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Algebraic Properties of Implication-Based Anti-Fuzzy Subgroup over a Finite Group

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***Abstract:** This article deals with few algebraic characteristics of implication-based anti-fuzzy subgroup of a finite group.In addition, the implication-based anti-fuzzy direct product of implication-based anti-fuzzy subgroups over finite groups is developed and studied elaborately. The condition for an implication-based anti-fuzzy subgroup of a finite group to be a conjugate to another implication-based anti-fuzzy subgroup is conceptualized. Some of their characteristics are investigated in this paper.*

***Keywords:** Implication-based anti-fuzzy subgroup, Implication-based anti-fuzzy normal subgroup, Implication-based anti-fuzzy direct product.*

I. INTRODUCTION

Zadeh[16] first introduced the concept of *fuzzy* set in 1965. In 1971, Rosenfeld [10] applied the theory of fuzzy sets developed by Zadeh to groups and framed the elementary theory of groupoids and groups. Henceforth, intensive research was elaborately done in this field. The algebraic structures of *fuzzy subgroups* were studied by [7], [3], [4] and many other researchers. In 1990, Biswas [1] postulated the theory of *anti-fuzzy subgroups* of groups [8], [5], [15], [9], [2], [13] and many others studied *anti-fuzzy subgroups* in detail. In 2015, M. Selvarathi [11], [12] studied about *implication-based fuzzy normal subgroup* and *implication-based anti-fuzzy subgroup* over a finite group. In this paper, further research had been done on *implication-based anti-fuzzy subgroups* and their direct product. Few basic properties of them are also proved.

II. PRELIMINARIES

Definition II.1[10]

Let$(\Omega,\cdot)$ be a group. Let a fuzzy set in Ω be a function A from Ω to [0,1]. A will be known as *fuzzy subgroup* of Ω if for all ξ_1,ξ_2 in Ω

(i) $A(\xi_1\xi_2) \geq min(A(\xi_1),A(\xi_2))$

(ii) $A({\xi_1}^{-1}) \geq A(\xi_1)$

Letχ be an universe of discourse and $(\Omega,\cdot)$be a group. In *fuzzy* logic, truth value of *fuzzy* proposition $\xi$is denoted by $[\xi]$. The *fuzzy* logical and the corresponding set theoretical notations used in this paper are

(i) $(\xi \in A) = A(\xi)$;

(ii) $(\psi \wedge \tau) = min\{[\psi],[\tau]\}$;

(iii) $(\psi \to \tau) = min\{1,1-[\psi]+[\tau]\}$;

(iv) $(\forall \xi \psi(\xi)) = inf_{\xi\in\chi}[\psi(\xi)]$;

(v) $(\exists \xi \psi(\xi)) = sup_{\xi\in\chi}[\psi(\xi)]$;

Definition II.2[14]

If a *fuzzy* subset A of a group Ω satisfies for any $\xi_1,\xi_2 \in \Omega$

(i) $\vDash (\xi_1 \in A) \wedge (\xi_2 \in A) \to (\xi_1\xi_2 \in A)$

(ii) $\vDash (\xi_1 \in A) \to ({\xi_1}^{-1} \in A)$

Then A is called a *fuzzifying* subgroup.

The concept of λ - tautology is $\vDash_\lambda \psi$ if and only if $(\psi) \geq \lambda$ for all valuation by Ying [6].

Definition II.3[14]

Let A be a *fuzzy* subset of a finite group Ω and $\lambda \in (0,1]$ is a fixed number. If for any $\xi_1,\xi_2 \in \Omega$

(i) $\vDash_\lambda (\xi_1 \in A) \wedge (\xi_2 \in A) \to (\xi_1\xi_2 \in A)$

(ii) $\vDash_\lambda (\xi_1 \in A) \to ({\xi_1}^{-1} \in A)$

Then A is called an *implication-based fuzzy subgroup* of Ω.

