International Journal of Engineering and Advanced Technology (IJEAT)
ISSN: 2249 – 8958, Volume-9, Issue-1S4, December 2019

Geometric Mean Cordial Labeling of Transformation Graph of a Star Graph

K. Nagarajan, K. Chitra Lakshmi

Abstract: *Let $G = (V, E)$ be a graph and f be a mapping from $V(G) \rightarrow \{0, 1, 2\}$. For each edge uv, assign the label $\left|\sqrt{f(u)f(v)}\right|$, f is called a geometric mean cordial labeling if $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x, $x \in \{0, 1, 2\}$ respectively.*

A graph with a geometric mean cordial labeling is called geometric mean cordial graph. In this paper, the geometric mean cordiality of transformation graph of star is discussed.

Keywords: *Cordial labeling, cordial graph, geometric mean cordial labeling, geometric mean cordial graph, transformation graph.*

I. INTRODUCTION

We use the symbol $\lceil x \rceil$ denotes the smallest integer greater than or equal to x. Some terms are used in the sense of Harary, Bondy and Murthy [2, 5]. The concept of cordial labeling [3] was introduced by Cahit in the year 1987. The labeled graphs are applied mostly in the areas of radar, circuit design, communication network, astronomy, cryptography etc [4].

For a graph $G = (V,E)$, let $f : V(G) \rightarrow \{0,1\}$ be a function. For each edge uv, assign the label $|f(u) - f(v)|$, f is called a cordial labeling if $|v_f(i) - v_f(j)| \le 1$ and $|e_f(i) - e_f(j)| \le 1$, where $v_f(x)$ and $e_f(x)$ denote the number of vertices and edges labeled with x, $x \in \{0, 1\}$ respectively.

A graph which admits cordial labeling is called is a cordial graph. Mean cordial labeling was introduced by R. Ponraj, M. Sivakumar and M. Sundaram [6].

We defined geometric mean cordial labeling in [7]. In the previous six papers [7,8, 9, 10, 11, 12], we have checked the geometric mean cordiality of standard graphs, subdivision of standard graphs, corona of standard graphs, transformation graphs of paths and cycles.

In this paper, the geometric mean cordiality is checked for the transformation graph of a star.

Revised Manuscript Received on December 15, 2019.

Dr. K. Nagarajan, Head and Associate Professor, Department of mathematics at Sri S. Ramasamy Naidu Memorial College, Sattur, India.

K. Chitra Lakshmi, Assistant Professor, Department of Mathematics, V. V. Vanniaperumal College for Women, Virudhunagar.

II. PRELIMINARIES

First we define a transformation graph of a graph.

Definition 2.1.[11] A transformation graph G^{xyz} is the graph whose vertex set is $V(G) \cup E(G)$. Two of its vertices are adjacent if and only if their associativity is consistent with the corresponding elements of xyz. Let α and β be two elements of $V(G) \cup E(G)$. Then the associativity of α and β is taken as + if they are adjacent or incident in G, otherwise - . Let xyz be the 3 - permutation of the set { +, - }. The pair α and β is said to correspond to x or y or z if both α and β are in $V(G)$ or both in $E(G)$ or one is in $V(G)$ and the other is in $E(G)$ respectively.

Theorem 1.2: P_n^{+++} is geometric mean cordial if and only if $n \equiv 1 \ (mod\ 3)$.
Theorem 1.3: P_n^{+-+} is geometric mean cordial if and only if $n \equiv 1 \ (mod\ 3)$.
Theorem 1.4: P_n^{-++} is geometric mean cordial if and only if $n \equiv 1, 2 \ (mod\ 3)$.
Theorem 1.5: P_n^{--+} is geometric mean cordial.

Theorem 1.6: C_n^{+++} is not geometric mean cordial.
Theorem 1.7: C_n^{+--} is not geometric mean cordial.
Theorem 1.8: C_n^{--+} is geometric mean cordial.
Theorem 1.9: C_n^{-++} is not geometric mean cordial.

III. MAIN RESULT

Here, we will prove transformation graph of a star is not geometric mean cordial.

Theorem 3.1. *$K_{1,n}^{+++}$ is not geometric mean cordial.*

Proof: Let $K_{1,n}$ be the star graph of $n+1$ vertices and n edges. Then $K_{1,n}^{+++}$ is a graph of $2n + 1$ vertices and $\frac{5n+n^2}{2}$ edges.

This graph can be drawn so that it is decomposed into 3 edge disjoint graphs G_1, G_2 and G_3 such that G_1 and G_2 are isomorphic to $K_{1,n}$ and G_3 is isomorphic to $K_n \Theta K_1$.

Retrieval Number: A12121291S419/2019©BEIESP
DOI: 10.35940/ijeat.A1212.1291S419

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Case (i) : $n \equiv 0 \ (mod \ \ 3\ )$. Let $n = 3t$.

