

# More on Coprime Irregular Graphs

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**Abstract:** An  $k$ -edge-weighting of a graph  $G = (V, E)$  is a map  $\varphi: E(G) \rightarrow \{1, 2, 3, \dots, k\}$ , where  $k$  is a positive integer. Denote  $S_\varphi(v)$  is the sum of edge-weights presenting on the edges incident at the vertex  $v$  under the edge-weighting  $\varphi$ . An  $k$ -edge-weighting of  $G$  is coprime irregular edge-weighting of  $G$  if  $\gcd(S_\varphi(u), S_\varphi(v)) = 1$  for every pair of adjacent vertices  $u$  and  $v$  in  $G$ . A graph  $G$  is coprime irregular if  $G$  admits a coprime irregular edge-weighting. In this paper, we discuss about coprime irregular edge-weighting for some families of graphs

**Keywords :** Irregular edge-weighting, coprime, corona graphs

## I. INTRODUCTION

By a graph  $G = (V, E)$ , we mean a finite, undirected graph with neither loops nor multiple edges. For graph theoretic terminology we refer to Chartrand and Lesniak [1]. The concept of labeling of graph is one of the fastest growing areas within graph theory which has been extensively studied. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Graph labelings were first introduced in the late 1960's. In the intervening years dozens of graph labeling have been studied in over 800 papers. An excellent updated survey of graph labeling is given in [2]. In this paper we discuss about coprime irregular edge-weighting for some families of graphs..

## II. COPRIME IRREGULAR GRAPHS

This section aims to prove some special classes of graphs such as ladder graphs, double triangular snakes and the corona of paths and cycles are coprime irregular graphs.

**Definition 2.1.** The Ladder graph  $L_n$  is the Cartesian product of  $P_n$  and  $P_2$ . (i.e),  $L_n = P_n * P_2$ , where  $*$  denotes the Cartesian product of graphs.

**Theorem 2.2.** The Ladder graph  $L_n$  ( $n \geq 3$ ) is coprime irregular graph for all  $n$ .

**Proof.** Let  $V(L_n) = \{u_1, u_2, u_3, \dots, u_n, u_1', u_2', \dots, u_n'\}$  and  $E(L_n) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i' u_{i+1}' : 1 \leq i \leq n-1\} \cup \{u_i u_i' : 1 \leq i \leq n\}$ . Now let us

Define an edge-weighting  $\varphi$  of  $L_n$  as follows.

Consider the following cases.

**Case(i):**  $n$  is even

For all  $i = 1, 2, 3, \dots, n-1$ ,

$$\varphi(u_i u_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \text{ (or)} 2 \pmod{4} \\ 2 & \text{otherwise} \end{cases}$$

and

$$\varphi(u_i' u_{i+1}') = \begin{cases} 2 & \text{if } i \equiv 1 \text{ (or)} 2 \pmod{4} \\ 4 & \text{otherwise} \end{cases}$$

Further, assign  $\varphi(u_i u_i') = 1$  for all  $1 \leq i \leq n$  and

$$\varphi(u_n v_n) = \begin{cases} 2 & \text{if } n \equiv 2 \text{ (or)} 3 \pmod{4} \\ 1 & \text{otherwise} \end{cases}$$

Then  $S_\varphi(u_1) = 2$  and  $S_\varphi(u_n) = 3$  and for all  $1 \leq i \leq n-1$ , we have

$$S_\varphi(u_i) = \begin{cases} 3 & \text{if } i \equiv 2 \pmod{4} \\ 4 & \text{if } i \equiv 1 \text{ (or)} 3 \pmod{4} \\ 5 & \text{otherwise} \end{cases}$$

and

$$S_\varphi(u_i') = \begin{cases} 5 & \text{if } i \equiv 2 \pmod{4} \\ 7 & \text{if } 1 \text{ (or)} 3 \pmod{4} \\ 9 & \text{otherwise} \end{cases}$$

Also,  $S_\varphi(u_1') = 3$  and

$$S_\varphi(u_n') = \begin{cases} 4 & \text{if } n \equiv 2 \pmod{4} \\ 5 & \text{otherwise} \end{cases}$$

Clearly, for any two adjacent vertices of  $L_n$  their weights are relatively prime and so  $L_n$  is coprime irregular (The graph  $L_4$  and it is coprime irregular edge-weighting  $\varphi$  is illustrated in Figure 1).

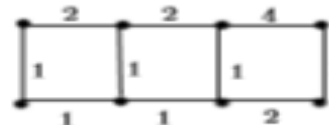


Figure 1.

**Case(ii):**  $n$  is odd

For all  $i = 1, 2, 3, \dots, n-1$ ,

$$\varphi(u_i u_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 2 \text{ (or)} 3 \pmod{4} \\ 2 & \text{otherwise} \end{cases}$$

and

$$\varphi(u_i' u_{i+1}') = \begin{cases} 2 & \text{if } i \equiv 2 \text{ (or)} 3 \pmod{4} \\ 4 & \text{otherwise} \end{cases}$$

Further, assign

$$\varphi(u_n v_n) = \begin{cases} 2 & \text{if } i \equiv 2 \text{ (or)} 3 \pmod{4} \\ 1 & \text{otherwise} \end{cases}$$

and also

$\varphi(u_i u_i') = 1$  for all  $1 \leq i \leq n$ .