Definition II.4[1]

Let Ω be a group. A fuzzy subset μ of Ω is called an *anti-fuzzy subgroup* of Ω if for $\xi,\psi \in\Omega$

(i) $\mu(\xi\psi) \leq \mu(\xi)\mu(\psi)$

(ii) $\mu(\xi^{-1}) \leq \mu(\xi)$

Definition II.5 [12]

Let A be a *fuzzy* subset of a finite group Ω and $\lambda \in (0,1]$ is a fixed number. If for any $\xi,\psi \in \Omega$

$\vDash_\lambda (\xi\psi \in A) \to ((\xi \in A) \vee (\psi \in A))$

$\vDash_\lambda (\xi^{-1} \in A) \to (\xi \in A)$

Then A is called an *implication-based anti-fuzzy* subgroup of Ω.

Example II.1 [12]

Consider the finite group $\Omega = \{\varepsilon,\alpha,\beta,\gamma\}$ along with the binary operation ' $*$ ' whose closure table is as follows.

$*$	ε	α	β	γ
ε	ε	α	β	γ
α	α	ε	γ	β
β	β	γ	ε	α
γ	γ	β	α	ε

For the *fuzzy* set $A:\Omega \to [0,1]$ defined by $A(\varepsilon) = 0.2, A(\alpha) = 0.3, A(\beta) = 0.4, A(\gamma) = 0.5$ with $\lambda = .4$and the *implication* operator is that of Kleene-Dienes, we have

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$\vee$	$[\varepsilon]$	$[\alpha]$	$[\beta]$	$[\gamma]$
$[\varepsilon]$	0.2	0.3	0.4	0.5
$[\alpha]$	0.3	0.3	0.4	0.5
$[\beta]$	0.4	0.4	0.4	0.5
$[\gamma]$	0.5	0.5	0.5	0.5

Then A is an *implication-based anti-fuzzy* subgroup of Ω.

Definition II.6 [12]
Let A is an *implication-based anti-fuzzy* subgroup of a group Ω. A is called an *implication-based anti-fuzzy* normal subgroup of a group Ω if $\vDash_\lambda (\xi\psi \in A) \to (\psi\xi \in A)$ for all $\xi, \psi \in \Omega$

Definition II.7 [12]
Let A and B be two *implication-based anti-fuzzy* subgroups of the finite group Ω, then the internal product $A \cdot B$ is defined by

$$\vDash_\lambda (\forall\psi, \tau\{(\psi \in A) \vee (\tau \in B)\}; \psi\tau = \xi; \psi, \tau \in \Omega) \to (\xi \in A \cdot B), \quad \xi \in \Omega$$

Theorem II.1 [12]
A is an implication-based fuzzy subgroup of the group Ω if and only if its complement A^C is an implication-based anti-fuzzy subgroup of Ω.

Theorem II.2 [12]
Intersection of two implication-based anti-fuzzy subgroups of a group Ω is again an implication-based anti-fuzzy subgroup of Ω.

Theorem II.3 [12]
Intersection of two implication-based fuzzy subgroups of a group Ω is an implication-based fuzzy subgroup of Ω.

Theorem II.4 [12]
A is an implication-based anti-fuzzy subgroup of Ω if and only if for all $\xi, \psi \in \Omega$

$$\vDash_\lambda (\xi\psi^{-1} \in A) \to ((\xi \in A) \vee (\psi \in A))$$

Theorem II.5 [12]
If A is an implication-based anti-fuzzy subgroup of Ω with the identity element 'ε' then for all $\xi \in \Omega$

(i) $\vDash_\lambda (\varepsilon \in A) \to (\xi \in A)$

(ii) $\vDash_\lambda (\xi \in A) \to (\xi^{-1} \in A)$

Theorem II.6 [12]
If B is an implication-based anti-fuzzy normal subgroup of the finite group Ω and C is an implication-based anti-fuzzy subgroups of Ω then

(i) $B \cdot C = C \cdot B$

(ii) $B \cdot C$ *is an implication-based anti-fuzzy subgroup of Ω*

Hereafter let $(\Omega, \cdot)$ denote a group and 'ε' be its identity element. And let $\lambda \in (0,1]$ be a fixed number.

III. RESULT ANALYSIS OF IMPLICATION-BASED ANTI-FUZZY SUBGROUPS OVER A FINITE GROUP

Theorem III.1
Let A and B be implication-based anti-fuzzy subgroups of a finite group Ω. Then $A \cup B$ is also an implication-based anti-fuzzy subgroup of a finite group.

