Abstract: An l-edge-weighting of a graph G is a map \( \mathcal{E}: E(G) \rightarrow \{1, 2, 3, \ldots, l\} \), where \( l \) is a positive integer. For a vertex \( v \in V(G) \), the weight \( S_G(v) \) is the sum of edge-weights appearing on the edges incident at \( v \) under the edge-weighting \( \mathcal{E} \). An l-edge-weighting of G is coprime irregular edge-weighting of G if \( \gcd(S_G(u), S_G(v)) = 1 \) for every pair of adjacent vertices \( u \) and \( v \) in \( G \). A graph \( G \) is coprime irregular if \( G \) admits a coprime irregular edge-weighting.

Example 2.2. A graph \( G \) given in Figure 1.1 is coprime irregular.

Theorem 2.3. Path \( P_n \) on \( n \geq 3 \) vertices is coprime irregular for all \( n \).

Proof. Let \( P_n = (v_1, v_2, v_3, \ldots, v_n) \). We now prove this theorem by considering the following cases.

Case (i) \( n \) is even.

Define an edge-weighting \( \mathcal{E} \) of \( P_n \) as follows. For all \( 1 \leq i \leq n-1 \), let \( \mathcal{E}(v_i, v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \text{ or } 2 \pmod{4} \\ 2 & \text{otherwise} \end{cases} \)

Then \( S_G(v_i) = 1 \), \( S_G(v_n) = 1 \) or 2 according as \( n \equiv 2 \pmod{4} \) or \( n \equiv 0 \pmod{4} \); and for each \( 2 \leq i \leq n-1 \), we have

\[ S_G(v_i) = \mathcal{E}(v_{i-1}, v_i) + \mathcal{E}(v_i, v_{i+1}) = \begin{cases} 4 & \text{if } i \equiv 0 \pmod{4} \\ 2 & \text{if } i \equiv 2 \pmod{4} \\ 3 & \text{otherwise} \end{cases} \]

It is not difficult to see that any two adjacent vertices of \( P_n \) whose weights are coprime under \( \mathcal{E} \) and thus \( P_n \) admits a coprime irregular edge-weighting.

Case (ii) \( n \) is odd.

Now, define for each \( 1 \leq i \leq n-1 \),

\[ \mathcal{E}(v_i, v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 2 \pmod{4} \\ 2 & \text{otherwise} \end{cases} \]

Then \( S_G(v_1) = 2 \), \( S_G(v_n) = 1 \) or 2 according as \( n \equiv 3 \pmod{4} \) or \( n \equiv 1 \pmod{4} \); and for all \( i = 2, 3, \ldots, n-1 \), we have

\[ S_G(v_i) = \mathcal{E}(v_{i-1}, v_i) + \mathcal{E}(v_i, v_{i+1}) = \begin{cases} 3 & \text{if } i \equiv 0 \pmod{2} \\ 4 & \text{otherwise} \end{cases} \]

Obviously, the weights of any two adjacent vertices of \( P_n \) are coprime and thus \( P_n \) is coprime irregular.

Theorem 2.4. For all \( n \geq 4 \), the cycle \( C_n \) is coprime irregular.

Proof. We prove this result in the following two cases.

Case (i) \( n \) is odd.

Considering the edge-weighting \( \mathcal{E} \) of \( C_n \) defined as follows. For all \( i = 1, 2, 3, \ldots, n-1 \), let

\[ \mathcal{E}(v_i, v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{4} \\ 2 & \text{if } i \equiv 2 \pmod{4} \\ 3 & \text{if } i \equiv 3 \pmod{4} \\ 4 & \text{if } i \equiv 0 \pmod{4} \end{cases} \]

and let \( \mathcal{E}(v_n, v_1) = 1 \) or 3 according as \( n \equiv 1 \pmod{4} \) or \( n \equiv 3 \pmod{4} \). Then for any two adjacent vertices of \( C_n \), their weights are coprime. Thus \( \mathcal{E} \) is coprime irregular edge-weighting of \( C_n \) and hence \( C_n \) is coprime irregular.