Then  $S_\varphi(u_1) = S_\varphi(u_n) = 3$  and for all  $1 \leq i \leq n-1$ , we have

$$S_\varphi(u_i) = \begin{cases} 3 & \text{if } i \equiv 3 \pmod{4} \\ 4 & \text{if } i \equiv 0 \pmod{2} \\ 5 & \text{if } i \equiv 1 \pmod{4} \end{cases}$$

and

$$S_\varphi(u_i') = \begin{cases} 5 & \text{if } i \equiv 3 \pmod{4} \\ 7 & \text{if } i \equiv 0 \pmod{2} \\ 9 & \text{if } i \equiv 1 \pmod{4} \end{cases}$$

Also,  $S_\varphi(u_1') = 5$  and

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$$S_\varphi(u_n') = \begin{cases} 4 & \text{if } n \equiv 3 \pmod{4} \\ 5 & \text{otherwise} \end{cases}$$

Obviously, the weights of any two adjacent vertices of  $L_n$  are relatively prime (The graph  $L_5$ , the coprime irregular edge-weighting  $\varphi$  is illustrated in Figure 2). Hence  $L_n$  is coprime irregular.

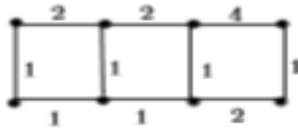


Figure 1.

**Definition 2.3.** A double triangular snake  $DT_n$  consist of two triangular snakes that have a common path.

**Theorem 2.4.** For all  $n \geq 3$ , the double triangular snake graph  $DT_n$  is coprime irregular graph.

**Proof:** Let

$$V(DT_n) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, v_1', v_2', \dots, v_n'\}$$

$$E(DT_n) = \{u_i u_{i+1} : 1 \leq i \leq n\}$$

$$\cup \{u_i v_i, u_i v_i' : 1 \leq i \leq n\} \cup \{u_{i+1} v_i, u_{i+1} v_i' : 1 \leq i \leq n\}.$$

Define an edge-weighting  $\varphi$  of  $DT_n$  as follows, for all

$$1 \leq i \leq n, \varphi(u_i u_{i+1}) = i \text{ and}$$

$$\varphi(u_i v_i) = \varphi(u_{i+1} v_i) = \varphi(u_i v_i') = 1 \text{ for all}$$

$$1 \leq i \leq n \text{ and } \varphi(u_{i+1} v_i') = 1 \text{ for all } 1 \leq i \leq n-1 \text{ and}$$

Moreover,

$$\varphi(v_n' u_{n+1}) = \begin{cases} 1 & \text{if } n \text{ is odd} \\ n-2 & \text{if } n \text{ is even} \end{cases}$$

$$\text{Then } S_\varphi(u_1) = 3 \text{ and } S_\varphi(u_{i+1}) = 2i + 3 \text{ for all}$$

$$2 \leq i \leq n \text{ and also}$$

$$S_\varphi(u_{n+1}) = \begin{cases} n+2 & \text{if } n \text{ is odd} \\ 2n-1 & \text{if } n \text{ is even} \end{cases}$$

$$\text{and } S_\varphi(v_i) = 2 \text{ for all } 1 \leq i \leq n \text{ and } S_\varphi(v_i') = 2 \text{ for all}$$

$$1 \leq i \leq n-1 \text{ and } S_\varphi(v_n') = n-1. \text{ It is clear that, the}$$

weighted degrees of any two adjacent vertices of  $DT_n$  under  $\varphi$  and so  $DT_n$  is coprime irregular edge weighting. (The graph  $DT_4$ , the coprime irregular edge-weighting  $\varphi$  is illustrated in Figure 3).

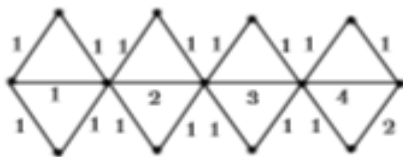


Figure 3.

**Definition 2.5.** The corona of a graph G is a graph obtain from G by attaching exactly one pendent edges at all vertices of G and its denoted by  $G^+$ .

**Theorem 2.6.** For all  $n \geq 3$ , the graph  $P_n^+$  is coprime irregular graph.

**Proof:** Let

$$V(P_n^+) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$$

$$E(P_n^+) = \{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{v_i u_i : 1 \leq i \leq n\}.$$

Define an edge-weighting  $\varphi$  of  $P_n^+$  as follows.

Consider the following cases.