Proof:
Let A and B be *implication-based anti-fuzzy* subgroups of a finite group Ω.
By Theorem II.1, A^C and B^C are *implication-based fuzzy* subgroups of a finite group Ω.
By Theorem II.3, $A^C \cap B^C$ are *implication-based fuzzy* subgroups of a finite group Ω.

$$\vDash_\lambda (\xi \in A^C \cap B^C) \wedge (\psi \in A^C \cap B^C) \to (\xi\psi^{-1} \in A^C \cap B^C)$$

By De Morgan's Law

$$\vDash_\lambda (\xi \in (A \cup B)^C) \wedge (\psi \in (A \cup B)^C) \to (\xi\psi^{-1} \in (A \cup B)^C)$$

$$\vDash_\lambda (1 - (\xi \in (A \cup B))) \wedge (1 - (\psi \in (A \cup B))) \to (1 - (\xi\psi^{-1} \in (A \cup B)))$$

$$\vDash_\lambda (\xi\psi^{-1} \in (A \cup B))) \to 1 - ((1 - (\xi \in (A \cup B))) \wedge (1 - (\psi \in (A \cup B))))$$

$$\vDash_\lambda (\xi\psi^{-1} \in (A \cup B))) \to (\xi \in (A \cup B)) \vee (\psi \in (A \cup B))$$

By Theorem II.4, Therefore $A \cup B$ is an *implication-based anti-fuzzy* subgroup of a finite group Ω.

Definition III.1
Let A be an *implication-based anti-fuzzy* subgroup of a finite group Ω and $\varphi: \Omega \to \Omega$ be a function defined on Ω. Then the *implication-based anti-fuzzy* subgroup B of $\varphi(\Omega)$ is defined by $\vDash_\lambda (\forall\xi\{(\xi \in A)\}; \xi = \varphi^{-1}(\psi)) \to (\psi \in B)$ for all $\psi \in \varphi(\Omega)$.
Similarly, if B is an *implication-based anti-fuzzy* subgroup of $\varphi(\Omega)$ then the *implication-based anti-fuzzy* subgroup $A = \varphi \circ B$ in Ω is defined as $\vDash_\lambda (\varphi(\xi) \in B) \to (\xi \in A)$ for all $\xi \in \Omega$, and is called the pre-image of B under φ.

Definition III.2
An *implication-based anti-fuzzy* subgroup A of a finite group Ω is said to have infremum - property if for any $B \subseteq \Omega$, there exists $\eta_0 \in B$ such that $\vDash_\lambda (\forall\eta\{(\eta \in A)\}; \eta \in B) \to (\eta_0 \in A)$.

Theorem III.2
Let A be an implication-based anti-fuzzy subgroup of a finite group Ω. Then A is an implication-based anti-fuzzy normal subgroup of Ω if and only if $\vDash_\lambda (\xi^{-1}\psi^{-1}\xi\psi \in A) \to (\xi \in A)$ for all $\xi, \psi \in \Omega$.

Proof:
Let $\xi, \psi \in \Omega$.
$(i) \Rightarrow (ii)$
A is an *implication-based fuzzy* normal subgroup of the group Ω.

$$\vDash_\lambda (\xi^{-1}\psi^{-1}\xi\psi \in A) \to (\psi^{-1}\xi\psi.\xi^{-1} \in A) \to (\psi^{-1}\xi\psi \in A) \vee (\xi^{-1} \in A) \to (\xi \in A) \vee (\xi \in A) \to (\xi \in A)$$

Conversely $(ii) \Rightarrow (i)$

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Assume that
$\vDash_\lambda (\xi^{-1}\psi^{-1}\xi\psi \in A) \to (\xi \in A)$ for all $\xi, \psi \in \Omega$.
Let $\xi, \psi \in \Omega$.
$\vDash_\lambda (\xi^{-1}\psi\xi \in A)$

$$\to (\psi\psi^{-1}.\xi^{-1}\psi\xi \in A)$$
$$\to (\psi.\psi^{-1}\xi^{-1}\psi\xi \in A)$$
$$\to (\psi \in A) \vee (\psi^{-1}\xi^{-1}\psi\xi \in A)$$
$$\to (\psi \in A)$$

