\hat{\Omega}}^{\star}C(X);\ A \subseteq F\}$. In particular, $\gamma_{\hat{\Omega}}^{\star}cl(A) \subseteq X \setminus U$, a contradiction to $x \in \gamma_{\hat{\Omega}}^{\star}cl(A)$.

Conversely,on contrary $x \notin \gamma_{\hat{\Omega}}^{\star}cl(A) = \bigcap\{F / F \in \gamma_{\hat{\Omega}}^{\star}C(X);\ A \subseteq F\}$. Then $x \notin F$ for some $\gamma_{\hat{\Omega}}^{\star}$-closed set F such that $A \subseteq F$. If U is chosen as $X \setminus F$, then U is a $\gamma_{\hat{\Omega}}^{\star}$-open subset of X such that $U \cap A = \emptyset$, a contradiction to hypothesis.

IV. MINIMAL $\gamma_{\hat{\Omega}}^{\star}$–OPEN SETS AND $\gamma_{\hat{\Omega}}^{\star}$–LOCALLY FINITE SPACE

Definition 4.1. *A $\gamma_{\hat{\Omega}}^{\star}$- open subset U of a space X is said to be a* **minimal $\gamma_{\hat{\Omega}}^{\star}$-open set** *if $\emptyset$ and U are the only $\gamma_{\hat{\Omega}}^{\star}$-open subsets of U.*

Example 4.2 *Let $X = \{a,b,c\}$ and $\tau = P(X) = \hat{\Omega}O(X,\tau)$. Define an operation $\gamma : \hat{\Omega}O(X) \rightarrow P(X)$ as follows:*

$$\gamma(W) = \begin{cases} W & \text{if } W = \{a,b\} \text{ or } \{a,c\} \text{ or } \emptyset \\ X & \text{Otherwise} \end{cases}$$

$\gamma_{\hat{\Omega}}^{\star}O(X) = \{\emptyset,\{a,b\},\{a,c\},X\} = \gamma_{\hat{\Omega}}O(X)$. *Here $\{a,b\}$ and $\{a,c\}$ are minimal $\gamma_{\hat{\Omega}}^{\star}$-open sets in X.*

Proposition 4.3. *Let (X,τ) be any topological space. If U is a non-empty $\gamma_{\hat{\Omega}}^{\star}$ -open subset of X such that $\gamma_{\hat{\Omega}}^{\star}cl(U) = \gamma_{\hat{\Omega}}^{\star}cl(V)$ for any non-empty subset V of U, then U is a minimal $\gamma_{\hat{\Omega}}^{\star}$-open set.*

Proof. On contrary, U is assumed to be a non-minimal $\gamma_{\hat{\Omega}}^{\star}$-open subset of X such that $\gamma_{\hat{\Omega}}^{\star}cl(U) = \gamma_{\hat{\Omega}}^{\star}cl(V)$ for any non-empty subset V of U. By the definition of minimal open set, there exists a non-empty $\gamma_{\hat{\Omega}}^{\star}$-open set W such that $W \subsetneq U$. Choose $x \in U \setminus W$. .Now W is a $\gamma_{\hat{\Omega}}^{\star}$ *-open set such that $x \notin W$ .Therefore,* $W \cap \gamma_{\hat{\Omega}}^{\star}cl(\{x\}) = \emptyset$ *or* $\gamma_{\hat{\Omega}}^{\star}cl(\{x\}) \subseteq X \setminus W$. *Now $\{x\}$ is a non-empty subset of U. By hypothesis,* $\gamma_{\hat{\Omega}}^{\star}cl(U) = \gamma_{\hat{\Omega}}^{\star}cl(\{x\}) \subseteq X \setminus W$. *However,* $W \subsetneq U \subseteq \gamma_{\hat{\Omega}}^{\star}cl(U) = \gamma_{\hat{\Omega}}^{\star}cl(\{x\}) \subseteq X \setminus W$ Which implies that $W \subseteq X \setminus W$, a contradiction.

