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# The Radial Radio Number and the Clique Number of a Graph

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***Abstract:*** ***Let*** *$G(V(G),E(G))$* ***be a graph. A radial radio labeling, f, of a connected graph G is an assignment of positive integers to the vertices satisfying the following condition:*** $d(u,v)+|f(u)-f(v)|\geq 1+r(G)$***, for any two distinct vertices*** $u,v\in V(G)$***, where*** $d(u,v)$ ***and*** $r(G)$ ***denote the distance between the vertices u and v and the radius of the graph G, respectively. The span of a radial radio labeling f is the largest integer in the range of f and is denoted by span(f). The radial radio number of G,*** $r(G)$***, is the minimum span taken over all radial radio labelingsof G. In this paper, we construct a graph a graph for which the difference between the radial radio number and the clique number is the given non negative integer.***

***Keywords: diameter, frequency assignment problem, radius, radio labeling, radio number, radial radio number, radial radio number.AMS Subject Classification Code(2010):05C78***

## I. INTRODUCTION

In this paper, by a graph, we mean only finite, simple, undirected and connected graph. For basic notations and terminology, we follow [4]. Let $G=(V(G),E(G))$ be a graph. The *distance* $d(u,v)$ between any two vertices $u$ and $v$, is the length of a shortest $(u,v)-$ path in $G$. The *eccentricity*, $e(u)$, of a vertex $u$ in $V(G)$ is the distance of a vertex farthest from $u$. The *radius* of a graph $G$ is the minimum eccentricity among all the vertices and is denoted by $r(G)$ or $r$. The *diameter* of $G$ is the maximum eccentricity among all the vertices and is denoted by $diam(G)$ or $d$. The relation between $r(G)$ and $diam(G)$ is given by the inequality $r(G)\leq diam(G)\leq 2r(G)$[8]. For further details on distance in graphs, one can refer [5].

For a subset $S$ of $V(G)$, let $<S>$ denote the induced subgraph of $G$ induced by $S$. A *clique* $C$ is a subset of $V(G)$ with maximum number of vertices such that $<C>$ is complete. The *clique number* of a graph $G$, denoted by $\omega(G)$ or $\omega$, is the number of vertices in a clique of $G$.

In 1960's Rosa[12] introduced the concept of graph labeling. A *graph labeling* is an assignment of numbers to the vertices or edges or both, satisfying some constraints. Rosa named the labeling introduced by him as $\beta-valuation$ and later on it becomes a very famous interesting graph labeling called *graceful labeling*, which is the origin for any graph labeling problem. Motivated by the real life problems, many mathematicians introduced various labeling concepts[9]. Here, we see one of the familiar graph labelings in graph theory.

The problem of assigning frequencies to the channels for the FM radio stations is known as *Frequency Assignment Problem* (FAP). This problem was studied by W. K. Hale[10].

In a telecommunication system, the assignment of channels to FM radio stations play a vital role. Motivated by the FAP, Chartrand et al.[6] introduced the concept of radio labeling. For a given $k$, $1\leq k\leq diam(G)$,a *radio k-coloring*, $f$, is an assignment of positive integers to the vertices satisfying the following condition:

$$d(u,v)+|f(u)-f(v)|\geq 1+k \qquad (1)$$

for any two distinct vertices $u,v\in V(G)$. Whenever, $diam(G)=k$, the radio k- coloring is called a *radio labeling*[7] of $G$. The *span* of a radio labeling $f$ is the largest integer in the range of $f$ and is denoted by $span(f)$. The *radio number* of $G$ is the minimum span taken over all radio labelings of $G$ and is denoted by $rn(G)$.

Motivated by the work of Chartrand et al., on radio labeling, KM. Kathiresan and S. Vimalajenifer[11] introduced the concept of radial radio labeling. A *radial radio labeling* $f$ of $G$ is a function $f:V\rightarrow\{1,2,...\}$ satisfying the condition,

$$d(u,v)+|f(u)-f(v)|\geq 1+r(G) \qquad (2)$$

for any two distinct vertices $u,v\in V(G)$. This condition is obtained by taking $k=r(G)$ in (1). The above condition is known as *radial radio condition*. The *span* of a radial radio labeling $f$ is the largest integer in the range of $f$. The *radial radio number* is the minimum span taken over all radial radio labelings of $G$ and is denoted by $rr(G)$.