Also
$\vDash_\lambda (\psi \in A)$

$$\to (\xi\xi^{-1}.\psi.\xi\xi^{-1} \in A)$$
$$\to (\xi.\xi^{-1}\psi\xi.\xi^{-1} \in A)$$
$$\to (\xi \in A) \vee (\xi^{-1}\psi\xi \in A) \vee (\xi^{-1} \in A)$$
$$\to (\xi \in A) \vee (\xi^{-1}\psi\xi \in A)$$

Therefore $\vDash_\lambda (\psi \in A) \to (\xi \in A) \vee (\xi^{-1}\psi\xi \in A)$
Case (i): Suppose
$\vDash_\lambda (\xi \in A) \to (\xi \in A) \vee (\xi^{-1}\psi\xi \in A)$
$\Rightarrow \vDash_\lambda (\xi \in A) \to (\psi \in A)$ for all $\xi, \psi \in \Omega$
$\Rightarrow \vDash_\lambda (\xi\psi \in A) \to (\psi\xi \in A)$ for all $\xi, \psi \in \Omega$
Case(ii): Suppose
$\vDash_\lambda (\xi^{-1}\psi\xi \in A) \to (\xi \in A) \vee (\xi^{-1}\psi\xi \in A)$
$\Rightarrow \vDash_\lambda (\xi^{-1}\psi\xi \in A) \to (\psi \in A)$ for all $\xi, \psi \in \Omega$
$\Rightarrow \vDash_\lambda (\xi\psi \in A) \to (\psi\xi \in A)$ for all $\xi, \psi \in \Omega$
Therefore, A is an *implication-based anti-fuzzy* normal subgroup of the group Ω.

Theorem III.3
A homomorphic image or pre-image of an implication-based anti-fuzzy subgroup of a finite group Ω is also an implication-based anti-fuzzy subgroup provided in the former case the infremum property holds.
Proof:
Let A be an *implication-based anti-fuzzy* subgroup of a finite group Ω for which the infremum property holds. Let $f: G \to G$ be a homomorphism.
For images:
Let $\varphi(\xi), \varphi(\psi) \in \Omega$.
Also let $\xi_0 \in \varphi^{-1}(\varphi(\xi))$ and $\psi_0 \in \varphi^{-1}(\varphi(\psi))$ be such that
$\vDash_\lambda (\forall\eta\{(\eta \in A)\}; \eta \in \varphi^{-1}(\varphi(\xi))) \to (\xi_0 \in A)$
$\vDash_\lambda (\forall\eta\{(\eta \in A)\}; \eta \in \varphi^{-1}(\varphi(\psi))) \to (\psi_0 \in A)$

$$\vDash_\lambda (\varphi(\xi).\varphi(\psi) \in B)$$
$$\to (\forall\tau\{(\tau \in A)\}; \tau \in \varphi^{-1}(\varphi(\xi).\varphi(\psi)))$$
$$\to (\xi_0\psi_0 \in A)$$
$$\to (\xi_0 \in A) \vee (\psi_0 \in A)$$
$$\to (\varphi(\xi) \in B) \vee (\varphi(\psi) \in B)$$

For pre-images: Let $\xi, \psi \in \Omega$.

$$\vDash_\lambda (\xi\psi \in A)$$
$$\to (\varphi(\xi\psi) \in B)$$
$$\to (\varphi(\xi).\varphi(\psi) \in B)$$

Since φ is a homomorphism

$$\to (\varphi(\xi) \in B) \vee (\varphi(\psi) \in B)$$
$$\to (\xi \in A) \vee (\psi \in A)$$

$$\vDash_\lambda (\xi^{-1} \in A) \to (\varphi(\xi^{-1}) \in B)$$
$$\to (\varphi(\xi)^{-1} \in B)$$
$$\to (\varphi(\xi) \in B)$$
$$\to (\xi \in A)$$

