

Some Properties of Generalized Star Semi Λ -Closed Sets in Topological Spaces

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Abstract: In this article, we proposed and examined topological characters of $g^*s\Lambda$ -closed, open sets. Its rapport with more generalized closed sets also inspected.

Keywords: $g^*s\Lambda$ -closed sets, $g^*s\Lambda$ -open sets, λ -closed sets, Λ -closed sets.

I. INTRODUCTION

Generalized open sets hits significant aspect in general topology. In 1986, Author [8] advanced Authors [6], [7], [1], [2] work and popularized the perception of Λ -sets in topological spaces. “ Λ -set is a set A which is equal to its kernel (=saturated set)”. Authors [5], [3], [4], [9], [10], [11] deliberated various closed and open sets concept by embroil Λ -sets and closed sets.

II. PRELIMINARIES

In this article (N, τ) , (or simply N) consistently stands for topological spaces. We recognize some established definitions mandatory for this paper.

Lemma 2.1

$M \subset N$, succeeding allegations are identical.

1. M is λ -closed.
2. $M = L \cap cl(M)$, L is Λ -set.
3. $M = M^\Lambda \cap cl(M)$.

Lemma 2.2

1. Λ -set is λ -set.
2. Open, closed are λ -closed.

Lemma 2.3

In $T_{1/2}$ space every subset of N is λ closed.

Lemma 2.4

Subset of $T_{1/4}$ space is λ closed.

Revised Manuscript Received on December 15, 2019.

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III. PROPERTIES OF $g^*s\Lambda$ - CLOSED SETS

Definition 3.1 $M \subseteq N$ is $g^*s\Lambda$ -closed if $cl_\lambda(M) \subset Q$ whenever $M \subset Q$, Q is gs -open in N .

Theorem 3.2 λ -closed $\Rightarrow g^*s\Lambda$ -closed.

Proof. Consider $M \subset Q$, Q is gs -open.

$\therefore M$ is λ -closed. Hence $cl_\lambda(M) = M \subset Q$.

$\Rightarrow M$ is $g^*s\Lambda$ -closed.

Remark 3.3 $g^*s\Lambda$ -closed $\neq \lambda$ -closed by subsequent example.

Example 3.4 Take $N = \{\gamma, \eta, \lambda, \sigma\}$ with $\tau = \{N, \emptyset, \{\gamma\}, \{\eta, \lambda\}, \{\gamma, \eta, \lambda\}, \{\eta, \lambda, \sigma\}\}$. Here $\{\eta, \sigma\}$ is not λ closed but $g^*s\Lambda$ -closed.

Theorem 3.5 Closed $\Rightarrow g^*s\Lambda$ closed.

Proof. Proof pursued from “lemma 2.2 and theorem 3.2”.

Remark 3.6 $g^*s\Lambda$ -closed is not closed by ensuing example.

Example 3.7 Let $N = \{\psi, \mu, \eta, \gamma\}$ and $\tau = \{N, \emptyset, \{\psi\}, \{\mu, \eta\}, \{\psi, \mu, \eta\}, \{\mu, \eta, \gamma\}\}$. Here $\{\mu, \gamma\}$ is $g^*s\Lambda$ closed but not closed.

Theorem 3.8 Open $\Rightarrow g^*s\Lambda$ -closed.

Proof. Proof conspicuous by definitions.

Theorem 3.9 gs -open $\Rightarrow \lambda$ -closed if it is $g^*s\Lambda$ -closed.

Proof. Consider M be $g^*s\Lambda$ -closed with gs -open.

$\therefore M \subset M$, $cl_\lambda(M) \subset M$.

$\Rightarrow M$ is λ closed.

Theorem 3.10 $g^*s\Lambda$ -closed $\Rightarrow g\Lambda$ -closed.

Proof. Certify M be $g^*s\Lambda$ -closed, $M \subset Q$, Q is open.

Open $\Rightarrow gs$ -open, also M is $g^*s\Lambda$ -closed.

$\Rightarrow cl_\lambda(M) \subset Q$

$\Rightarrow M$ is $g\Lambda$ -closed.

Remark 3.11 $g\Lambda$ -closed $\neq g^*s\Lambda$ -closed by coming example.

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Example 3.12 Take $N = \{v, \sigma, \rho\}$ and $\tau = \{N, \varphi, \{v\}, \{v, \rho\}\}$. Then $\{v, \sigma\}$ is not $g^*s\Lambda$ -closed but $g\Lambda$ -closed.

Remark 3.13 Generalized closed and $g^*s\Lambda$ -closed are self-reliant.

Remark 3.14 $g^*s\Lambda$ -closed, ω -closed are autonomous.

Theorem 3.15 ω -closed $\Rightarrow g^*s\Lambda$ -closed, if gs open is semi open.

Proof. Consider $M \subset Q$, Q is gs -open.

$\therefore M$ is ω -closed.

$\Rightarrow cl(M) \subset Q$. But $Cl_\lambda(M) \subset Cl(M) \subset Q$.