That is, $rr(G)=\min_{f}\max_{v\in V(G)} f(v)$, where the minimum runs over all radial radio labelings of $G$.

Let $f$ be a radial radio labeling of a graph $G$ and let $C$ be a clique in $G$. Then the minimum label in $C$ under $f$ is denoted by $m_f(C)$. That is, $m_f(C)=\min_{v\in C} f(v)$.

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Also, the maximum of all such $m_f(C)$, where the maximum runs over all cliques in $G$ is denoted by $Cmm_f(G)$ and is called the *clique min max of G* under $f$.

In this paper, we construct some families of graphs with radial radio number $\omega + m$, for any given $m \geq 0$ and $\omega \geq 3$.

Now, we present some basic results, which are helpful for further investigation.

The following three theorems have been proved in [2].

**Theorem 1.1** Let $G$ be a simple connected graph with a full vertex. Then $rr(G) \geq \omega(G)$.

**Theorem 1.2** Let $G$ be any simple connected graph. Then $rr(G) \geq \Delta(r-1)+2$, where $\Delta$ and $r$ are the maximum degree and the radius of $G$, respectively.

**Theorem 1.3** Let $f$ be a radial radio labeling of a graph $G$. Then $span(f) \geq Cmm_f(G) + (\omega - 1)r$, where $\omega$ is the clique number of $G$ and $r$ is the radius of $G$.

The next theorem has been established in [3].

**Theorem 1.4** For any $G$, $rr(G) = 2$ if and only if $G \cong K_{1,n}$, $n \geq 1$.

And we stated the following theorem which has been proved in [11].

**Theorem 1.5** Let $W_n$, $n \geq 3$ be the wheel graph. Then

$$rr(W_n) = \begin{cases} 3, & \text{if n is even} \\ 4, & \text{if n is odd} \end{cases}$$

Throughout this paper, let $\oplus$ denote the addition modulo $n-1$.

## II. RADIAL RADIO NUMBER AND CLIQUE NUMBER

In this section, we construct graphs, for which the radial radio number is $\omega + m$, where $\omega \geq 3$ is the clique number and $m \geq 0$ is any given integer.

When $m = 0$, it is trivial that, $K_\omega$ is the required graph with radial radio number $\omega$. Therefore, we assume that, $m \geq 1$.

**Theorem 2.1** *For any given* $m \geq 1$, *there is a graph* $G$ *with* $\omega = 3$ *and* $rr(G) = \omega + m$.

**Proof**. Given that $\omega = 3$.

When $m = 1$, $W_n$, $n$ is odd is the required graph. We have, by Theorem 1.5 $rr(W_n) = 4$.

When $m = 2$, consider the graph $G'$ with vertex set $V(G') = \{x, y, z, w, v\}$ andedgeset $E(G') = \{xy, yz, xz, xw, vw\}$.

Define $g: V(G') \rightarrow \{1,2,3,...\}$ such that

$g(x) = 1$
$g(y) = 3$
$g(z) = 5$
$g(w) = 4$
$g(v) = 2$

It is obvious that, $g$ is a radial radio labeling for $G'$ and so $span\, g = 5$ and $rr(G) \leq 5$. Also $\Delta(G') = 3$ and $r = 2$, by Theorem 1.2, we have $rr(G') \geq 5$. Thus $rr(G') = 5$. Therefore, assume that $m \geq 3$. Consider a graph $G$ with vertex set $V(G) = \{x, y, z, w, v_1, v_2, ..., v_m\}$ and edge set $E(G) = \{xy, xz, yz, xw\} \cup \{wv_i : 1 \leq i \leq m\}$. We have $r = 2$. Define $f: V(G) \rightarrow \{1,2,3,...\}$ such that

$f(x) = 1$
$f(y) = 3$
$f(z) = 5$
$f(v_i) = i+1$, $1 \leq i \leq m$
$f(w) = f(v_m) + 2 = m+3$

Now, we have to show that $f$ is a radial radio labeling of $G$. That is, to prove that $f$ satisfies the following condition:

$$d(u,v) + |f(u) - f(v)| \geq 3 \qquad (3)$$

for any two distinct vertices $u, v \in V(G)$.

Since $d(x, y) = 1$ and $d(x, y) + |f(x) - f(y)| = 1 + |1-3| \geq 3$. Hence the pair $(x, y)$ satisfies (3). Similarly, the pairs $(x, z)$, $(y, z)$ and $(x, w)$ satisfy (3).