Theorem III.4
Let $\varphi: \Omega \to \Omega$ be a homomorphism on a finite group Ω. Let B be an implication-based anti-fuzzy normal subgroup of $\varphi(\Omega)$. Then $A = \varphi \circ B$ is also an implication-based anti-fuzzy normal subgroup of Ω.
Proof:
Let B be an implication-based fuzzy normal subgroup of $\varphi(\Omega)$.By Theorem III.3, $A = \varphi \circ B$is also an *implication-based anti-fuzzy* subgroup of Ω.
Let $\xi, \psi \in \Omega$

$$\vDash_\lambda (\xi\psi \in A)$$
$$\to (\varphi(\xi\psi) \in B)$$
$$\to (\varphi(\xi)\varphi(\psi) \in B)$$

Since φ is a homomorphism

$$\to (\varphi(\psi)\varphi(\xi) \in B)$$

Since B is an implication-based anti-fuzzy normal subgroup.

$$\to (\varphi(\psi\xi) \in B)$$
$$\to (\psi\xi \in A)$$

Therefore $A = \varphi \circ B$ is also an *implication-based anti-fuzzy* normal subgroup of Ω.

IV. PRODUCT OF IMPLICATION-BASED ANTI-FUZZY SUBGROUPS OVER A FINITE GROUP

Definition IV.1
Let Ω_1 and Ω_2 be two finite groups. $\Omega = \Omega_1 \times \Omega_2$$ be the direct product of Ω_1 and Ω_2. Then the *implication-based anti-fuzzy* direct product of the two *implication-based anti-fuzzy* subgroups $A_1$and A_2 of Ω_1 and Ω_2 respectively, is denoted by $A_1 \times A_2$ and is defined as follows
$\vDash_\lambda ((\alpha \in A_1) \vee (\beta \in A_2)) \to ((\alpha, \beta) \in A_1 \times A_2) \forall\alpha \in \Omega_1$ and $\forall\beta \in \Omega_2$.

Theorem IV.1
Let A and B be two implication-based anti-fuzzy subgroups of two finite groups Ω_1 and Ω_2 respectively. Then $A \times B$ is also an implication-based anti-fuzzy subgroup of $\Omega_1 \times \Omega_2$.

Proof:
Let $\xi = (\alpha_1, \beta_1)$ and $\psi = (\alpha_2, \beta_2)$ be any two elements of the group $\Omega_1 \times \Omega_2$.
$\vDash_\lambda (\xi\psi^{-1} \in A \times B)$
$\to ((\alpha_1\alpha_2{}^{-1}, \beta_1\beta_2{}^{-1}) \in A \times B)$
$\to (\alpha_1\alpha_2{}^{-1} \in A) \vee (\beta_1\beta_2{}^{-1} \in B)$
$\to (\alpha_1 \in A) \vee (\alpha_2{}^{-1} \in A) \vee (\beta_1 \in B) \vee (\beta_2{}^{-1} \in B)$
$\to (\alpha_1 \in A) \vee (\alpha_2 \in A) \vee (\beta_1 \in B) \vee (\beta_2 \in B)$
$\to (\alpha_1 \in A) \vee (\beta_1 \in B) \vee (\alpha_2 \in A) \vee (\beta_2 \in B)$
$\to ((\alpha_1, \beta_1) \in A \times B) \vee ((\alpha_2, \beta_2) \in A \times B)$
$\to (\xi \in A \times B) \vee (\psi \in A \times B)$
By Theorem II.4, $A \times B$ is also an *implication-based anti-fuzzy subgroup of $\Omega_1 \times \Omega_2$.*
Theorem IV.2
Let A and B be two implication-based anti-fuzzy normal subgroups of two finite groups Ω_1 and Ω_2 respectively. Then $A \times B$ is also an implication-based anti-fuzzy normal subgroup of $\Omega_1 \times \Omega_2$.
Proof:
By Theorem IV.1,$A \times B$ is also an *implication-based anti-fuzzy* subgroup of $\Omega_1 \times \Omega_2$.
Let $\xi = (\alpha_1, \beta_1)$ and $\psi = (\alpha_2, \beta_2)$ be any two elements of the group $\Omega_1 \times \Omega_2$.

$\vDash_{\lambda} (\xi\psi \in A \times B) \to ((\alpha_1\alpha_2, \beta_1\beta_2) \in A \times B)$
$\to (\alpha_1\alpha_2 \in A) \vee (\beta_1\beta_2 \in B)$
$\to (\alpha_2\alpha_1 \in A) \vee (\beta_2\beta_1 \in B)$
$\to ((\alpha_2\alpha_1, \beta_2\beta_1) \in A \times B)$
$\to (\psi\xi \in A \times B)$
Therefore $A \times B$ is also an *implication-based anti-fuzzy* subgroup of $\Omega_1 \times \Omega_2$.

