On Δ Generalised Star Semi-Closed Sets in Topological Spaces

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***Abstract:** A modern form of sets labeled δg*s -closed sets is imported in this material. A part of utilization of δg*s -closed sets and its resources are explained. A modernistic space called δg*s-T*$_{3/4}$–space is also popularized.*

***Keywords:** δ-closed sets, δ-open sets, δg*s-open sets, δg*s-closed sets and δg*s-T*$_{3/4}$–space.*

I INTRODUCTION

In 1968, Velicko [13] introduced the δ closed sets. Some authors([1], [4], [8], [10], [11]) continuing their studies on semi-closed sets and continuous maps on topological spaces. g*s-closed sets were made known by the topologist [14]. δg*s-closed sets and *δg*s*-T*$_{3/4}$–space are popularized in this note by the author.

II PRELIMINARIES

In every place of this text (N, τ) or N perform topological spaces. Definitions of g, sg, gs, αg, δg, αg-closed sets are collected from [5], [2], [7], [6], [3], [12], [9], [14].

III. MAIN RESULTS

Definition 3.1 A division M of (N, τ) is termed as , δg*s -closed if cl$_\delta$(M) ⊆ V whenever M ⊆ V , V is g*s—open.

Theorem 3.2 Each one δ-closed ⇒ δg*s -closed set.

Proof. Approve U⊆δ-closed, V⊆g*s-open consisting of U. As cl$_\delta$(U)=U for each ⊂ U of N. Thus cl$_\delta$(U)⊆U ⟹ U is δg*s-closed.

Note 3.3, δg*s -closed ⟹ δ-closed.

Example 3.4 Let N = {m, n, o, p} with the topology τ = {N, φ,{m}, {n}, {m, n}, { m, n, o}}. Though the set {o, p} is δg*s --closed it is not δ-closed.

Proposition 3.5 δg*s closed ⇒ g-closed set.

Proof. Authorize U = δg*s -closed, V = open set consists of U.

Here U is δg*s closed, cl$_\delta$ (U) ⊆ U for each U of N. As cl(U) ⊆cl$_\delta$(U)⊆U, cl(U)⊆U, consequently U is g-closed.

Example 3.6 Let N= {s, t, u, v}, τ ={ N, φ, {s} , {t},{s, t},{t, u, v}}. Then {v} is g-closed set and ≠ δg*s –closed in N. This example proves that g-closed ≠ δg*s –closed.

Theorem 3.7 Every δg*s -closed ⇒ gs-closed.

Proof. Presume U = δg*s –closed. V = open set consists of U. As cl (U) ⊆ U for every ⊂ U of N . As scl(U) ⊆ cl$_\delta$(U) ⊆ V , scl(U) ⊆ V , ⇒ U is gs-closed.

Example 3.8 Let N={α, κ, θ, ψ}, τ = {N, φ, {α}, {κ}, {α, κ} , {κ, θ, ψ}}. Though {θ} is gs-closed ≠ δg*s –closed. This shows that gs-closed ≠ δg*s –closed.

Theorem 3.9 δg*s -closed ⇒ αg-closed set.

Proof. For each U of N , It is clear that αcl(U) ⊆ cl$_\delta$(U).

Example 3.10 Let N = { 1, 2, 3, 4} , τ = { N, φ, { 1}, { 2}, {1, 2} ,{ 2, 3, 4}}. Then {1, 3} is αg-closed set δg*s -closed. However converse part fails in Theorem 3.9.

Theorem 3.11 δg*s -closed ⇒ δg-closed set.

Proof. Suppose U ⊆ δg*s -closed, V the open set consists of U. As cl$_\delta$(A) ⊆ U , when A ⊆ U , U is g*s -open. There upon cl$_\delta$(U) ⊆ U, U is open. Thus U is δg-closed.

Example 3.12 Take N = {χ, ψ, γ, δ}, τ = { N, φ, {χ} , {ψ} , {χ, ψ} , {ψ, γ, δ}}. Then {χ, δ} is δg- closed set ≠ δg*s -closed in N . It proves that confer gets blunder in Theorem 3.11.

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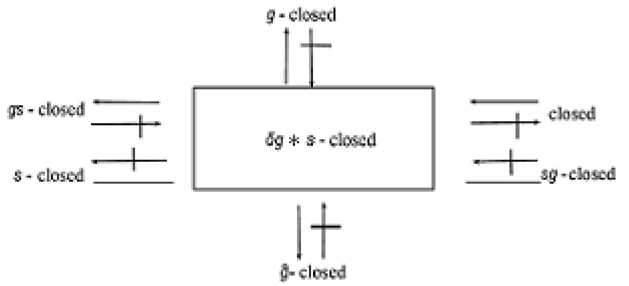
Theorem 3.13 δg*s -closed ⇒ αg^-closed set.

Proof. For every D of (L, τ), αcl(D) ⊆ cl_δ(D) is clear.

Example 3.14 Let N = { η, κ, λ, μ }, with the topology τ ={ N, φ, {η }, { κ} , { η, κ}, { κ, λ, μ} }. Then { η, κ, λ } is αg^-closed set ≠ δg*s --closed in N . Hence exchange is not perfect in Theorem 3.13.

Note 3.15 Examples for relationships of δg*s -closed with more noted sets.



Remark 3.16 Self-reliant results with δg*s -closed are given below.