Case (ii) \( n \) is even.

Assign \( \mathcal{E}(v_1, v_2) = 1 \), \( \mathcal{E}(v_2, v_3) = 2 \), \( \mathcal{E}(v_3, v_4) = 3 \) and \( \mathcal{E}(v_4, v_5) = 4 \) and for all the remaining vertices,
Coprime Irregular graphs

One can easily verify that the weights of any two adjacent vertices of \( C_n \) are coprime and so \( \mathcal{O} \) is a coprime irregular edge-weighting of \( C_n \). Hence \( C_n \) is coprime irregular.

**Definition 2.5.** A triangular snake graph \( TS_n \) is obtained from a path \((u_1, u_2, \ldots, u_{n-1}, u_n)\) by joining \( u_i \) and \( u_{i+1} \) to a new vertex \( v_i \) for \( i = 1, 2, \ldots, n \). A quadrilateral snake \( QS_n \) is obtained from the path \((u_1, u_2, \ldots, u_{n-1}, u_n)\) by introducing \( n \) copies of \( K_2 \), say \( v_1w_1, v_2w_2, \ldots, v_nw_n \) and joining \( v_i \) to \( u_i \) and \( w_i \) to \( u_{i+1} \) for \( i = 1, 2, \ldots, n \).

**Theorem 2.6.** The triangular snake graph \( TS_n \) is coprime irregular for all \( n \).

**Proof.** Suppose \( n \) is even. Now we define an edge-weighting \( \mathcal{O} \) as follows. For all \( 1 \leq i \leq n \), let \( \mathcal{O}(u_i, v_i) = 1 \), \( \mathcal{O}(v_i, u_{i+1}) = 2 \) and \( \mathcal{O}(u_{i+1}, u_i) = 4 \) and \( \mathcal{O}(u_{i+1}, v_i) = \begin{cases} 2 & \text{if } i \equiv 0 \text{ or } 1 \pmod{4} \\ 4 & \text{otherwise} \end{cases} \).

For \( n = 8 \), the graph \( TS_8 \) and edge-weighting \( \mathcal{O} \) are shown in Figure 2.

![Figure 2](image2)

Certainly, by the definition of \( \mathcal{O} \), we have

\[
S_{\mathcal{O}}(u_i) = \begin{cases} 3 & \text{if } i \equiv 0 \pmod{4} \\ 4 & \text{if } i \equiv 1 \pmod{4} \\ 5 & \text{if } i \equiv 2 \pmod{4} \\ 7 & \text{if } i \equiv 3 \pmod{4} \end{cases}
\]

Clearly, the weights of any two adjacent vertices of \( TS_n \) are coprime and hence \( \mathcal{O} \) is a coprime irregular edge-weighting of \( TS_n \).

On the other hand, let us assume that \( n \) is odd. Consider the edge-weighting \( \mathcal{O} \) of \( TS_n \) defined as follows.

For all \( i = 1, 2, \ldots, n \), \( \mathcal{O}(u_i, v_i) = \mathcal{O}(v_i, u_{i+1}) = 1 \) and \( \mathcal{O}(u_{i+1}, v_i) = \begin{cases} 2 & \text{if } i \equiv 1 \pmod{4} \\ 3 & \text{if } i \equiv 0 \pmod{4} \\ 4 & \text{otherwise} \end{cases} \).

For \( n = 9 \), the graph \( TS_9 \) and its edge-weighting given in the following Figure 3.

![Figure 3](image3)

Then \( S_{\mathcal{O}}(u_1) = 3, S_{\mathcal{O}}(u_{n+1}) = 3 \) or \( 5 \) according as \( n \equiv 1 \pmod{4} \) or \( n \equiv 3 \pmod{4} \); and for all \( i = 1, 2, \ldots, n \), we have

\[
S_{\mathcal{O}}(u_i) = \mathcal{O}(u_i, v_i) + \mathcal{O}(u_i, u_{i+1}) + \mathcal{O}(u_{i+1}, u_i) + \mathcal{O}(u_{i+1}, v_i)
\]

Obviously, the weights of any two adjacent vertices of \( TS_n \) is a coprime irregular.