For the pairs $(v_i, v_j)$, $1 \leq i \neq j \leq m$, we have $d(v_i, v_j) + |f(v_i) - f(v_j)| = 2 + |i+1-(j+1)| \geq 3$.

Thus the pairs $(v_i, v_j)$, $1 \leq i \neq j \leq m$, satisfies (3).Since $d(w, v_i) + |f(w) - f(v_i)| = 1 + |m+3-(i+3)| > 3$, the pairs $(w, v_i)$, $1 \leq i \leq m$ satisfy (3).

From the above discussion, we conclude that $f$ is a radial radio labeling of $G$. This implies that, $span\ f = m+3$ and hence

$rr(G) \leq m+3$ (4)

Also, we have $\Delta(G) = m+1$ and $r = 2$, by Theorem 1.2,

$rr(G) \geq m+3$ (5)Combining the inequalities (4) and (5), we get $rr(G) = 3+m$. ■

**Theorem 2.2** For any given $m \geq \omega - 1 \geq 3$, there exists a graph $G$ with $rr(G) = \omega + m$.

**Proof**. For $m = 3$ and $\omega = 4$, consider the graph $G'$ with vertex set $V(G') = \{v_1, v_2, v_3, v_4, u_1, u_2\}$ and the edge set $E(G') = \{v_i v_j : 1 \leq i \neq j \leq 4\} \cup \{v_4 u_i : i = 1,2\}$. It is easy to verify that, $rr(G') = \omega + m = 7$.

Assume that, $\omega \geq 5$ and $m \geq 4$.

Consider the graph $G$ with vertex set $V(G) = \{v_1, v_2, ..., v_\omega, u_1, u_2, ..., u_{m-1}\}$ and the edge set $E(G) = \{v_i v_j : 1 \leq i \neq j \leq \omega\} \cup$

$\{v_i u_j : 1 \leq i \leq \lfloor \omega/2 \rfloor + 1 \text{ and } \lfloor \omega/2 \rfloor + 1 \leq j \leq m-1\} \cup$

$\{v_i u_j : \lfloor \omega/2 \rfloor + 2 \leq i \leq \omega \text{ and } 1 \leq j \leq \lfloor \omega/2 \rfloor\}$.

Here, $r(G) = 2$ .Define

$f : V(G) \to \{1,2,3,...\}$ such that

$f(v_i) = 2i-1$, $1 \le i \le \omega$

$f(u_i) = f(v_i)+1$, $1 \le i \le \lfloor \omega/2 \rfloor$

$f(u_j) = f(v_i)+1$, $\lfloor \omega/2 \rfloor + 2 \le i \le \omega$ and $\lfloor \omega/2 \rfloor + 1 \le j \le \omega - 1$

$f(u_j) = f(u_{j-1})+1$, $\omega \le j \le m$

Now,we have to show that $f$ satisfies the following condition:

$$d(u,v) + | f(u) - f(v) | \ge 3 \qquad (6)$$

for any two distinct vertices $u$ and $v$ of $G$.

**Case 1** Consider the pair $(v_i, v_j)$, $1 \le i \ne j \le \omega$.

Now, $d(v_i, v_j) + | f(v_i) - f(v_j) | = 1 + | (2i-1) - (2j-1) | \ge 3$.

Thus the pair $(v_i, v_j)$, $1 \le i \ne j \le \omega$ satisfies (6).

**Case 2** Consider the pair $(u_i, u_j)$, $1 \le i \ne j \le m-1$.

We have $d(u_i, u_j) \ge 2$, $1 \le i \ne j \le m-1$.

**Subcase 2a** If $1 \le i \ne j \le \omega - 1$, then $d(u_i, u_j) = 2$.

Now, $d(u_i, u_j) + | f(u_i) - f(u_j) | =$

$2 + | f(v_i) + 1 - (f(v_j)+1) | \ge 3$

**Subcase 2b** If $\lfloor \omega/2 \rfloor + 1 \le i \ne j \le \omega - 1$, then $d(u_i, u_j) = 2$.

Also, $d(u_i, u_j) + | f(u_i) - f(u_j) | =$

$2 + | f(v_s) + 1 - (f(v_t)+1) | \ge 3$, where $\lfloor \omega/2 \rfloor + 2 \le s \ne t \le \omega$.

