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More on Coprime Irregular Graphs

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***Abstract:** An k−edge-weighting of a graph G = (V,E) is a map φ: E(G) → {1,2,3,...k}, where k is a positive integer. Denote Sφ(v) is the sum of edge-weights presenting on the edges incident at the vertex v under the edge-weighting φ. An k−edge-weighting of G is coprime irregular edge-weighting of G if gcd(Sφ(u),Sφ(v)) = 1 for every pair of adjacent vertices u and v in G. A graph G is coprime irregular if G admits a coprime irregular edge-weighting. In this paper, we discuss about coprime irregular edge-weighting for some families of graphs*

Keywords : Irregular edge-weighting, coprime, corona graphs

I. INTRODUCTION

By a graph G = (V,E), we mean a finite, undirected graph with neither loops nor multiple edges. For graph theoretic terminology we refer to Chartrand and Lesniak [1]. The concept of labeling of graph is one of the fastest growing areas within graph theory which has been extensively studied. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Graph labelings were first introduced in the late 1960's. In the intervening years dozens of graph labeling have been studied in over 800 papers. An excellent updated survey of graph labeling is given in [2]. In this paper we discuss about coprime irregular edge-weighting for some families of graphs..

II. COPRIME IRREGULAR GRAPHS

This section aims to prove some special classes of graphs such as ladder graphs, double triangular snakes and the corona of paths and cycles are coprime irregular graphs.

Definition 2.1. The Ladder graph L_n is the Cartesian product of P_n and P_2. (i.e), $L_n = P_n * P_2$, where $*$ denotes the Cartesian product of graphs.

Theorem 2.2. The Ladder graph L_n $(n \geq 3)$ is coprime irregular graph for all n.

Proof. Let V (L_n)={ $u_1, u_2, u_3, \dots, u_n, u_1', u_2', \dots, u_n'$ } and E(L_n) = { $u_i u_{i+1}$: $1 \leq i \leq n-1$} ∪ {$u_i' u_{i+1}'$: $1 \leq i \leq n-1$} ∪ {$u_i u_i'$: $1 \leq i \leq n$}. Now let us Define an edge-weighting φ of L_n as follows.

Consider the following cases.

Case(i): n is even

For all $i = 1,2,3,\dots,n-1,$

$$\varphi(u_i u_{i+1}) = \begin{cases} 1 & if\ i \equiv 1 (or) 2\ (mod 4) \\ 2 & otherwise \end{cases}$$

and

$$\varphi(u_i' u_{i+1}') = \begin{cases} 2 & if\ i \equiv 1 (or) 2\ (mod 4) \\ 4 & otherwise \end{cases}$$

Further, assign $\varphi(u_i u_i')$ = 1 for all $1 \leq i \leq n$ and

$$\varphi(u_n v_n) = \begin{cases} 2 & if\ n \equiv 2 (or) 3\ (mod 4) \\ 1 & otherwise \end{cases}$$

Then $S_\varphi(u_1) = 2$ and $S_\varphi(u_n)$=3 and for all $1 \leq i \leq n-1$, we have

$$S_\varphi(u_i) = \begin{cases} 3 & if\ i \equiv 2 (mod 4) \\ 4 & if\ i \equiv 1 (or) 3\ (mod 4) \\ 5 & otherwise \end{cases}$$

and

$$S_\varphi(u_i') = \begin{cases} 5 & if\ i \equiv 2 (mod 4) \\ 7 & if\ 1 (or) 3\ (mod 4) \\ 9 & otherwise \end{cases}$$

Also. $S_\varphi(u_1')$ = 3 and

$$S_\varphi(u_n') = \begin{cases} 4 & if\ n \equiv 2 (mod 4) \\ 5 & otherwise \end{cases}$$

Clearly, for any two adjacent vertices of L_n their weights are relatively prime and so L_n is coprime irregular (The graph L_4 and it is coprime irregular edge-weighting φ is illustrated in Figure 1).

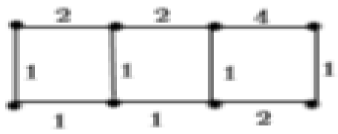

Figure 1.

