On $\Delta$ Generalised Star Semi-Closed Sets in Topological Spaces

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Abstract: A modern form of sets labeled $\delta g*s$-closed sets is imported in this material. A part of utilization of $\delta g*s$-closed sets and its resources are explained. A modernistic space called $\delta g*s-T^*_3/4$-space is also popularized.

Keywords: $\delta$-closed sets, $\delta$-open sets, $\delta g*s$-open sets, $\delta g*s$-closed sets and $\delta g*s-T^*_3/4$-space.

I INTRODUCTION

In 1968, Velicko [13] introduced the $\delta$ closed sets. Some authors([1], [4], [8], [10], [11]) continuing their studies on semi-closed sets and continuous maps on topological spaces. $g^s$-closed sets were made known by the topologist [14]. $\delta g*s$-closed sets and $\delta g*s$-space are popularized in this note by the author.

II PRELIMINARIES

In every place of this text (N, $\tau$) or N perform topological spaces. Definitions of g, sg, gs, ag, $\delta g$, ag-closed sets are collected from [5], [2], [7], [6], [3], [12], [9], [14].

III. MAIN RESULTS

Definition 3.1 A division $M$ of (N, $\tau$) is termed as $\delta g*s$-closed if $clg(M) \subseteq V$ whenever $M \subseteq V$, V is $g^s$—open.

Theorem 3.2 Each one $\delta$-closed $\Rightarrow$ $\delta g*s$-closed set.

Proof. Approve $U \subseteq \delta$-closed, $V \supseteq g^s$-open consisting of U. As $clg(U)=U$ for each $U \subseteq U$ of N. Thus $clg(U) \subseteq U \Rightarrow \delta g*s$-closed.

Note 3.3, $\delta g*s$ -closed $\Rightarrow$ $\delta$-closed.

Example 3.4 Let $N = \{m, n, o, p\}$ with the topology $\tau = \{N, \varphi, \{m\}, \{n\}, \{m, n\}, \{m, n, o\}\}$. Though the set $\{o, p\}$ is $\delta g*s$--closed it is not $\delta$-closed.

Proposition 3.5 $\delta g*s$ closed $\Rightarrow$ open set.

Proof. Authorize $U = \delta g*s$-closed, $V =$ open set consists of $U$.

Here U is $\delta g*s$ closed, $clg(U) \subseteq U$ for each U of N. As $clg(U) \subseteq clg(U) \subseteq U$, consequently U is gclosed.

Example 3.6 Let $N = \{s, t, u, v\}$, $\tau = \{N, \varphi, \{s\}, \{t\}, \{s, t\}, \{t, u, v\}\}$. Then $\{v\}$ is g-closed set and $\neq \delta g*s$--closed in N. This example proves that g-closed $\neq \delta g*s$--closed.

Theorem 3.7 Every $\delta g*s$ -closed $\Rightarrow$ gs-closed.

Proof. Presume $U = \delta g*s$-closed, $V =$ open set consists of $U$.

As cg (U) $\subseteq U$ for every $U \subseteq U$ of N. As $scg(U) \subseteq cg(U) \subseteq V$, $scg(U) \subseteq V \Rightarrow U$ is gs-closed.

Example 3.8 Let N=${a, \kappa, \theta, \varphi}$, $\tau = \{N, \varphi, \{a\}, \{\kappa\}, \{\kappa, \theta\}, \{\varphi\}\}$. Though $\{\theta\}$ is gs-closed $\neq \delta g*s$ --closed. This shows that gs-closed $\neq \delta g*s$ --closed.

Theorem 3.9 $\delta g*s$ -closed $\Rightarrow$ ag-closed set.

Proof. For each U of N, It is clear that $aclg(U) \subseteq cg(U)$.

Example 3.10 Let $N = \{1, 2, 3, 4\}$, $\tau = \{N, \varphi, \{1\}, \{2\}, \{1, 2\}, \{2, 3, 4\}\}$. Then (1, 3) is ag-closed set $\delta g*s$-closed. However converse part fails in Theorem 3.9.

Theorem 3.11 $\delta g*s$ -closed $\Rightarrow$ $\delta$-closed set.