**Subcase 2c** If $\omega \le i \ne j \le m-1$, then $d(u_i, u_j) = 2$.

We have, $d(u_i, u_j) + | f(u_i) - f(u_j) | =$

$2 + | f(u_{i-1}) + 1 - (f(u_{j-1})+1) | \ge 3$.

**Subcase 2d** If $1 \le i \le \lfloor \omega/2 \rfloor$ and $\lfloor \omega/2 \rfloor + 1 \le j \le \omega - 1$, then $d(u_i, u_j) = 3$.

Since $d(u_i, u_j) = 3$, it is obvious that the pair $(u_i, u_j)$, $1 \le i \le \lfloor \omega/2 \rfloor$ and $\lfloor \omega/2 \rfloor + 1 \le j \le \omega - 1$, satisfies (6).

Thus the pair $(u_i, u_j)$, $1 \le i \ne j \le m-1$ satisfies (6).

**Case 3** Consider the pair $(v_i, u_j)$, $1 \le i \le \omega$ and $1 \le j \le m-1$.It is easy to verify that the pair $(v_i, u_j)$, $1 \le i \le \omega$ and $1 \le j \le m-1$ satisfies (6).

Thus every pair of vertices of $G$ satisfies (6) and so $f$ is a radial radio labeling of $G$. Also, $span\ f = \omega + m$ and hence $rr(G) \le \omega + m$.

Also, by Theorem 1.3, we have $rr(G) \ge m + \omega$. Thus $rr(G) = \omega + m$. ■

**Theorem 2.3** For any given $\omega \ge 4$, there exists a graph $G$ with $rr(G) = \omega + 1$.

**Proof.**

Take $V(G) = \{x, v_1^{(1)}, v_2^{(1)}, ..., v_{\omega-1}^{(1)}, v_1^{(2)}, v_2^{(2)}, ..., v_{\omega-1}^{(2)}, ..., v_1^{(\omega)}, v_2^{(\omega)}, ..., v_{\omega-1}^{(\omega)}\}$ and

$E(G) = \{x v_i^{(j)} : 1 \le i \le \omega - 1, 1 \le j \le \omega\} \cup$

$\{v_i^{(j)} v_k^{(j)}, 1 \le i \ne k \le \omega - 1, 1 \le j \le \omega\} \cup$

$\{v_1^{(1)} v_1^{(j)} : 2 \le j \le \omega\} \cup$

$\{v_i^{(2)} v_k^{(i+1)} : 2 \le i \le \omega - 1, 2 \le k \le \omega - 1\}$.

Here, $rad(G) = 1$.Define $f : V(G) \to \{1,2,3,...\}$ such that

$f(x) = \omega$

$f(v_i^{(2)}) = i$, $1 \le i \le \omega - 1$

For $1 \le j \le \omega - 1$ and $2 \le i \le \omega - 1$,

$$f(v_j^{(i+1)}) = \begin{cases} f(v_j^{(i)}) + 1, & if\ f(v_j^{(i)}) + 1 \le \omega - 1 \\ f(v_j^{(i)}) \oplus 1, & \text{otherwise} \end{cases}$$

$f(v_1^{(1)}) = \omega + 1$

$f(v_j^{(1)}) = j - 1$, $2 \le j \le \omega - 1$

Now,we have to show that $f$ is a radial radio labeling for $G$. The radial radio condition for $G$ is

$$d(u,v) + | f(u) - f(v) | \ge 2 \quad (7)$$

for any two distinct vertices $u$ and $v$ of $G$.

**Case 1** Consider the pair $(x, v_1^{(1)})$. Since $d(x, v_1^{(1)}) = 1$, we have $d(x, v_1^{(1)}) + | f(x) - f(v_1^{(1)}) | = 1 + | \omega - (\omega+1) | \ge 2$.

Therefore, the pair $(x, v_1^{(1)})$ satisfies (7).

**Case 2** Consider the pair $(x, v_i^{(1)})$, $2 \le i \le \omega - 1$.

Since $d(x, v_i^{(1)}) = 1$, we have $d(x, v_i^{(1)}) + | f(x) - f(v_i^{(1)}) | = 1 + | \omega - (\omega+1) | \ge 2$. and hence the pair $(x, v_i^{(1)})$, $2 \le i \le \omega - 1$ satisfies (7).