**Theorem 2.7.** For all \( n \), the quadrilateral snake graph \( QS_n \) is coprime irregular.

**Proof.** Consider the following cases.

**Case 1.** \( n \) is even.

Consider the edge-weighting \( \mathcal{O} \) of quadrilateral snake defined as follows.

For all \( i = 1, 2, \ldots, n \),

\[
\mathcal{O}(u_i, v_i) = \mathcal{O}(v_i, w_i) = 2, \mathcal{O}(v_i, u_{i+1}) = 3 \text{ and } \mathcal{O}(u_i, u_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 2 \pmod{4} \\ 3 & \text{if } i \equiv 0 \text{ or } 1 \pmod{4} \\ 5 & \text{otherwise} \end{cases}
\]

For \( n = 6 \), the graph \( QS_6 \) and its edge-weighting given in Figure 4.

![Figure 4](image4)

Then \( S_{\mathcal{O}}(u_1) = 5, S_{\mathcal{O}}(u_{n+1}) = 4 \) or \( 6 \) according as \( n \equiv 3 \pmod{4} \) or \( n \equiv 1 \pmod{4} \); and for all \( i = 1, 2, \ldots, n \), we have

\[
S_{\mathcal{O}}(u_i) = \mathcal{O}(u_i, v_i) + \mathcal{O}(u_i, u_{i+1}) + \mathcal{O}(u_{i+1}, u_i) + \mathcal{O}(u_{i+1}, v_i)
\]

Clearly, the weights of any two adjacent vertices are coprime and so \( \mathcal{O} \) is a coprime irregular edge-weighting of \( QS_n \). Therefore the graph \( QS_n \) is coprime irregular.

**Case 2.** \( n \) is odd

Consider the edge-weighting \( \mathcal{O} \) of the quadrilateral snake defined as follows.

For all \( i = 1, 2, \ldots, n \),

\[
\mathcal{O}(u_i, v_i) = \mathcal{O}(v_i, w_i) = 3, \mathcal{O}(u_i, v_{i+1}) = 4 \text{ and } \mathcal{O}(u_{i+1}, u_i) = \begin{cases} 1 & \text{if } i \equiv 2 \pmod{4} \\ 2 & \text{if } i \equiv 1 \pmod{4} \\ 3 & \text{if } i \equiv 0 \pmod{4} \\ 4 & \text{if } i \equiv 3 \pmod{4} \end{cases}
\]

For \( n = 5 \), the graph \( QS_5 \) and its edge-weighting illustrated in Figure 5.

![Figure 5](image5)
Then \( S_G(u_1) = 4 \), \( S_G(u_{n+1}) = 6 \) or 8 according as \( n \equiv 2 \pmod{4} \) or \( n \equiv 0 \pmod{4} \) and for all \( i=2, 3, \ldots, n \), we have
\[
S_G(u_i) = \{ u_{i-1}v_i + \mathcal{O}(u_{i-1}u_i) + \mathcal{O}(u_{i-1}u_i') + \mathcal{O}(u_{i-1}v_i') \\
\{ \begin{array}{ll}
9 & \text{if } i \equiv 2 \pmod{4} \\
11 & \text{if } i \equiv 1 \pmod{4} \\
13 & \text{otherwise}.
\end{array} \}
\]

Certainly from the above, the weights of any two adjacent vertices are coprime and so \( \mathcal{O} \) is a coprime irregular edge-weighting of \( QS_n \). Hence \( QS_n \) is coprime irregular.

**Definition 2.8.** The corona \( G^* \) of a graph \( G \) is the graph obtained from \( G \) by attaching exactly one pendant edge at each of the vertices of \( G \).