Corollary IV.1
By Theorem II.6 and by Theorem III.3, if A is an implication-based anti-fuzzy normal subgroup of Ω*.and B be implication-based anti-fuzzy subgroups of* Ω*. Then* $A \cdot B$ *and its pre-image are also an implication-based anti-fuzzy normal subgroups of* Ω.

Corollary IV.2
By Theorem IV.1, Theorem IV.2, Theorem III.3 and Theorem III.4, if A and B are {implication-based anti-fuzzy normal subgroups of Ω*, then* $A \times B$ *and its pre-image are also an implication-based anti-fuzzy normal subgroups of* Ω.

Definition IV.2
An implication-based anti-fuzzy subgroup A of a finite group Ω *is said to be conjugate to an implication-based anti-fuzzy subgroup B of* Ω *if there exists* $\psi \in \Omega$ *such that* $\vDash_{\lambda} (\xi \in A) \to (\psi^{-1}\xi\psi \in B)$ *for all* $\xi \in \Omega$.

Theorem IV.3
Let an implication-based anti-fuzzy subgroup A_1 *of a finite group* Ω_1 *be conjugate to an implication-based anti-fuzzy subgroup* B_1 *of* Ω_1*. Also let an implication-based anti-fuzzy subgroup* A_2 *of a finite group* Ω_2 *be conjugate to an implication-based anti-fuzzy subgroup* B_2 *of* Ω_2*. Then the implication-based anti-fuzzy subgroup* $A_1 \times A_2$ *of* $\Omega_1 \times \Omega_2$ *is conjugate to the implication-based anti-fuzzy subgroup*$B_1 \times B_2$of $\Omega_1 \times \Omega_2$.
Proof:
Let $(\xi, \psi) \in \Omega_1 \times \Omega_2$
$\vDash_{\lambda} ((\xi, \psi) \in A_1 \times A_2)$
$\to (\xi \in A_1) \vee (\psi \in A_2)$
$\to (\gamma^{-1}\xi\gamma \in B_1) \vee (\delta^{-1}\psi\delta \in B_2)$
$\to ((\gamma, \delta)^{-1}.(\xi, \psi).(\gamma, \delta) \in B_1 \times B_2)$
Therefore the*implication-based anti-fuzzy* subgroupsubgroup $A_1 \times A_2$ of $\Omega_1 \times \Omega_2$ is conjugate to the implication-based anti-fuzzy subgroup $B_1 \times B_2$of $\Omega_1 \times \Omega_2$.

Theorem IV.4
Let A and B be two fuzzy subsets of the finite groupsΩ_1 *and* Ω_2 *respectively. Suppose that* ε_1 *and* ε_2 *are the identity elements of* Ω_1 *and* Ω_2 *respectively. If* $A \times B$ *is an implication-based anti-fuzzy subgroup of* $\Omega_1 \times \Omega_2$*, then atleast one of the following statement holds.*

(i) $\vDash_{\lambda} (\varepsilon_2 \in B) \to (\xi \in A)$ for all $\xi \in \Omega_1$
(ii) $\vDash_{\lambda} (\varepsilon_1 \in A) \to (\psi \in B)$ for all $\psi \in \Omega_2$

Proof:
$A \times B$ is an *implication-based anti-fuzzy* subgroup of $\Omega_1 \times \Omega_2$such that $(i)$and (ii)conditions does not hold.
Then there exists $\xi \in \Omega_1$and all $\psi \in \Omega_2$such that
$\vDash_{\lambda} (\xi \in A) \to (\varepsilon_2 \in B)$ for all $\xi \in \Omega_1$
$\vDash_{\lambda} (\psi \in B) \to (\varepsilon_1 \in A)$ for all $\psi \in \Omega_2$
Let $(\xi, \psi) \in \Omega_1 \times \Omega_2$
$\vDash_{\lambda} ((\xi, \psi) \in A \times B) \to (\xi \in A) \vee (\psi \in B)$
$\to (\varepsilon_2 \in B) \vee (\varepsilon_1 \in A)$
$\to (\varepsilon_1 \in A) \vee (\varepsilon_2 \in B)$
$\to ((\varepsilon_1, \varepsilon_2) \in A \times B)$
By Theorem II.5,$A \times B$ is not an *implication-based anti-fuzzy* subgroup of $\Omega_1 \times \Omega_2$.This is a contradiction to the assumption. Hence either (i)or (ii)should hold.