**Case 3** Consider the pair $(x, v_i^{(j)})$, $2 \le i \le \omega - 1$ and $2 \le j \le \omega$. Here, $d(x, v_i^{(j)}) = 1$, we have $d(x, v_i^{(j)}) + | f(x) - f(v_i^{(j)}) | \ge 2$, since $f(v_i^{(j)}) \le \omega - 1$, for all $2 \le i \le \omega - 1$ and $2 \le j \le \omega$.Thus the pair $(x, v_i^{(j)})$, $2 \le i \le \omega - 1$ and $2 \le j \le \omega$ satisfies (7).

**Case 4** Consider the pair $(v_i^{(j)}, v_s^{(t)})$, $2 \le j, t \le \omega$, $1 \le i, s \le \omega - 1$.

**Subcase 4a** If $i = s$ and $j \ne t$, then $d(v_i^{(j)}, v_i^{(t)}) = 2$ and so it is easy to verify that the pair $(v_i^{(j)}, v_i^{(t)})$ satisfies (7).

**Subcase 4b** If $i \ne s$ and $j = t$, then $d(v_i^{(j)}, v_s^{(j)}) = 1$. Now,

$d(v_i^{(j+1)}, v_s^{(j+1)}) + | f(v_i^{(j+1)}) - f(v_s^{(j+1)}) | =$

$$\begin{cases} 1 + | \{ f(v_i^{(j)}) + 1\} - \{ f(v_s^{(j)}) + 1\} |, \text{ if } f(v_i^{(j)}) + 1, \\ \qquad f(v_s^{(j)}) + 1 \le \omega - 1 \\ 1 + | \{ f(v_i^{(j)}) \oplus 1\} - \{ f(v_s^{(j)}) + 1\} |, \text{if } f(v_i^{(j)}) + 1 > \omega - 1 \\ \qquad f(v_s^{(j)}) + 1 \le \omega - 1 \\ 1 + | \{ f(v_i^{(j)}) + 1\} - \{ f(v_s^{(j)}) \oplus 1\} |, \text{if } f(v_i^{(j)}) + 1 \le \omega - 1, \\ \qquad f(v_s^{(j)}) + 1 > \omega - 1 \\ 1 + | \{ f(v_i^{(j)}) \oplus 1\} - \{ f(v_s^{(j)}) \oplus 1\} |, \text{ if } f(v_i^{(j)}) + 1 > \omega - 1, \\ \qquad f(v_s^{(j)}) + 1 > \omega - 1 \end{cases}$$

This implies that, $d(v_i^{(j+1)}, v_s^{(j+1)}) + | f(v_i^{(j+1)}) - f(v_s^{(j+1)}) | \ge 2$ and hence the pair $(v_i^{(j)}, v_s^{(j)})$ satisfies (7).

Hence the pair $(v_i^{(j)}, v_s^{(t)})$,

$2 \le j,t \le \omega$, $1 \le i,s \le \omega-1$ satisfies (7).

**Case 5** Consider the pair $(v_i^{(1)}, v_k^{(j)})$, $2 \le j \le \omega$, $1 \le i,k \le \omega-1$.Since $d(v_i^{(1)}, v_k^{(j)}) = 2$, for all $2 \le j \le \omega$ and $1 \le i,k \le \omega-1$, it is easy to verify that the pair $(v_i^{(1)}, v_k^{(j)})$, $2 \le j \le \omega$, $1 \le i,k \le \omega-1$ satisfies (7).

From the above discussion, we conclude that $f$ is a radial radio labeling of $G$ and hence $span\ f = \omega+1$. This forces that, $rr(G) \le \omega+1$.

We can see that $G$ contains $\omega$ copies of $K_\omega$.

Let $V(K_\omega^{(i)}) = \{x, v_1^{(i)}, v_2^{(i)}, \ldots v_{\omega-1}^{(i)}\} : 1 \le i \le \omega$. Then from the definition of $G$, we have $d(x,v) = 1$, for all $v \in V(G) - \{x\}$. By Theorem 1.1, we have $rr(G) \ge \omega$. Suppose $rr(G) = \omega$. Then there exists a radial radio labeling $c$ such that $span\, c = \omega = rr(G)$.