**Theorem 2.9.** Corona of a triangular snake graph is coprime irregular.

**Proof.** Consider the following cases.

**Case 1.** \( n \) is even.

Define an edge-weighting \( \mathcal{O} \) of the corona of a triangular snake as follows.

For all \( i=1, 2, 3, \ldots, n \), define
\[
\mathcal{O}(u_i) = \mathcal{O}(u_{i-1}v_i) + \mathcal{O}(v_iu_i') = \mathcal{O}(u_{i+1}v_i) = 1, \mathcal{O}(u_{i+1}u_i') = 2 \text{ if } i \equiv 0 \pmod{4} \\
3 \text{ if } i \equiv 1 \pmod{4} \\
5 \text{ if } i \equiv 2 \pmod{4} \\
6 \text{ if } i \equiv 3 \pmod{4}
\]

and for all \( i=1, 2, 3, \ldots, n-1 \), \( \mathcal{O}(u_i) = \mathcal{O}(u_{i-1}v_i) = \mathcal{O}(v_iu_i') = 1, \mathcal{O}(u_{i+1}v_i) = \mathcal{O}(u_{i+1}u_i') = 1 \).

For the corona of a triangular snake \( TS_n \) as given in Figure 6.

![Figure 6](image)

Then \( S_G(v_1) = 3 \), \( S_G(v_i') = 1 \) and \( \mathcal{O}(v_i) = 1 \), for all \( i=1, 2, 3, 4, \ldots, n \), we have
\[
S_G(u_i) = \{ u_{i-1}v_i + \mathcal{O}(u_{i-1}u_i) + \mathcal{O}(u_{i-1}u_i') + \mathcal{O}(u_{i-1}v_i') \\
\{ \begin{array}{ll}
8 & \text{if } i \equiv 1 \pmod{4} \\
11 & \text{if } i \equiv 0 \pmod{4} \\
13 & \text{otherwise}.
\end{array} \}
\]

Clearly, the weights of any two adjacent vertices are relatively prime and hence \( \mathcal{O} \) is a coprime irregular edge-weighting of corona of a triangular snake. So that the corona of a triangular snake is coprime irregular edge-weighting. Hence case 2.

**Theorem 2.7.** Corona of a quadrilateral snake graph is coprime irregular for all \( n \).

**Proof.** We prove the theorem in the following cases.

**Case 1.** \( n \) is odd

Define an edge-weighting \( \mathcal{O} \) of the corona of a quadrilateral snake as follows.

For all \( i=1, 2, 3, \ldots, n-1 \), assign
\[
\mathcal{O}(u_iu_i') = \mathcal{O}(v_iu_i') = \mathcal{O}(u_iu_i') = 1, \mathcal{O}(u_iu_i') = 2 \text{ and } \mathcal{O}(u_iu_i') = 3
\]

and for all \( i=1, 2, 3, \ldots, n-1 \), \( \mathcal{O}(u_i) = \mathcal{O}(u_{i+1}v_i) = \mathcal{O}(v_iu_i') = 1, \mathcal{O}(u_{i+1}v_i) = 2 \) and \( \mathcal{O}(u_{i+1}u_i') = 3 \).

Then \( S_G(v_1) = 4 \), \( S_G(v_i') = 5 \) and \( \mathcal{O}(v_i) = 1 \), for all \( i=1, 2, 3, 4, \ldots, n-1 \), we have
\[
S_G(u_i) = \{ u_{i-1}v_i + \mathcal{O}(u_{i-1}u_i) + \mathcal{O}(u_{i-1}u_i') + \mathcal{O}(u_{i-1}v_i') \\
\{ \begin{array}{ll}
8 & \text{if } i \equiv 1 \pmod{6} \\
10 & \text{if } i \equiv 2 \pmod{6} \\
12 & \text{if } i \equiv 3 \pmod{6} \\
14 & \text{if } i \equiv 4 \pmod{6} \\
16 & \text{if } i \equiv 5 \pmod{6} \\
18 & \text{if } i \equiv 0 \pmod{6}
\end{array} \}
\]

