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Some Properties of Generalized Star Semi Λ-Closed Sets in Topological Spaces

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***Abstract:** In this article, we proposed and examined topological characters of g*sΛ-closed, open sets. Its rapport with more generalized closed sets also inspected.*

***Keywords:** g*sΛ-closed sets, g*sΛ-open sets, λ- closed sets, Λ-closed sets.*

I. INTRODUCTION

Generalized open sets hits significant aspect in general topology. In 1986, Author [8] advanced Authors [6], [7], [1], [2] work and popularized the perception of Λ-sets in topological spaces. "Λ-set is a set A which is equal to its kernel (=saturated set)". Authors [5], [3], [4], [9], [10], [11] deliberated various closed and open sets concept by embroil Λ-sets and closed sets.

II. PRELIMINARIES

In this article (N, τ), (or simply N) consistently stands for topological spaces. We recognize some established definitions mandatory for this paper.

Lemma 2.1
$M \subset N$, succeeding allegations are identical.
1. M is λ-closed.
2. $M = L \cap cl(M)$, L is Λ-set.
3. $M = M^{\wedge} \cap cl(M)$.

Lemma 2.2
1.Λ-set is λ-set.
2.Open, closed are λ-closed.

Lemma 2.3
In $T_{1/2}$ space every subset of N is λ closed.

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Lemma 2.4
Subset of $T_{1/4}$ space is λ closed.

III. PROPERTIES OF $g^*s\Lambda$ - CLOSED SETS

Definition 3.1 $M \subseteq N$ is $g^*s\Lambda$-closed if $cl_\lambda(M) \subset Q$ whenever $M \subset Q$, Q is gs-open in N.

Theorem 3.2 λ-closed $\Rightarrow g^*s\Lambda$-closed.
Proof. Consider $M \subset Q$, Q is gs -open.
$\therefore M$ is λ-closed. Hence $cl_\lambda(M) = M \subset Q$.
$\Rightarrow M$ is $g^*s\Lambda$-closed.

Remark 3.3 $g^*s\Lambda$-closed $\neq \lambda$-closed by subsequent example.

Example 3.4 Take $N = \{\gamma, \eta, \lambda, \sigma\}$ with $\tau = \{N, \phi, \{\gamma\}, \{\eta, \lambda\}, \{\gamma, \eta, \lambda\}, \{\eta, \lambda, \sigma\}\}$. Here $\{\eta, \sigma\}$ is not λ closed but $g^*s\Lambda$-closed.

Theorem 3.5 Closed $\Rightarrow g^*s\Lambda$ closed.

Proof. Proof pursued from "lemma 2.2 and theorem 3.2".

Remark 3.6 $g^*s\Lambda$-closed is not closed by ensuing example.

Example 3.7 Let $N = \{\psi, \mu, \eta, \gamma\}$ and $\tau = \{N, \phi, \{\psi\}, \{\mu, \eta\}, \{\psi, \mu, \eta\}, \{\mu, \eta, \gamma\}\}$. Here $\{\mu, \gamma\}$ is $g^*s\Lambda$ closed but not closed.

Theorem 3.8 Open $\Rightarrow g^*s\Lambda$-closed.

Proof. Proof conspicuous by definitions.

Theorem 3.9 gs-open $\Rightarrow \lambda$-closed if it is $g^*s\Lambda$-closed.

Proof. Consider M be $g^*s\Lambda$-closed with gs-open.
$\because M \subset M$, $cl_\lambda(M) \subset M$.
$\Rightarrow M$ is λ closed.

Theorem 3.10 $g^*s\Lambda$-closed $\Rightarrow g\Lambda$-closed.

Proof. Certify M be $g^*s\Lambda$-closed, $M \subset Q$, Q is open. Open $\Rightarrow gs$-open, also M is $g^*s\Lambda$-closed.
$\Rightarrow cl_\lambda(M) \subset Q$
$\Rightarrow M$ is $g\Lambda$-closed.

Remark 3.11 $g\Lambda$-closed $\neq g^*s\Lambda$-closed by coming example.

Example 3.12 Take $N = \{v, \sigma, \rho\}$ and $\tau = \{N, \varphi, \{v\}, \{v, \rho\}\}$. Then $\{v, \sigma\}$ is not $g^*s\Lambda$- closed but $g\Lambda$-closed.

Remark 3.13 Generalized closed and $g^*s\Lambda$-closed are self-reliant.

Remark 3.14 $g^*s\Lambda$-closed, ω-closed are autonomous.

Theorem 3.15 ω-closed $\Rightarrow g^*s\Lambda$-closed, if *gs* open is semi open.

Proof. Consider $M \subset Q$, Q is *gs*-open.
$\because M$ is ω-closed.

$\Rightarrow cl(M) \subset Q$. But $Cl_\lambda(M) \subset Cl(M) \subset Q$.
$\Rightarrow M$ is $g^*s\Lambda$-closed.