Proposition 4.4. *Let γ be an $\hat{\Omega}$-regular operation in a space X. If U is a minimal $\gamma_{\hat{\Omega}}^{\star}$-open set and V is a $\gamma_{\hat{\Omega}}^{\star}$-open set then either $U \cap V = \emptyset$ or $U \subseteq V$.*

Proof. *Assume that $U \cap V = \emptyset$. By Proposition 3.9, $U \cap V$ is a $\gamma_{\hat{\Omega}}^{\star}$-open set. By the definition of minimal $\gamma_{\hat{\Omega}}^{\star}$-open sets, $U \cap V = V$. Thus $U \subseteq V$.*

Proposition 4.5. *Let γ be an $\hat{\Omega}$-regular operation in a space X. If U and V are any two minimal $\gamma_{\hat{\Omega}}^{\star}$-open sets, then either $U \cap V = \emptyset$ or $U = V$*

Proof. Assume that $U \cap V = \emptyset$. Apply Proposition 4.4 for both minimal $\gamma_{\hat{\Omega}}^{\star}$-open sets U and V. Then $U = V$.

Proposition 4.6. *Let γ be an $\hat{\Omega}$-regular operation in a space X and U be a minimal $\gamma_{\hat{\Omega}}^{\star}$-open set of X. Then for any $x \in U$, $U \subseteq V$ where V is any $\gamma_{\hat{\Omega}}^{\star}$-open set containing x.*

Proof. Assume the contrary that there exists $x \in U$ such that $U \not\subseteq V$ for any $\gamma_{\hat{\Omega}}^{\star}$-open subset V containing x. By Proposition 3.9, $U \cap V$ is a $\gamma_{\hat{\Omega}}^{\star}$ -open subset of X. As $x \in U \cap V, U \cap V \neq \emptyset$ such that $U \cap V \subseteq U$, a contradiction to U is a minimal $\gamma_{\hat{\Omega}}^{\star}$-open subset.

Proposition 4.7. *If U is a minimal $\gamma_{\hat{\Omega}}^{\star}$-open subset of a space X*, then $U = \bigcap_{x \in U}\{V / V \in \gamma_{\hat{\Omega}}^{\star}O(X,x)\}$.

Proof. By Proposition 4.6, for any $x \in U$, $U \subseteq V$ for any $\gamma_{\hat{\Omega}}^{\star}$ -open subset V containing x. Then $U \subseteq \bigcap\{V / V \in \gamma_{\hat{\Omega}}^{\star}O(X,x)\}$. In particular, $\bigcap\{V / V \in \gamma_{\hat{\Omega}}^{\star}O(X,x)\} \subseteq U$ .Now $U = \bigcap\{V / V \in \gamma_{\hat{\Omega}}^{\star}O(X,x)\}$.

Proposition 4.8. *Let γ be an $\hat{\Omega}$-regular operation of a space X. If U is a minimal $\gamma_{\hat{\Omega}}^{\star}$-open subset of X, then for any $x \in X \setminus U$, and for any $\gamma_{\hat{\Omega}}^{\star}$-open set V containing x, either $V \cap U = \emptyset$ or $U \subseteq V$.*

Proof. It follows from Proposition 4.4.

Proposition 4.9. *Let γ be an $\hat{\Omega}$-regular operation in a space X. If U is a non-empty minimal $\gamma_{\hat{\Omega}}^{\star}$-open subset of X, then $U \subseteq \gamma_{\hat{\Omega}}^{\star}cl(V)$ for any non-empty subset V of U.*

Proof. Let $x \in U$ be arbitrary and W be any $\gamma_{\hat{\Omega}}^{\star}$ -open set containing x. By Proposition 4.4, either $U \cap W = \emptyset$ or $U \subseteq W$. Since $x \in U \cap W$, the only possibility is $U \subseteq W$. Then, $V = U \cap V \subseteq W \cap V$. As $V \neq \emptyset, W \cap V \neq \emptyset$. By Proposition 3.13, $x \in \gamma_{\hat{\Omega}}^{\star} cl(V)$.

Characterization of minimal $\gamma_{\hat{\Omega}}^{\star}$-open set in a space is given as follows.