Consider the copies $K_\omega^{(i)}$, $2 \le i \le \omega$. Since $d(v_j^{(i)}, v_k^{(i)}) = 1$, $1 \le j \ne k \le \omega-1$ and $r = 1$, the integers $1,2,3,\ldots,\omega$ are enough to label the vertices of $K_v^{(i)}$, for some $i$. First, if we label the vertices of $K_\omega^{(3)}$ with integers $1,2,3,\ldots,\omega$. Since $d(v_i^{(2)}, v_k^{(i+1)}) = 1$, $c(v_i^{(2)}) \ne c(v_k^{(i+1)})$, $2 \le i \le \omega-1$, $1 \le k \le \omega-1$. Also, $d(v_i^{(2)}, v_1^{(j)}) = 1$, $1 \le i \le \omega-1$, $3 \le j \le \omega$, so that $c(v_i^{(2)}) = c(v_1^{(j)})$. This happens only when $c(v_i^{(2)}) \ne c(v_1^{(k)})$, $3 \le j \ne k \le \omega$.

Suppose $c(v_i^{(2)}) = c(v_1^{(k)})$, for some $j$ and $k$, then we need a new label other than $1,2,3,\ldots\omega$, which is a contradiction to our assumption.

Consider $K_\omega^{(2)}$. Since $d(v_i^{(2)}, v_j^{(i+1)}) = 1$, $2 \le i \le \omega-1$, $2 \le j \le n$ and $d(v_i^2, v_j^{(k)}) = 2$, $2 \le i,k \le \omega-1$ and $k \ne i+1$, $c(v_i^{(2)}) = c(v_j^{(k)})$, $2 \le i,k \le \omega-1$, $k \ne i+1$. Also, $d(v_i^{(2)}, v_1^{(i+1)}) = 2$, $c(v_i^{(2)}) = c(v_1^{(i+1)})$, $2 \le i \le \omega-1$. Finally, we have to label $K_\omega^{(1)}$. Since $d(x, v_1^{(1)}) = 1$ and $d(v_1^{(1)}, v_i^{(1)}) = 1$, $2 \le i \le \omega-1$ and label of each $v_1^{(i)}$ is distinct, we can not label $v_1^{(1)}$ with integers $1,2,3,\ldots,\omega$, which is a contradiction. From the above discussion, we conclude that $rr(G) > \omega$, which implies $rr(G) \ge \omega$. This completes the proof. ■

The following corollary is the generalization of the above theorem.

**Corollary 2.4** For any given $\omega \ge 4$, and $2 \le m < \omega$, there exists a simple connected graph $G$ with $rr(G) = \omega + m$.

**Proof**. Consider the graph $G$ with $V(G)$ and $E(G)$ are defined as follows:

$$V(G) = \{x, v_i^{(j)},\ 1 \le i \le \omega-1,\ 1 \le j \le \omega\}$$

$$E(G) = \{xv_i^{(j)} : 1 \le i \le \omega-1, 1 \le j \le \omega\} \cup \{v_i^{(j)} v_k^{(j)}, 1 \le i \ne k \le \omega-1, 1 \le j \le \omega\} \cup \{v_i^{(1)} v_1^{(j)} : 1 \le i \le m, 2 \le j \le \omega\} \cup \{v_i^{(2)} v_k^{(i+1)} : 2 \le i \le \omega-1, 2 \le k \le \omega-1\}.$$

Here, $rad(G) = 1$.Define $f : V(G) \to \{1,2,3,\ldots\}$ such that

$$f(x) = \omega$$
$$f(v_i^{(1)}) = \omega + i,\ 1 \le i \le m$$
$$f(v_{m+i}^{(1)}) = i,\ 2 \le i \le \omega - m$$
$$f(v_i^{(2)}) = i,\ 1 \le i \le \omega-1$$

For $1 \le j \le \omega-1$ and $2 \le i \le \omega-1$,

$$f(v_j^{(i+1)}) = \begin{cases} f(v_j^{(i)}) + 1, & if\ f(v_j^{(i)}) + 1 \le \omega-1 \\ f(v_j^{(i)}) \oplus 1, & \text{otherwise} \end{cases}$$

By the above theorem, we can easily show that $rr(G) = \omega + m$, $\omega \ge 4$ and $m < \omega$. ■

By combining all the above results, we conclude the following:

**Theorem 2.5** For any given $m \ge 0$, there exists a graph $G$ for which $rr(G) = \omega + m$, where $\omega \ge 3$. ■

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