**Case(i):** n is even

$$\text{For all } i = 1, 2, 3, \dots, n-1,$$

$$\varphi(v_i v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \text{ (or) } 2 \pmod{4} \\ 2 & \text{otherwise} \end{cases}$$

and  $\varphi(v_i u_i) = 1$  and also

$$\varphi(v_n u_n) = \begin{cases} 2 & \text{if } n \equiv 2 \pmod{4} \\ 1 & \text{if } n \equiv 0 \pmod{4} \end{cases}$$

Then  $S_\varphi(v_1) = 2$  and  $S_\varphi(v_n) = 3$  and for all

$1 \leq i \leq n-1$  we have

$$\varphi(v_i) = \begin{cases} 3 & \text{if } i \equiv 2 \pmod{4} \\ 4 & \text{if } i \equiv 3 \pmod{4} \\ 5 & \text{if } i \equiv 0 \pmod{4} \end{cases}$$

and  $S_\varphi(u_i) = 1$ . Moreover  $S_\varphi(u_n) = 1$  (or) 2 according as  $n \equiv 0$  (or) 2 (mod 4).

Hence, any two adjacent vertices of  $P_n^+$  whose weighting degrees are coprime and then  $\varphi$  is coprime irregular edge weighting of  $P_n^+$ .

**Case(ii):** n is odd

$$\text{For all } i = 1, 2, 3, \dots, n-1,$$

$$\varphi(v_i v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 2 \text{ (or) } 3 \pmod{4} \\ 2 & \text{otherwise} \end{cases}$$

and  $\varphi(v_i u_i) = 1$  and also

$$\varphi(v_n u_n) = \begin{cases} 2 & \text{if } n \equiv 3 \pmod{4} \\ 1 & \text{if } n \equiv 1 \pmod{4} \end{cases}$$

Then  $S_\varphi(v_1) = S_\varphi(v_n) = 3$  and for all

$1 \leq i \leq n-1$  we have

$$\varphi(v_i) = \begin{cases} 3 & \text{if } i \equiv 3 \pmod{4} \\ 4 & \text{if } i \equiv 0 \pmod{2} \\ 5 & \text{if } i \equiv 1 \pmod{4} \end{cases}$$

and  $S_\varphi(u_i) = 1$ . Moreover  $S_\varphi(u_n) = 1$  (or) 2 according as  $n \equiv 1$  (or) 3 (mod 4).

Clearly any two adjacent vertices of  $P_n^+$  whose weighted degrees are relatively prime and then  $\varphi$  is coprime irregular edge weighting of  $P_n^+$  (For the graph  $P_4^+$  and  $P_5^+$  the edge-weighting  $\varphi$  is illustrated in Figure 4)

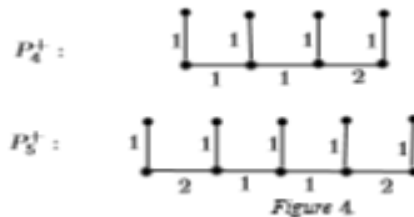


Figure 4.

**Theorem 2.8.** For each  $n \geq 3$ , the corona cycle  $C_n^+$  is coprime irregular graph.

**Proof.** Let  $V(C_n^+) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$  and  $E(C_n^+) = \{u_i u_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\} \cup \{u_1 u_n\}$ . Now let us Define an edge-weighting  $\varphi$  of  $C_n^+$  as follows.

Let  $\varphi(u_i u_{i+1}) = 1$  for all  $1 \leq i \leq n-1$  and  $\varphi(u_1 u_n) = 1$

Moreover, for all  $1 \leq i \leq n$ ,

$$\varphi(u_i v_i) = \begin{cases} 2i-1 & \text{if } n \equiv 0 \text{ or } 2 \pmod{3} \\ 2i+1 & \text{if } n \equiv 1 \pmod{3} \end{cases}$$

Then for all  $1 \leq i \leq n$ ,

$$S_\varphi(u_i) = \begin{cases} 2i+1 & \text{if } n \equiv 0 \text{ or } 2 \pmod{3} \\ 2i+3 & \text{if } n \equiv 1 \pmod{3} \end{cases}$$

$$S_\varphi(v_i) = \begin{cases} 2i-1 & \text{if } n \equiv 0 \text{ or } 2 \pmod{3} \\ 2i+1 & \text{if } n \equiv 1 \pmod{3} \end{cases}$$

Then the weighted degrees of any two adjacent vertices of  $C_n^+$  is coprime irregular. (For instance the graph  $C_4^+$  is coprime irregular the edge-weighting  $\varphi$  is given Figure 5)..

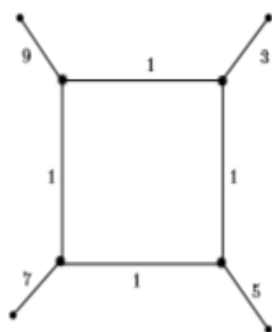


Figure 5.

### III. CONCLUSION

In this paper, we proved that some special cases of graphs such as ladder graphs, double triangular snakes and the corona of paths and cycles are coprime irregular graphs. Even there is a wide scope for further research in this topic.

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