Now, the graph $K_{1,n}^{+++}$has $6t + 1$ vertices and $\frac{15t+9t^2}{2}$ edges. We have 3 subcases.

(i) $v_f(0) = v_f(2) = 2t$, $v_f(1) = 2t + 1$.
(ii) $v_f(0) = v_f(1) = 2t$, $v_f(2) = 2t + 1$.
(iii) $v_f(1) = v_f(2) = 2t$, $v_f(0) = 2t + 1$ and

$$e_f(0) = e_f(1) = e_f(2) = \frac{5t+3t^2}{2}. \text{———}(1)$$

Subcase (i): $v_f(0) = v_f(2) = 2t$, $v_f(1) = 2t + 1$.

Now, we consider the subgraphs G_1, $G_2$and G_3 of $K_{1,n}^{+++}$. We label the central vertex as 1, label all the other vertices of G_1 and G_2 of $K_{1,n}^{+++}$ as in the case (i) of Theorem 2.7[7],. It follows that, the graphs $G_1$and G_2 together gives the edge contributions as

$$e_{f_{G_1\cup G_2}}(0) = e_{f_{G_1\cup G_2}}(1) = e_{f_{G_1\cup G_2}}(2) = 2t$$

and also from case (i) of Theorem,2.1 [11], it follows that the graph G_3gives the edge contributions as

$$e_{f_{G_3}}(0) = \binom{t}{2} + t^2 + t^2 + t,\ e_{f_{G_3}}(1) = \binom{t}{2} + t,$$

$$e_{f_{G_3}}(2) = \binom{t}{2} + t^2 + t. \text{ Totally, we have}$$

$$e_f(0) = \frac{5t+5t^2}{2}\quad e_f(1) = \frac{5t+t^2}{2}\quad e_f(2) = \frac{5t+3t^2}{2}.$$

In this subcase, if we reduce the contribution t^2 from $e_f(0)$ and add the contribution t^2 to $e_f(1)$, we get the condition (1). But, this removal affects the vertex labeling condition (i).

Subcase (ii): $v_f(0) = v_f(1) = 2t$, $v_f(2) = 2t + 1$.

Suppose the central vertex is labeled as 2. We apply the similar argument in subcase (i) of case (i), we get

$$e_{f_{G_1\cup G_2}}(0) = 2t,\ e_{f_{G_1\cup G_2}}(1) = 0,\ e_{f_{G_1\cup G_2}}(2) = 4t,$$

$$e_{f_{G_3}}(0) = \binom{t}{2} + t^2 + t^2 + t,\ e_{f_{G_3}}(1) = \binom{t}{2} + t,$$

$$e_{f_{G_3}}(2) = \binom{t}{2} + t^2 + t. \text{ Totally, we have}$$

$$e_f(0) = \frac{5t+5t^2}{2}\quad e_f(1) = \frac{t+t^2}{2}\quad e_f(2) = \frac{9t+3t^2}{2}.$$

In this subcase, if we reduce the contribution t^2 from $e_f(0)$ and $2t$ from $e_f(2)$ add the contribution$t^2 + 2t$ to $e_f(1)$, we get the condition (1). But, this removal affects the vertex labelingcondition (ii).

Subcase (iii):$v_f(1) = v_f(2) = 2t$, $v_f(0) = 2t + 1$.

Suppose the central vertex is labeled 0. We apply the similar argument in subcase (ii) of case (i), we get

$$e_{f_{G_1\cup G_2}}(0) = 6t,\ e_{f_{G_1\cup G_2}}(1) = 0,\ e_{f_{G_1\cup G_2}}(2) = 0,$$

$$e_{f_{G_3}}(0) = \binom{t}{2} + t^2 + t^2 + t,\ e_{f_{G_3}}(1) = \binom{t}{2} + t,$$

$$e_{f_{G_3}}(2) = \binom{t}{2} + t^2 + t. \text{ Totally, we have}$$

$$e_f(0) = \frac{13t+5t^2}{2}\quad e_f(1) = \frac{t+t^2}{2}\quad e_f(2) = \frac{t+3t^2}{2}.$$

In this subcase, if we reduce the contribution $t^2 + 4t$ from $e_f(0)$ add the contribution $t^2 + 2t$ to $e_f(1)$ and $2t$ to e_f (2), we get the condition (1). But, this removal affects the vertex labeling condition (iii).

Case (ii):$n \equiv 1 (mod\ 3\ )$. Let $n = 3t + 1$.