Theorem IV.5
Let A and B be two fuzzy subsets of two finite groups Ω_1 *and* Ω_2 *respectively such that* $\vDash_{\lambda} (\varepsilon_2 \in B) \to (\xi \in A)$ *for all* $\xi \in \Omega_1$ *where* ε_2 *is the identity element of* Ω_2*. If* $A \times B$ *is an implication-based anti-fuzzy subgroup of* $\Omega_1 \times \Omega_2$*,then A is an implication-based anti-fuzzy subgroup of* Ω_1.
Proof:
Let $A \times B$ be an *implication-based anti-fuzzy subgroup* of $\Omega_1 \times \Omega_2$. Let $\xi, \psi \in \Omega_1$.
$\Rightarrow (\xi, \varepsilon_2), (\psi, \varepsilon_2) \in \Omega_1 \times \Omega_2$
$\vDash_{\lambda} (\xi\psi \in A)$
$\to (\xi\psi \in A) \vee (\varepsilon_2\varepsilon_2 \in B)$
$\to ((\xi\psi, \varepsilon_2\varepsilon_2) \in A \times B)$
$\to ((\xi, \varepsilon_2).(\psi, \varepsilon_2) \in A \times B)$
$\to ((\xi, \varepsilon_2) \in A \times B) \vee ((\psi, \varepsilon_2) \in A \times B)$
$\to (\xi \in A) \vee (\varepsilon_2 \in B) \vee (\psi \in A) \vee (\varepsilon_2 \in B)$
$\to (\xi \in A) \vee (\psi \in A)$

$\vDash_{\lambda} (\xi^{-1} \in A) \to (\xi^{-1} \in A) \vee ({\varepsilon_2}^{-1} \in B)$
$\to ((\xi^{-1}, {\varepsilon_2}^{-1}) \in A \times B)$
$\to ((\xi, \varepsilon_2)^{-1} \in A \times B)$
$\to ((\xi, \varepsilon_2) \in A \times B)$
$\to (\xi \in A) \vee (\varepsilon_2 \in B)$
$\to (\xi \in A)$
Therefore A is an implication-based anti-fuzzy subgroup of Ω_1.

Corollary IV.3
Let A and B be two fuzzy subsets of two finite groups Ω_1 *and* Ω_2 *respectively such that* $\vDash_{\lambda} (\varepsilon_1 \in A) \to (\xi \in B)$ *for all* $\xi \in \Omega_2$*where* ε_1 *is the identity element of* Ω_1*. If* $A \times B$ *is an implication-based anti-fuzzy subgroup of* $\Omega_1 \times \Omega_2$*, then B is an implication-based anti-fuzzy subgroup of* Ω_2 .

Corollary IV.4
By Theorem IV.4, Theorem IV.5 and Corollary IV.3 we have, let A and B be fuzzy subsets of the finite groups Ω_1 *and* Ω_2 *respectively. If* $A \times B$ *is an implication-based anti-fuzzy subgroup of* $\Omega_1 \times \Omega_2$*, then either A is an implication-based anti-fuzzy subgroup of* Ω_1*, or B is an implication-based anti-fuzzy subgroup of* Ω_2 .

V. CONCLUSION

In this paper, the notion of *implication-based anti-fuzzy*direct product of *implication-based anti-fuzzy* subgroup of a finite group is introduced. It is proved that union of two *implication-based anti-fuzzy* subgroups of a finite group is also an *implication-based anti-fuzzy* subgroup. Further, the homomorphic pre-image of *implication-based anti-fuzzy normal* subgroup of a finite group is also an *implication-based anti-fuzzy normal* subgroup. Certain algebraic properties of *implication-based anti-fuzzy* direct product of *implication-based anti-fuzzy* subgroups of a finite group are also proved.

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