$\Rightarrow M$ is $g^*s\Lambda$ -closed.

Remark 3.16 $g^*s\Lambda$ -closed, Λg -closed are nonpartisan.

Remark 3.17 $g^*s\Lambda$ -closed, gs -closed are self-reliant.

Remark 3.18 $g^*s\Lambda$ -closed, gp -closed are autonomous.

Remark 3.19 M, L are $g^*s\Lambda$ -closed but $M \cap L$ is not $g^*s\Lambda$ -closed.

Remark 3.20 $M \cup L$ is not $g^*s\Lambda$ -closed even though M, L are $g^*s\Lambda$ -closed.

IV. APPLICATIONS OF $g^*s\Lambda$ - CLOSED SETS

Theorem 4.1 M is $g^*s\Lambda$ -closed $\Rightarrow cl_\lambda(M) \setminus M$ contains empty closed set.

Proof. Consider M be $g^*s\Lambda$ -closed.

Suppose $\phi \neq P \subset cl_\lambda(M) \setminus M$.

$\Rightarrow M \subseteq P^c$, P^c is open.

$\therefore M$ is $g^*s\Lambda$ -closed, open $\Rightarrow gs$ -open, $cl_\lambda(M) \subseteq P^c$.

$\therefore P \subseteq N \setminus cl_\lambda(M)$, also $P \subseteq cl_\lambda(M)$.

$\Rightarrow P \subseteq [N \setminus cl_\lambda(M)] \cap cl_\lambda(M) = \phi$.

$\Rightarrow cl_\lambda(M) \setminus M$ contains empty closed set.

Remark 4.2 Reverse part of theorem 4.1 fails by the coming example.

Example 4.3 Assume $N = \{\eta, \theta, \psi, \chi\}$ with $\tau = \{N, \phi, \{\eta\}, \{\theta\}, \{\eta, \theta\}, \{\theta, \psi, \chi\}\}$. If $M = \{\eta, \chi\}$, $cl_\lambda(M) = \{\eta, \psi, \chi\}$, $cl_\lambda(M) \setminus M = \{\psi\} \supset$ closed set $\neq \phi$ however M is $g^*s\Lambda$ -closed.

Theorem 4.4 M is $g^*s\Lambda$ -closed $\Rightarrow cl_\lambda(M) \setminus M$ contains empty gs -closed set.

Proof. Take M be $g^*s\Lambda$ -closed.

Suppose P is a gs -closed $\subset cl_\lambda(M) \setminus M$.

$\Rightarrow M \subseteq P^c$, where P^c is gs -open.

Since M is $g^*s\Lambda$ -closed $cl_\lambda(M) \setminus P^c$.

$\Rightarrow P \subseteq N \setminus cl_\lambda(M)$, also $P \subseteq cl_\lambda(M)$.

$\Rightarrow P \subseteq [N \setminus cl_\lambda(M)] \cap cl_\lambda(M) = \phi$.

$\therefore cl_\lambda(M) \setminus M$ contains empty gs -closed set.

Theorem 4.5 Subset of $T_{1/2}$ space is $g^*s\Lambda$ -closed.

Proof. Proof evident from lemma 2.3 and theorem 3.2

Theorem 4.6 Subset of $T_{1/4}$ space is $g^*s\Lambda$ closed.

Proof. Proof results from "lemma 2.4 and theorem 3.2"

Theorem 4.7 In T_1 space Λg -closed is $g^*s\Lambda$ -closed.

Proof. In T_1 space Λg -closed \Rightarrow closed.

Closed $\Rightarrow g^*s\Lambda$ -closed "by theorem 3.5"

\Rightarrow In T_1 space Λg -closed is $g^*s\Lambda$ -closed.

Theorem 4.8 Certify M be $g^*s\Lambda$ -closed. Then M is λ -closed if and only if $cl_\lambda(M) \setminus M$ is closed.

Proof.

Necessity: Consider M be $g^*s\Lambda$ closed, λ closed.

M is λ closed $\Rightarrow cl_\lambda(M) = M$.

$\Rightarrow cl_\lambda(M) \setminus M = \phi$ is closed.

Sufficiency: Take M is $g^*s\Lambda$ -closed and $cl_\lambda(M) \setminus M$ is closed.

$\Rightarrow cl_\lambda(M) \setminus M$ contains empty closed "by theorem 4.1"

$\Rightarrow cl_\lambda(M) \setminus M = \phi$.

$\Rightarrow Cl_\lambda(M) = M$. Therefore, M is λ -closed.

Theorem 4.9 If $g^*s\Lambda$ -closed is λ -closed then $\{x\}$ is gs -closed or λ -open.

Proof. Assume $\{x\}$ is not gs -closed.

$\Rightarrow N \setminus \{x\}$ is not gs -open.

$\Rightarrow N$ is the only gs -open set $\supset N \setminus \{x\}$.