Clearly, the weights of any two adjacent vertices are relatively prime and hence \( \mathcal{O} \) is a coprime irregular edge-weighting of corona of a quadrilateral snake. So that the corona of a quadrilateral snake is coprime irregular edge-weighting.
Case 2. \( n \) is even

Define an edge-weighting \( \mathcal{O} \) of the corona of a quadrilateral snake as follows. For all \( i=1,2,\ldots,n-1 \),
\[
\mathcal{O}(v_iw_i) = \mathcal{O}(v'_i w'_i) = \mathcal{O}(u_i t_i) = \mathcal{O}(u_i t'_i) = \mathcal{O}(v'_i v'_i) = 1,
\]
\[
\mathcal{O}(u_i v_i) = 2 \quad \text{and} \quad \mathcal{O}(u_i+1 v'_i) = 3 \quad \text{and}
\]
\[
\mathcal{O}(u_i u_{i+1}) = \begin{cases} 
1 & \text{if } i \equiv 2 \pmod{4} \\
2 & \text{if } i \equiv 1 \pmod{4} \\
3 & \text{if } i \equiv 0 \pmod{4} \\
4 & \text{if } i \equiv 3 \pmod{4}
\end{cases}
\]
and for all \( i=1,2,3,\ldots,n-1 \), \( \mathcal{O}(v_i w_i) = \mathcal{O}(v'_i w'_i) = \mathcal{O}(u_i t_i) = \mathcal{O}(u_i t'_i) = \mathcal{O}(v'_i v'_i) = 1, \mathcal{O}(u_i v_i) = 2 \) and \( \mathcal{O}(u_i+1 v'_i) = 3 \).

The corona of a quadrilateral snake \( QS_n \) as given in Figure 9.

![Figure 9](image-url)

Then \( S_0(v_i) = 4, S_0(v'_i) = 5, S_0(w_i) = S_0(w'_i) = S_0(t_i) = S_0(t'_i) = 1 \), for all \( i=1,2,3,4,\ldots,n-1 \), \( S_0(u_i) = 5 \), \( S_0(u_{n+1}) = 6 \) (or) \( 8 \) according as \( n \equiv 2 \pmod{6} \) or \( n \equiv 1 \pmod{6} \) or \( n \equiv 0 \pmod{6} \) where \( k \geq 0 \) is an integer and for all \( i=2,3,4,\ldots,n-1 \), we have
\[
S_0(u_i) = \mathcal{O}(u_i v_i) + \mathcal{O}(u_i v'_i) + \mathcal{O}(u_i u_{i+1}) + \mathcal{O}(u_i u_{i+1}) + \mathcal{O}(u_i u_{i+1}) = \\
S_0(u_i) = 9 \quad \text{if } i \equiv 2 \pmod{4} \quad \text{or} \quad i \equiv 1 \pmod{4} \quad \text{or} \quad S_0(u_i) = 11 \quad \text{if } i \equiv 0 \pmod{4} \quad \text{or} \quad S_0(u_i) = 13 \quad \text{otherwise}.
\]

Clearly, the weights of any two adjacent vertices are coprime and hence \( \mathcal{O} \) is a coprime irregular edge-weighting of corona of a quadrilateral snake. So that the corona of a quadrilateral snake is coprime irregular edge-weighting.

III. CONCLUSION AND SCOPE

In this paper, we have introduced a coprime irregular edge weighting of graphs and proved some classes of graphs are coprime irregular. Even though, there is a wide scope for further research on this parameter. The following are some interesting directions for future research.

1. Find more classes of coprime irregular graphs.
2. Obtaining a necessary or sufficient condition for a graph to be coprime irregular is worthy trying.
3. It seems to us that the problem of characterizing trees which are coprime irregular would be very interesting.

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