Theorem 4.10. *Let γ be an $\hat{\Omega}$-regular operation in a space X and U be any non-empty $\gamma_{\hat{\Omega}}^{\star}$-open subset of X. Then the following statements are equivalent:*

i) U is a minimal $\gamma_{\hat{\Omega}}^{\star}$-open set.

ii) $U \subseteq \gamma_{\hat{\Omega}}^{\star} cl(V)$ for any non-empty subset V of U.

iii) $\gamma_{\hat{\Omega}}^{\star} cl(U) = \gamma_{\hat{\Omega}}^{\star} cl(V)$ for any non-empty subset V of U.

Proof. $i) \Rightarrow ii)$ It follows from Proposition 4.9.

$ii) \Rightarrow iii)$ Let V be any non-empty subset of U. By hypothesis $U \subseteq \gamma_{\hat{\Omega}}^{\star} cl(V)$ and hence $\gamma_{\hat{\Omega}}^{\star} cl(U) \subseteq \gamma_{\hat{\Omega}}^{\star}(\gamma_{\hat{\Omega}}^{\star} cl(V)) = \gamma_{\hat{\Omega}}^{\star} cl(V)$. On the other hand, $V \subseteq U \Rightarrow \gamma_{\hat{\Omega}}^{\star} cl(V) \subseteq \gamma_{\hat{\Omega}}^{\star} cl(U)$.

$iii) \Rightarrow i)$ On contrary, U is assumed to be a non-minimal $\gamma_{\hat{\Omega}}^{\star}$-open subset. Then, a non-empty $\gamma_{\hat{\Omega}}^{\star}$-open set V can be chosen such that $V \subsetneqq U$. Having chosen $x \in U \backslash V$, $V \cap \gamma_{\hat{\Omega}}^{\star} cl(\{x\}) = \emptyset$ and hence $\gamma_{\hat{\Omega}}^{\star} cl(\{x\}) \subseteq X \backslash V$. But by hypothesis, $\gamma_{\hat{\Omega}}^{\star} cl(\{x\}) = \gamma_{\hat{\Omega}}^{\star} cl(U)$ .It is concluded that $V \subseteq U \subseteq \gamma_{\hat{\Omega}}^{\star} cl(U) = \gamma_{\hat{\Omega}}^{\star} cl(\{x\}) \subseteq X \backslash V$, a contradiction. Therefore, U is a minimal $\gamma_{\hat{\Omega}}^{\star}$-open set.

Definition 4.11. *A space X is said to be **$\gamma_{\hat{\Omega}}^{\star}$-locally finite space**, if for every $x \in X$ there exists a finite $\gamma_{\hat{\Omega}}^{\star}$-open set U in X such that $x \in U$.Every finite topological space is always $\gamma_{\hat{\Omega}}^{\star}$-locally finite space.*

Proposition 4.12. *Every non-empty finite $\gamma_{\hat{\Omega}}^{\star}$-open set in a space X contains some non-empty finite minimal $\gamma_{\hat{\Omega}}^{\star}$-open set.*

Proof. Let V be any non-empty finite $\gamma_{\hat{\Omega}}^{\star}$-open set in X. If V is a minimal $\gamma_{\hat{\Omega}}^{\star}$-open set in X, then it meets our requirement. Suppose that V is not a minimal $\gamma_{\hat{\Omega}}^{\star}$-open set in X. Then, there exists a non-empty finite $\gamma_{\hat{\Omega}}^{\star}$-open set W_1 such that $W_1 \subsetneqq V$. If W_1 is a minimal $\gamma_{\hat{\Omega}}^{\star}$-open set, then the condition is satisfied. If not, by proceeding like this we would arrive at a sequence of non-empty finite $\gamma_{\hat{\Omega}}^{\star}$-open sets $W_1, W_2, \ldots W_n \ldots$ such that $\ldots \subsetneqq W_n \ldots \subsetneqq W_2 \subsetneqq W_1 \subsetneqq V$. Since V is finite, there exists $m \in \mathbb{N}$ such that W_m is a minimal $\gamma_{\hat{\Omega}}^{\star}$-open set such that $W_m \subseteq V$. Hence the result holds.