Now the graph $K_{1,n}^{+++}$has $6t + 3$ vertices and $\frac{6+21t+9t^2}{2}$ edges. We have only one subcase.

$$v_f(0) = v_f(1) = v_f(2) = 2t + 1.\text{----------}(2)$$

$$e_f(0) = e_f(1) = e_f(2) = \frac{2+7t+3t^2}{2}. \text{———}(3)$$

From subcase (iii) of case (ii) of Theorem 2.1 [11] it follows that the graph G_3 gives the edge contributions as

$$e_{f_{G_3}}(0) = \binom{t}{2} + t^2 + t^2 + t + t + 1,\ e_{f_{G_3}}(1) = \binom{t}{2} + t,$$

$e_{f_{G_3}}(2) = \binom{t+1}{2} + t^2 + t + t.$ Also, the graphs G_1 and G_3gives $e_{f_{G_1}}(0) = t+1$, $e_{f_{G_1}}(1) = t$, $e_{f_{G_1}}(2) = t$ and $e_{f_{G_2}}(0) = t$, $e_{f_{G_2}}(1) = t$, $e_{f_{G_2}}(2) = t+1$. Totally, we have $e_f(0) = \frac{4+7t+5t^2}{2}$, $e_f(1) = \frac{5t+t^2}{2}$,

$$e_f(2) = \frac{2+9t+3t^2}{2}.$$

In this subcase, if we reduce the contribution t^2+1 from $e_f(0)$ and t from $e_f(2)$, and add the contribution $t^2 + t + 1$ to $e_f(1)$, we get the condition (3). But, this removal affects the vertex labeling condition (2)..

Case (iii): $n \equiv 2(mod\ 3\ )$. Let $n = 3t + 2$.

Now the graph $K_{1,n}^{+++}$ has $6t + 5$ vertices and $\frac{14+27t+9t^2}{2}$ edges. We have 3 subcases.

(i) $v_f(0) = v_f(2) = 2t+2$, $v_f(1) = 2t + 1$.
(ii) $v_f(0) = v_f(1) = 2t+2$, $v_f(2) = 2t + 1$.
(iii) $v_f(1) = v_f(2) = 2t+2$, $v_f(0) = 2t + 1$ and

(a) $e_f(0) = \frac{5+9t+3t^2}{2}$, $e_f(1) = \frac{5+9t+3t^2}{2}$,

$$e_f(2) = \frac{4+9t+3t^2}{2}.$$

(b) $e_f(0) = \frac{5+9t+3t^2}{2}$, $e_f(2) = \frac{5+9t+3t^2}{2}$,

$$e_f(1) = \frac{4+9t+3t^2}{2}.$$

Retrieval Number: A12121291S419/2019©BEIESP
DOI: 10.35940/ijeat.A1212.1291S419

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(c) $e_f(1) = \dfrac{5+9t+3t^2}{2}$, $e_f(2) = \dfrac{5+9t+3t^2}{2}$,

$e_f(0) = \dfrac{4+9t+3t^2}{2}$.

Subcase(i): $v_f(0) = v_f(2) = 2t+2,\ v_f(1) = 2t + 1.$

Suppose the central vertex islabeled as 1.From Theorem 2.7 [7], this labeling does not affect the vertex labeling of G_1 and G_2 , but from Theorem 2.1 [11], this labeling affects the vertex labeling of G_3.

Subcase (ii): $v_f(0) = v_f(1) = 2t+2,\ v_f(2) = 2t + 1.$

Suppose the central vertex is labeled as 1. From Theorem 2.7 [7] and from Theorem 2.1 [11], this labeling does not affect the vertex labeling of G_1 and G_3 , but from Theorem 2.7 [7], this labeling affects the vertex labeling of G_2.

Subcase (iii): $v_f(1) = v_f(2) = 2t+2,\ v_f(0) = 2t + 1.$

Suppose the central vertex islabeled as 1. From Theorem 2.7 [7] and from Theorem 2.1 [11], this labeling does not affect the vertex labeling of $G_1$and G_3 , but from Theorem 2.7 [7], this labeling affects the vertex labeling of G_2.

Thus in all the above subcases, we see that $K_{1,n}^{+++}$ is not geometric mean cordial.

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AUTHORS PROFILE

Dr. K. Nagarajan is working as a Professor of mathematics at Kalasalingamuniversity, Krishnankoil. He was working as Head and Associate Professorof Department of mathematics at Sri S.Ramasamy Naidu Memorial College, Sattur. He has guided 38 M.Phil. Scholars and 4 Ph.D. Scholars. He is currently guiding 5 Ph.D. Scholars. Having published 11 research papers in national level journals and 40 research papers in international level journals.He has completed Minor Research Project sponsored by UGC.He has 9 years experience in research and his area of interests in research is graph labelling, domination of graphs, distances in graphs and energy of graphs.

K. Chitra Lakshmi, \ Working as an Assistant Professor in Mathematics at V. V. Vanniaperumal College for Women, Virudhunagar. Working as Research Scholar for 4 years under the Guidance of Dr K. Nagarajan, Department of Mathematics Kalasalingam Academy of Research and Education Krishnankoil - 626 126.