Remark 3.16 $g^*s\Lambda$-closed, Λg-closed are nonpartisan.

Remark 3.17 $g^*s\Lambda$-closed, *gs*-closed are self-reliant.

Remark 3.18 $g^*s\Lambda$-closed, *gp*-closed are autonomous.

Remark 3.19 M, L are $g^*s\Lambda$-closed but $M \cap L$ is not $g^*s\Lambda$-closed.

Remark 3.20 $M \cup L$ is not $g^*s\Lambda$-closed even though M, L are $g^*s\Lambda$-closed.

IV. APPLICATIONS OF $g^*s\Lambda$ - CLOSED SETS

Theorem 4.1 M is $g^*s\Lambda$-closed $\Rightarrow cl_\lambda(M)\backslash M$ contains empty closed set.

Proof. Consider M be $g^*s\Lambda$-closed.
Suppose $\phi \neq P \subset cl_\lambda(M)\backslash M$.

$\Rightarrow M \subseteq P^c$, P^c is open.
$\because M$ is $g^*s\Lambda$-closed, open $\Rightarrow$ *gs*-open, $cl_\lambda(M) \subseteq P^c$.
$\therefore P \subseteq N\backslash cl_\lambda(M)$, also $P \subseteq cl_\lambda(M)$.
$\Rightarrow P \subseteq [N\backslash cl_\lambda(M)] \cap cl_\lambda(M) = \phi$.
$\Rightarrow cl_\lambda(M)\backslash M$ contains empty closed set.

Remark 4.2 Reverse part of theorem 4.1 fails by the coming example.

Example 4.3 Assume $N = \{\eta, \theta, \psi, \chi\}$ with $\tau = \{N, \phi, \{\eta\}, \{\theta\}, \{\eta, \theta\}, \{\theta, \psi, \chi\}\}$. If $M = \{\eta, \chi\}$, $cl_\lambda(M) = \{\eta, \psi, \chi\}$, $cl_\lambda(M)\backslash M = \{\psi\} \supset$ closed set $\neq \phi$ however M is $g^*s\Lambda$-closed.

Theorem 4.4 M is $g^*s\Lambda$-closed $\Rightarrow cl_\lambda(M)\backslash M$ contains empty *gs*-closed set.

Proof. Take M *be* $g^*s\Lambda$-closed.
Suppose P is a *gs*-closed $\subset cl_\lambda(M)\backslash M$.
$\Rightarrow M \subseteq P^c$, where P^c is *gs*-open.
Since M is $g^*s\Lambda$-closed $cl_\lambda(M)\backslash P^c$.

$\Rightarrow P \subseteq N\backslash cl_\lambda(M)$, also $P \subseteq cl_\lambda(M)$.
$\Rightarrow P \subseteq [N\backslash cl_\lambda(M)] \cap cl_\lambda(M) = \phi$.
$\therefore cl_\lambda(M)\backslash M$ contains empty *gs*-closed set.

Theorem 4.5 Subset of $T_{1/2}$ space is $g^*s\Lambda$-closed.

Proof. Proof evident from lemma 2.3 and theorem 3.2

Theorem 4.6 Subset of $T_{1/4}$ space is $g^*s\Lambda$ closed.

Proof. Proof results from "lemma 2.4 and theorem 3.2"

Theorem 4.7 In T_1 space Λg-closed is $g^*s\Lambda$-closed.

Proof. In T_1 space Λg-closed $\Rightarrow$ closed.

Closed $\Rightarrow g^*s\Lambda$-closed "by theorem 3.5"

$\Rightarrow$ In T_1 space Λg-closed is $g^*s\Lambda$-closed.

Theorem 4.8 Certify M be $g^*s\Lambda$-closed. Then M is λ-closed if and only if $cl_\lambda(M)\backslash M$ is closed.

Proof.
Necessity: Consider M be $g^*s\Lambda$ closed, λ closed.
M is λ closed $\Rightarrow cl_\lambda(M) = M$.
$\Rightarrow cl_\lambda(M)\backslash M = \phi$ is closed.
Sufficiency: Take M is $g^*s\Lambda$-closed and $cl_\lambda(M)\backslash M$ is closed.
$\Rightarrow cl_\lambda(M)\backslash M$ contains empty closed "by theorem 4.1"

$\Rightarrow cl_\lambda(M)\backslash M = \phi$.
$\Rightarrow Cl_\lambda(M) = M$. Therefore, M is λ-closed.

Theorem 4.9 If $g^*s\Lambda$-closed is λ-closed then $\{x\}$ is *gs*-closed or λ-open.

Proof. Assume $\{x\}$ is not *gs*-closed.
$\Rightarrow N\backslash\{x\}$ is not *gs*-open.

$\Rightarrow N$ is the only *gs*-open set $\supset N\backslash\{x\}$.