Proposition 4.13. *Let γ be an $\hat{\Omega}$-regular operation in a $\gamma_{\hat{\Omega}}^{\star}$-locally finite space X. Then every non-empty $\gamma_{\hat{\Omega}}^{\star}$-open set in a space X contains some non-empty finite minimal $\gamma_{\hat{\Omega}}^{\star}$-open set.*

Proof. Let V be any non-empty $\gamma_{\hat{\Omega}}^{\star}$-open subset of X. Then choose $x \in V$. By the definition of $\gamma_{\hat{\Omega}}^{\star}$-locally finite space, there exists a finite $\gamma_{\hat{\Omega}}^{\star}$-open set U such that $x \in U$. Since U is finite, $U \cap V$ is also finite. By Proposition 3.9, $U \cap V$ is a finite $\gamma_{\hat{\Omega}}^{\star}$-open set containing x. By Proposition 4.12, there exists non-empty finite minimal $\gamma_{\hat{\Omega}}^{\star}$-open set W such that $W \subseteq U \cap V \subseteq V$.

Proposition 4.14. *Let X be a topological space and γ be an $\hat{\Omega}$-regular operation on X. Then the following are true.*

i) Let V_α, $\alpha \in I$ be any $\gamma_{\hat{\Omega}}^{\star}$-open set and U be a non-empty finite $\gamma_{\hat{\Omega}}^{\star}$-open set in X. Then $U \cap \left(\bigcap_{\alpha \in I} V_\alpha\right)$ is a finite $\gamma_{\hat{\Omega}}^{\star}$-open set.

ii) Let V_α be a $\gamma_{\hat{\Omega}}^{\star}$-open set for any $\alpha \in I$ and V_β be a non-empty finite $\gamma_{\hat{\Omega}}^{\star}$-open set for any $\beta \in J$. Then $(\bigcup_{\beta \in J} V_\beta) \cap (\bigcap_{\alpha \in I} V_\alpha)$ is a $\gamma_{\hat{\Omega}}^{\star}$-open set.

Proof. $i)$ Let V_α be a $\gamma_{\hat{\Omega}}^{\star}$-open subset for any $\alpha \in I$ and U be a non-empty finite $\gamma_{\hat{\Omega}}^{\star}$-open set in X. Since U is finite, $U \cap \left(\bigcap_{\alpha \in I} V_\alpha\right)$ is a finite set. Then, there exists a positive integer n such that $U \cap \left(\bigcap_{\alpha \in I} V_\alpha\right) = U \cap (\bigcap_{i=1}^{n} V_{\alpha i})$. By Proposition 3.9, $U \cap \left(\bigcap_{\alpha \in I} V_\alpha\right)$ is $\gamma_{\hat{\Omega}}^{\star}$-open set.

$ii)$ Let U_α be a $\gamma_{\hat{\Omega}}^{\star}$-open set for any $\alpha \in I$ and V_β be a non-empty finite $\gamma_{\hat{\Omega}}^{\star}$-open set for any $\beta \in J$.By Theorem 3.6, $(\bigcup_{\beta \in J} V_\beta)$ is $\gamma_{\hat{\Omega}}^{\star}$-open set. By (i), $(\bigcup_{\beta \in J} V_\beta) \cap (\bigcap_{\alpha \in I} V_\alpha)$ is *$\gamma_{\hat{\Omega}}^{\star}$-open set in X.*

REFERENCES

1. M.Lellis Thivagar, and M.Anbuchelvi, "Note on $\hat{\Omega}$-closed sets in topological spaces", Mathematical Theory and Modeling, 2012, pp.50-58.
2. N.Levine, "Semi-open sets and semi-continuity in topological spaces", Amer. Math. Monthly, 70,1, 1963, pp.36-41.

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DOI:10.35940/ijeat.A1219.1291S419

3. S.M.Meenarani, K.Poorani, and M.Anbuchelvi, "Operation on $\hat{\Omega}$-closed sets", Malaya Journal of Matematik, Vol. S. No. 1, 2019, pp.7-11.
4. F.Nakaoka, and N.Oda, "Some applications of minimal open sets", Int. Math. Math. Sci. 27, 2001, No.8, pp.471-476.
5. S.F.Namiq, "New types of continuity and separation axioms based on operation in topological spaces", M.sc. Thesis, University of Sulaimani, 2011.

AUTHORS PROFILE

K. Poorani, Research Scholar, Department of Mathematics, V. V. Vanniaperumal college for women, Virudhunagar.