Illustration 3.17 Assume N = {η, σ, ρ}, τ = { N, φ, { η } {σ}, { η, σ}}. Then {η, σ} is δg*s –closed but ≠ g*s -closed and ≠ sg-closed

Example 3.18 Take N = {γ, ρ, σ, s} with the topology τ = {N,φ,{γ},{γ,ρ},{σ,s},{γ,σ,s}}. Then {γ,ρ} is δg*s closed but≠ gα ≠α-closed.

Illustration 3.19 Assume N = {θ, σ, ω, ψ} with the topology τ = {N, φ, {θ }, {σ} , { θ , σ}, { σ, ω, ψ }}. Then { θ, ω } is g*s ,sg, gα-closed and ≠ δg*s –closed.

Illustration 3.20 Consider N = { γ, η, π, } with the topology τ ={N, φ, {γ}, { γ, η }, { π, ρ }, { γ, π, ρ}}. Then {γ} is α-closed and ≠ δg*-closed set.

IV. CHARACTERIZATIONS

Theorem 4.1 The limited combination of δg*s -closed sets⟶ δg*s closed.

Proof. Endorse {X_i/i=1,2,....n } be a limited set of δg*s –closed subsets N. Moreover for N_i, $cl_\delta(X_i)$⊆ U_i,i ∈ {1, 2, ...n}. Hence N_i ⊆ ⋃U_i=V. Here ∪ of g*s-open = g*s-open in N. V is g*s-open in N. In addition , ⋃ $cl_\delta(N_i)$= cl_δ(⋃ N_i)⊆ V. Therefore ⋃ N_i is δg*s –closed in N.

Observation 4.2 ∩ of δg*s -closed sets in L ≠ δg*s -closed set. In example 3.14 { η, κ, λ, } ∩ { η, κ, μ } = {η, κ} is ≠ δg*s -closed.

Theorem 4.3 If E is a δg*s -closed ⊂ of N , then cl_δ(E) – E not ⊆ an g*s -closed set.

Proof. Take U as δg*s -closed and assume G be a g*s closed set contained in cl_δ(U)-U . Now G^c = g*s open set of N, likewise U ⊂ G^c. As U = δg*s -closed set of N, then cl_δ(U) ⊆ G^c. Thus G ⊆ $(cl_\delta(U))^c$. Also G ⊆ cl_δ (U)-U . Therefore G ⊆ $(cl_\delta (U))^c$ ∩ (cl_δ (U))=ϕ. Thus G=ϕ.

Theorem 4.4 In N, U = g*s open, δg*s -closed ⊂ of N then is δ-closed ⊂ of N .

Proof. As g*s -open and δg*s -closed =U , cl_δ(U)⊆ U . Hence U is δ-closed.

Theorem 4.5 In $T_{3/4}$-space each one δg*s -closed set ⟹ δ-closed.

Proof. Consider U be δg*s -closed set of N, where N is $T_{3/4}$-Space. Here, each one δg*s -closed set⟶ δg closed. Therefore N is $T_{3/4}$-Space and U is δ-closed.

Theorem 4.6 In N, U is δ-closed and δg*s -closed if and only if cl_δ(U)-U = g*s -closed.

Proof. Necessity. Assume U be a δ-closed ⊂ of N. Then cl_δ(U)=U and so cl_δ(U)-U= φ, a g*s -closed.

Adequate: As U is δg*s –closed by Theorem 4.3, cl_δ(A)-A ≠ g*s-closed. But cl_δ(U)-U = φ. That is cl_δ(U)=U . Hence U is δ-closed.

V. APPLICATIONS

Definition 5.1 N is termed as δg*s - $T^*_{3/4}$ -space if each δg*s-closed set ⇒ δ-closed.

Theorem 5.2 Every $T_{3/4}$-space is a δg*s-$T_{*/4}$-space.

Proof. Here each δg*s-closed ⟹ δg-closed, the argument is clear.

Remark 5.3 δg*s - $T^*_{3/4}$ -space ≠ $T^*_{3/4}$ –space.

Illustration 5.4 Consider N = {δ, π, θ} and τ = {N, φ, {δ}}. N is a δg*s - $T^*_{3/4}$ -space but not a $T^*_{3/4}$- space.

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Theorem 5.5 Every δg*s - $T^*_{3/4}$ -space is a $T_{\alpha g^\wedge}$ - space.

Proof. Concede N be a δg*s - $T^*_{3/4}$ -space, then each singleton is g*s -closed or δ-open. As each singleton is g*s -closed or α-open, N is a $T_{\alpha g^\wedge}$ - space

Note 5.6 $T_{\alpha g^\wedge}$ - space ≠ δg*s - $T^*_{3/4}$ -space.

Illustration 5.7 Consider N= { χ, π, ξ } and τ = { N, φ, {χ} , {ξ} , {χ, ξ} , {π, ξ}}. N is a $T_{\alpha g^\wedge}$ -space but ≠ δg*s – $T^*_{3/4}$ – space.

Remark 5. 8 δg*s - $T^*_{3/4}$ -space and $T_{1/2}$-space are self-reliant to one another

Illustration 5.9 Take N = {χ, ψ, ζ} and τ = {N, φ, {χ}}. (N, τ) is δg*s - $T^*_{3/4}$ -space and ≠ $T_{1/2}$-space.

Illustration 5.10 Assume N = {λ, ς, ϑ} and τ={ N, φ, {λ}, {ϑ}, {λ, ϑ}, {ς, ϑ}}. N is a $T_{1/2}$-space and ≠ δg*s - $T^*_{3/4}$ -space

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