Evidently $cl_\lambda(N\backslash\{x\}) \subseteq N$.

$\therefore N\backslash\{x\}$ is $g^*s\Lambda$-closed.

$\Rightarrow \{x\}$ is λ-open.

Theorem 4.10 Consider M be *gs*-open and $g^*s\Lambda$-closed. P is λ-closed then $M \cap P$ is $g^*s\Lambda$-closed.

Proof. M is both *gs*-open and $g^*s\Lambda$-closed
$\Rightarrow M$ is λ-closed "by theorem 3.9".

$\Rightarrow M \cap P$ is λ-closed as the intersection of λ-closed sets is λ-closed.

$\Rightarrow M \cap P$ is $g^*s\Lambda$-closed "by theorem 3.2".

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Theorem 4.11 M is $g^*s\Lambda$-closed $\Rightarrow gscl(\{x\}) \cap M \neq \phi$, $\forall\, x \in cl_\lambda(M)$.

Proof. Assume M be $g^*s\Lambda$-closed with $gscl(\{x\}) \cap M = \phi$
$\exists\, x \in cl_\lambda(M)$.
$\Rightarrow N \setminus gscl(\{x\})$ is a gs open $\supset M$.
Also, $x \in cl_\lambda(M)$ and $x \notin N \setminus gscl(\{x\})$.
$\Rightarrow x \in (cl_\lambda(M)) \notin N \setminus gscl(\{x\})$.
$\Rightarrow cl_\lambda(M) \not\subset X \setminus gscl(\{x\})$.
$\Rightarrow \Leftarrow M$ is a $g^*s\Lambda$ closed.

$\Rightarrow gscl(\{x\}) \cap M \neq \phi, \forall x \in Cl_\lambda(M)$

Theorem 4.12 M be gs-open. The ensuing's are identical

1. λ-closed.
2. $g^*s\Lambda$-closed.

Proof.
(1) $\Rightarrow$ (2)

Assume $M \subset N$ with $M \subseteq Q$, Q is gs-open.
$\Rightarrow cl_\lambda(M) \subseteq cl_\lambda(Q)$. Here Q is λ-closed.
$\Rightarrow cl_\lambda(M) \subseteq cl_\lambda(Q) = Q$.
$\Rightarrow M$ is $g^*s\Lambda$-closed.
(2) $\Rightarrow$ (1)

Consider M be gs-open with M is $g^*s\Lambda$-closed.
$\Rightarrow cl_\lambda(M) \subseteq M$.
$\Rightarrow$ *M is* λ-closed.

Theorem 4.13 In door space subset is $g^*s\Lambda$-closed.

Proof. Certify $M \subseteq N$.
Since in door space, subset is one among the choices, open or closed.
$\Rightarrow$ Subset is λ-closed.
$\Rightarrow$ Subset is $g^*s\Lambda$ closed "by theorem 3.2".

Theorem 4.14 In partition space, $g^*s\Lambda$-closed $\Rightarrow$ gs-closed.

Proof. Take M abide $g^*s\Lambda$ closed with $M \subseteq Q$, Q is open.
As open is gs-open
$\Rightarrow Q$ is gs-open.
But M is $g^*s\Lambda$-closed.
$\Rightarrow cl_\lambda(M) \subseteq Q$.
In partition space, closed set is open.
$\therefore cl(A) = cl_\lambda(M) \subseteq Q$.
$\Rightarrow M$ is gs-closed.

V.CHARACTERISTICS OF $g^*s\Lambda$ - OPEN SETS

Definition 5.1 $M \subseteq N$ is $g^*s\Lambda$ open if M^c is a $g^*s\Lambda$ closed.

Theorem 5.2 λ open $\Rightarrow g^*s\Lambda$-open.

Proof. Assume M be a λ open.
$\Rightarrow N \setminus M$ is λ-closed.
$\Rightarrow N \setminus M$ is $g^*s\Lambda$-closed "by theorem 3.2"'.

$\Rightarrow M$ is $g^*s\Lambda$-open.

Theorem 5.3 Closed $\Rightarrow g^*s\Lambda$ open.

Proof. Consider M be a closed.
$\Rightarrow N \setminus M$ is open.
$\Rightarrow N \setminus M$ is λ closed "by lemma 2.2".

$\Rightarrow M$ is λ open.
$\Rightarrow M$ is $g^*s\Lambda$ open "by theorem 5.1".

Theorem 5.4 Open entail $g^*s\Lambda$ open.
Proof. Assume M hold open.
$\Rightarrow N \setminus M$ closed.
$\Rightarrow N \setminus M$ is λ-closed "by lemma 2.2".

$\Rightarrow N \setminus M$ is $g^*s\Lambda$ closed "by theorem 3.2".

$\Rightarrow M$ is $g^*s\Lambda$ open.

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