Proof. Suppose $U \subseteq \delta g*s$-closed, $V$ the open set consists of $U$. As $clg(A) \subseteq U$, when $A \subseteq U$, $U$ is $g^s$--open. There upon $clg(U) \subseteq U$, $U$ is open. Thus U is $\delta$-closed.

Example 3.12 Take $N = \{x, \psi, \gamma, \delta\}$, $\tau = \{N, \varphi, \{x\}, \{\psi\}, \{\psi, \gamma\}, \{\psi, \gamma, \delta\}\}$. Then $\{x, \delta\}$ is $\delta$-closed set $\neq \delta g*s$-closed in N. It proves that confer gets blunder in Theorem 3.11.
Theorem 3.13 $\delta g^s$ -closed $\Rightarrow$ ag*-closed set.

Proof. For every D of $(L, \tau)$, $\text{cl}(D) \subseteq \text{cl}_D(D)$ is clear.

Example 3.14 Let $N = \{ \delta, \kappa, \lambda, \mu \}$, with the topology $\tau = \{ N, \varphi, \{ \delta \}, \{ \kappa \}, \{ \lambda, \mu \} \}$. Then $\{ \delta, \kappa, \lambda \}$ is $ag^*$-closed set $\neq \delta g^*$-closed in $N$. Hence exchange is not perfect in Theorem 3.13.

Note 3.15 Examples for relationships of $\delta g^*$ -closed with more noted sets.

Remark 3.16 Self-reliant results with $\delta g^*$ -closed are given below.

Illustration 3.17 Assume $N = \{ \delta, \sigma, \rho \}$, $\tau = \{ N, \varphi, \{ \delta \}, \{ \sigma, \rho \}, \{ \delta, \sigma, \rho \} \}$. Then $\{ \delta, \sigma \}$ is $\delta g^*$ -closed but $\neq g^*$ -closed.

Example 3.18 Take $N = \{ \gamma, \rho, \sigma, s \}$ with the topology $\tau = \{ N, \varphi, \{ \gamma \}, \{ \gamma, \rho \}, \{ \sigma, s \}, \{ \gamma, \sigma, s \} \}$. Then $\{ \gamma, \rho \}$ is $\delta g^*$ -closed but $\neq g^*$ -closed.

Illustration 3.19 Assume $N = \{ \delta, \sigma, \omega, \psi \}$ with the topology $\tau = \{ N, \varphi, \{ \delta \}, \{ \sigma \}, \{ \delta, \sigma, \omega, \psi \} \}$. Then $\{ \delta, \sigma \}$ is $g^*$ -closed, $ag$-closed and $\neq \delta g^*$ -closed.

Illustration 3.20 Consider $N = \{ \gamma, \eta, \pi \}$ with the topology $\tau = \{ N, \varphi, \{ \gamma \}, \{ \gamma, \eta \}, \{ \pi, \rho \}, \{ \gamma, \pi, \rho \} \}$. Then $\{ \gamma \}$ is $a$-closed and $\neq \delta g^*$ -closed set.

IV. CHARACTERIZATIONS

Theorem 4.1 The limited combination of $\delta g^*$ -closed sets $\Rightarrow \delta g^*$ -closed.

Proof. Endorse $\{ X_i \}_{i=1,2,...,n}$ be a limited set of $\delta g^*$ -closed subsets $N$. Moreover for $N_i, \text{cl}(X_i) \subseteq U, i \in \{ 1, 2,...,n \}$. Hence $N = \bigcup U = \bigvee N_i$. Here $U$ of $g^*$-open = $g^*$-open in $N$. $V$ is $g^*$-open in $N$. In addition, $\bigcup \text{cl}(N_i) = \text{cl}(\bigcup N_i) \subseteq \text{cl}(N)$. Therefore $\bigcup N_i$ is $\delta g^*$ -closed in $N$.

Observation 4.2 $\cap \delta g^*$ -closed sets in $L \neq \delta g^*$ -closed set. In example 3.14 $\{ \eta, \kappa, \lambda \} \cap \{ \eta, \kappa, \mu \} = \emptyset$ is $\neq \delta g^*$ -closed.

Theorem 4.3 If $E$ is a $\delta g^*$ -closed set contained in $\text{cl}_E(E) - E$, then $\text{cl}_E(E) - E \not\subseteq \text{ag^*} - \text{closed set}$.