Case(ii): n is odd

For all $i = 1,2,3,\dots,n-1,$

$$\varphi(u_i u_{i+1}) = \begin{cases} 1 & if\ i \equiv 2 (or) 3\ (mod 4) \\ 2 & otherwise \end{cases}$$

and

$$\varphi(u_i' u_{i+1}') = \begin{cases} 2 & if\ i \equiv 2 (or) 3\ (mod 4) \\ 4 & otherwise \end{cases}$$

Further, assign

$$\varphi(u_n v_n) = \begin{cases} 2 & if\ i \equiv 2 (or) 3\ (mod 4) \\ 1 & otherwise \end{cases}$$

and also

$\varphi(u_i u_i')$ = 1 for all $1 \leq i \leq n.$

Then $S_\varphi(u_1) = S_\varphi(u_n)$=3 and for all $1 \leq i \leq n-1$, we have

$$S_\varphi(u_i) = \begin{cases} 3 & if\ i \equiv 3 (mod 4) \\ 4 & if\ i \equiv 0\ (mod 2) \\ 5 & if\ i \equiv 1\ (mod 4) \end{cases}$$

and

$$S_\varphi(u_i') = \begin{cases} 5 & if\ i \equiv 3 (mod 4) \\ 7 & if\ i \equiv 0\ (mod 2) \\ 9 & if\ i \equiv 1\ (mod 4) \end{cases}$$

Also. $S_\varphi(u_1')$ = 5 and

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$S_\varphi(u_n') = \begin{cases} 4 & if\ n \equiv 3(mod4) \\ 5 & otherwise \end{cases}$

Obviously, the weights of any two adjacent vertices of L_n are relatively prime (The graphL_5, the coprime irregular edge-weighting φ is illustrated in Figure 2). Hence L_n is coprime irregular.

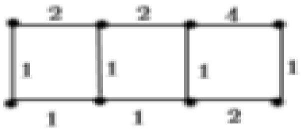

Figure 1.

Definition 2.3. A double triangular snake DT_nconsist of two triangular snakes that have a common path.

Theorem 2.4. For all $n \geq 3$, the double triangular snake graph DT_n is coprime irregular graph.

Proof: Let

V(DT_n)={ $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, v_1', v_2', \dots, v_n'$ } and E(DT_n) = { $u_iu_{i+1}: 1 \leq i \leq n$ } $\cup\{u_iv_i, u_iv_i': 1 \leq i \leq n\} \cup \{u_{i+1}v_i, u_{i+1}v_i': 1 \leq i \leq n\}$.

Define an edge-weighting φ of DT_n as follows, for all $1 \leq i \leq n$, $\varphi(u_iu_{i+1}) = i$ and $\varphi(u_iv_i) = \varphi(u_{i+1}v_i) = \varphi(u_iv_i') = 1$ for all $1 \leq i \leq n$ and $\varphi(u_{i+1}v_i') = 1$ for all $1 \leq i \leq n-1$ and Moreover,

$\varphi(v_n'u_{n+1}) = \begin{cases} 1 & if\ n\ is\ odd \\ n-2 & if\ n\ is\ even \end{cases}$

Then $S_\varphi(u_1) = 3$ and $S_\varphi(u_{i+1}) = 2i + 3$ for all $2 \leq i \leq n$ and also

$S_\varphi(u_{n+1}) = \begin{cases} n+2 & if\ n\ is\ odd \\ 2n-1 & if\ n\ is\ even \end{cases}$

and $S_\varphi(v_i) = 2$ for all $1 \leq i \leq n$ and $S_\varphi(v_i') = 2$ for all $1 \leq i \leq n-1$ and $S_\varphi(v_n') = n-1$. It is clear that, the weighted degrees of any two adjacent vertices of DT_n under φ and so DT_n is coprime irregular edge weighting.(The graph DT_4, the coprime irregular edge-weighting φ is illustrated in Figure 3).

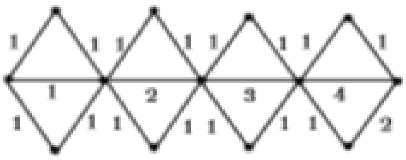

Figure 3.

Defintion 2.5. The corona of a graph G is a graph obtain from G by attaching exactly one pendent edges at all vertices of G and its denoted by G^+.

Theorem 2.6. For all $n \geq 3$, the graph P_n^+ is coprime irregular graph.

Proof: Let

V(P_n^+)={ $v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n$ } and E(P_n^+) = { $v_iv_{i+1}: 1 \leq i \leq n-1$ } $\cup\{v_iu_i: 1 \leq i \leq n$.

Define an edge-weighting φ of P_n^+ as follows.

Consider the following cases.