Evidently $cl_\lambda(N \setminus \{x\}) \subseteq N$.

$\therefore N \setminus \{x\}$ is $g^*s\Lambda$ -closed.

$\Rightarrow \{x\}$ is λ -open.

Theorem 4.10 Consider M be gs -open and $g^*s\Lambda$ -closed. P is λ -closed then $M \cap P$ is $g^*s\Lambda$ -closed.

Proof. M is both gs -open and $g^*s\Lambda$ -closed

$\Rightarrow M$ is λ -closed "by theorem 3.9".

$\Rightarrow M \cap P$ is λ -closed as the intersection of λ -closed sets is λ -closed.

$\Rightarrow M \cap P$ is $g^*s\Lambda$ -closed "by theorem 3.2".

Theorem 4.11 M is $g^*s\Lambda$ -closed $\Rightarrow gsc(\{x\}) \cap M \neq \phi$,

$\forall x \in cl_\lambda(M)$.

Proof. Assume M be $g^*s\Lambda$ -closed with $gscl(\{x\}) \cap M = \emptyset$
 $\exists x \in cl_\lambda(M)$.
 $\Rightarrow N \setminus gscl(\{x\})$ is a gs open $\supset M$.
 Also, $x \in cl_\lambda(M)$ and $x \notin N \setminus gscl(\{x\})$.
 $\Rightarrow x \in (cl_\lambda(M)) \notin N \setminus gscl(\{x\})$.
 $\Rightarrow cl_\lambda(M) \not\subset N \setminus gscl(\{x\})$.
 $\Rightarrow \Leftarrow M$ is a $g^*s\Lambda$ closed.
 $\Rightarrow gscl(\{x\}) \cap M \neq \emptyset, \forall x \in Cl_\lambda(M)$

Theorem 4.12 M be gs -open. The ensuing's are identical
 1. λ -closed.
 2. $g^*s\Lambda$ -closed.

Proof.

(1) \Rightarrow (2)

Assume $M \subset N$ with $M \subseteq Q$, Q is gs -open.
 $\Rightarrow cl_\lambda(M) \subseteq cl_\lambda(Q)$. Here Q is λ -closed.
 $\Rightarrow cl_\lambda(M) \subseteq cl_\lambda(Q) = Q$.
 $\Rightarrow M$ is $g^*s\Lambda$ -closed.

(2) \Rightarrow (1)

Consider M be gs -open with M is $g^*s\Lambda$ -closed.
 $\Rightarrow cl_\lambda(M) \subseteq M$.
 $\Rightarrow M$ is λ -closed.

Theorem 4.13 In door space subset is $g^*s\Lambda$ -closed.

Proof. Certify $M \subseteq N$.

Since in door space, subset is one among the choices, open or closed.
 \Rightarrow Subset is λ -closed.
 \Rightarrow Subset is $g^*s\Lambda$ closed “by theorem 3.2”.

Theorem 4.14 In partition space, $g^*s\Lambda$ -closed \Rightarrow gs -closed.

Proof. Take M abide $g^*s\Lambda$ closed with $M \subseteq Q$, Q is open.

As open is gs -open

$\Rightarrow Q$ is gs -open.

But M is $g^*s\Lambda$ -closed.

$\Rightarrow cl_\lambda(M) \subseteq Q$.

In partition space, closed set is open.

$\therefore cl(A) = cl_\lambda(M) \subseteq Q$.

$\Rightarrow M$ is gs -closed.

V. CHARACTERISTICS OF $g^*s\Lambda$ - OPEN SETS

Definition 5.1 $M \subseteq N$ is $g^*s\Lambda$ open if M^c is a $g^*s\Lambda$ closed.

Theorem 5.2 λ open \Rightarrow $g^*s\Lambda$ -open.

Proof. Assume M be a λ open.

$\Rightarrow N \setminus M$ is λ -closed.

$\Rightarrow N \setminus M$ is $g^*s\Lambda$ -closed “by theorem 3.2”.

$\Rightarrow M$ is $g^*s\Lambda$ -open.

Theorem 5.3 Closed \Rightarrow $g^*s\Lambda$ open.

Proof. Consider M be a closed.

$\Rightarrow N \setminus M$ is open.

$\Rightarrow N \setminus M$ is λ closed “by lemma 2.2”.

$\Rightarrow M$ is λ open.

$\Rightarrow M$ is $g^*s\Lambda$ open “by theorem 5.1”.

Theorem 5.4 Open entail $g^*s\Lambda$ open.

Proof. Assume M hold open.

$\Rightarrow N \setminus M$ closed.

$\Rightarrow N \setminus M$ is λ -closed “by lemma 2.2”.

$\Rightarrow N \setminus M$ is $g^*s\Lambda$ closed “by theorem 3.2”.

$\Rightarrow M$ is $g^*s\Lambda$ open.

ACKNOWLEDGMENT

The Authors indebted to the referees for their beneficial ideas and conclusions which will help to advance the paper.

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