Proof. Take $U$ as $\delta g^*$ -closed and assume $G$ be a $g^*$ -closed set contained in $\text{cl}_G(U) \subseteq G$. Now $G' = g^*$ -open set on $N$, likewise $G \subseteq G'$. As $U = \delta g^*$ -closed set, then $\text{cl}_G(U)$ $\subseteq G'$. Thus $G \subseteq (\text{cl}_G(U))^c$. Also $G \subseteq \text{cl}_G(U) = U$. Therefore $G \subseteq (\text{cl}_G(U))^c \cap (\text{cl}_G(U)) = \emptyset$ Thus $G = \emptyset$.

Theorem 4.4 In $N, U = g^*$ -open, $\delta g^*$ -closed $\subseteq N$ then is $\delta$-closed $\subseteq N$.

Proof. As $g^*$ -open and $\delta g^*$ -closed $= \cup U, \text{cl}_G(U) \subseteq U$. Hence $U$ is $\delta$-closed.

Theorem 4.5 In $T_{3/4}$-space each one $\delta g^*$ -closed set $\Rightarrow \delta$-closed.

Proof. Consider $U$ be $\delta g^*$ -closed set of $N$, where $N$ is $T_{3/4}$-Space. Here, each one $\delta g^*$ -closed set $\Rightarrow \delta g$ closed. Therefore $N$ is $T_{3/4}$-Space and $U$ is $\delta$-closed.

Theorem 4.6 In $N, U$ is $\delta$-closed and $\delta g^*$ -closed if and only if $\text{cl}_G(U) - U = g^*$ -closed.

Proof. Necessity. Assume $U$ be a $\delta$-closed $\subseteq N$. Then $\text{cl}_G(U) = U$ and so $\text{cl}_G(U) - U = \varphi$, a $g^*$ -closed.

Adequate As $U$ is $\delta g^*$ -closed by Theorem 4.3, $\text{cl}_G(A) - A \neq g^*$-closed. But $\text{cl}_G(U) - U = \varphi$. Thus $\text{cl}_G(U) = U$ Hence $U$ is $\delta$-closed.

V. APPLICATIONS

Definition 5.1 $N$ is termed as $\delta g^*$ - $T_{3/4}$-space if each $\delta g^*$-closed set $\Rightarrow \delta$-closed.

Theorem 5.2 Every $T_{3/4}$-space is a $\delta g^*$-T$_{3/4}$-space.

Proof. Here each $\delta g^*$-closed $\Rightarrow \delta g$-closed, the argument is clear.

Remark 5.3 $\delta g^*$ - $T_{3/4}$-space $\neq T_{3/4}$-space.

Illustration 5.4 Consider $N = \{ \delta, \pi, \theta \}$ and $\tau = \{ N, \varphi, \{ \delta \} \}$. $N$ is a $\delta g^*$ - $T_{3/4}$-space but not a $T_{3/4}$-space.

Further details and examples can be found in the referenced sources.
Theorem 5.5 Every δg*s - T*₃₄₃-space is a T₉₀₆-space.

Proof. Concede N be a δg*s - T*₃₄₃-space, then each singleton is g*s-closed or δ-open. As each singleton is g*s-closed or α-open, N is a T₉₀₆-space.

Note 5.6 T₉₀₆-space ≠ δg*s - T*₃₄₃-space.

Illustration 5.7 Consider N= { χ, π, ξ } and τ= { N, φ, {φ} }. N is a T₉₀₆-space but ≠ δg*s - T*₃₄₃-space.

Remark 5.8 δg*s - T*₃₄₃-space and T₁/₂-space are self-reliant to one another.

Illustration 5.9 Take N = {χ, ψ, ζ} and τ= {N, φ, {φ} }. (N, τ) is δg*s - T*₃₄₃-space and ≠ T₁/₂-space.

Illustration 5.10 Assume N = {λ, ζ, θ} and τ= {N, φ, {φ}, {φ}, {φ}, {φ}, {φ}}. N is a T₁/₂-space and ≠ δg*s - T*₃₄₃-space.

ACKNOWLEDGMENT

The biographer desire to give thanks to the conciliators for their impressive commentary and verdict which will guide to develop this publication.

REFERENCES


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