Case(i): n is even

For all $i = 1,2,3,\dots,n-1$,

$\varphi(v_iv_{i+1}) = \begin{cases} 1 & if\ i \equiv 1(or)2\ (mod4) \\ 2 & otherwise \end{cases}$

and $\varphi(v_iu_i) = 1$ and also

$\varphi(v_nu_n) = \begin{cases} 2 & if\ n \equiv 2\ (mod4) \\ 1 & if\ n \equiv 0\ (mod4) \end{cases}$

Then $S_\varphi(v_1) = 2$ and $S_\varphi(v_n) = 3$ and for all $1 \leq i \leq n-1$ we have

$\varphi(v_i) = \begin{cases} 3 & if\ i \equiv 2(mod4) \\ 4 & if\ i \equiv 3\ (mod2) \\ 5 & if\ i \equiv 0\ (mod4) \end{cases}$

and $S_\varphi(u_i) = 1$. Moreover $S_\varphi(u_n) = 1$(or)2 according as $n \equiv 0(or)2\ (mod\ 4)$.

Hence, any two adjacent vertices of P_n^+whose weighting degrees are coprime and then φ is coprime irregular edge weighting of P_n^+.

Case(ii): n is odd

For all $i = 1,2,3,\dots,n-1$,

$\varphi(v_iv_{i+1}) = \begin{cases} 1 & if\ i \equiv 2(or)3\ (mod4) \\ 2 & otherwise \end{cases}$

and $\varphi(v_iu_i) = 1$ and also

$\varphi(v_nu_n) = \begin{cases} 2 & if\ n \equiv 3\ (mod4) \\ 1 & if\ n \equiv 1\ (mod4) \end{cases}$

Then $S_\varphi(v_1) = S_\varphi(v_n) = 3$ and for all $1 \leq i \leq n-1$ we have

$\varphi(v_i) = \begin{cases} 3 & if\ i \equiv 3(mod4) \\ 4 & if\ i \equiv 0\ (mod2) \\ 5 & if\ i \equiv 1(mod4) \end{cases}$

and $S_\varphi(u_i) = 1$. Moreover $S_\varphi(u_n) = 1$(or)2 according as $n \equiv 1(or)3\ (mod\ 4)$.

Clearly any two adjacent vertices of P_n^+ whose weighted degrees are relatively prime and then φ is coprime irregular edge weighting of P_n^+ (For the graph $P_4^+$and $P_5^+$the edge-weighting φ is illustrated in Figure 4)

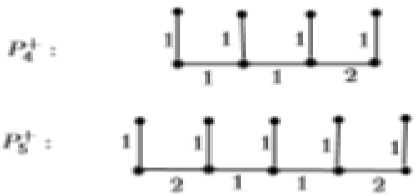

Figure 4.

Theorem 2.8. For each $n \geq 3$, the corona cycle C_n^+ is coprime irregular graph.

Proof. Let V (C_n^+)={ $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n$ } and E(C_n^+) = $\{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{u_1u_n\}$. Now let us Define an edge-weighting φ of C_n^+ as follows.

Let $\varphi(u_iu_{i+1}) = 1$ for all $1 \leq i \leq n-1$ and $\varphi(u_1u_n) = 1$

Moreover, for all $1 \leq i \leq n$.

$\varphi(u_iv_i) = \begin{cases} 2i-1 & if\ n \equiv 0\ or\ 2\ (mod3) \\ 2i+1 & if\ n \equiv 1(mod3) \end{cases}$

Then for all $1 \leq i \leq n$,

$S_\varphi(u_i) = \begin{cases} 2i+1 & if\ n \equiv 0\ or\ 2\ (mod3) \\ 2i+3 & if\ n \equiv 1(mod3) \end{cases}$

$S_\varphi(v_i) = \begin{cases} 2i-1 & if\ n \equiv 0\ or\ 2\ (mod3) \\ 2i+1 & if\ n \equiv 1(mod3) \end{cases}$

Then the weighted degrees of any two adjacent vertices of C_n^+ is coprime irregular.(For instance the graph C_4^+ is coprime irregular the edge-weighting φ is given Figure 5)..

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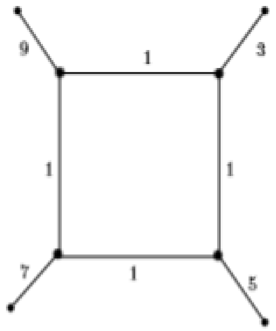

Figure 5.

III. CONCLUSION

In this paper, we proved that some special cases of graphs such as ladder graphs, double triangular snakes and the corona of paths and cycles are coprime irregular graphs. Even there is a wide scope for further research in this topic.

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