

Coprime Irregular graphs

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Abstract: An l -edge-weighting of a graph G is a map $\phi: E(G) \rightarrow \{1, 2, 3, \dots, l\}$, where l is a positive integer. For a vertex $v \in V(G)$, the weight $S_\phi(v)$ is the sum of edge-weights appearing on the edges incident at v under the edge-weighting ϕ . An l -edge-weighting of G is coprime irregular edge-weighting of G if $\gcd(S_\phi(u), S_\phi(v)) = 1$ for every pair of adjacent vertices u and v in G . A graph G is coprime irregular if G admits a coprime irregular edge-weighting. In this paper, we discuss this new irregular edge weighting of graphs.

Keywords: coprime, edge-weighting.

I. INTRODUCTION

All graphs considered in this article are simple, undirected and connected graphs. For graph theoretic terminology we refer to Chartrand and Lesniak [1].

Labeling of graphs is one of the fastest research areas which connects number theory and graph theory. Graph labeling was first introduced in the late 1960's and in recent years dozens of graphs labeling were introduced and studied by several authors. A graph labeling is an assignment of integers to the vertices or edges or both with respect to some conditions. A detailed survey of graph labeling is given in [2]. In this paper, we introduce a new variation of irregular labeling (irregular edge-weighting) of graphs namely coprime irregular edge-weighting of graphs.

II. CLASSES OF COPRIME IRREGULAR GRAPHS

In this section we prove that paths, cycles, triangular snakes, quadrilateral snakes and the corona of triangular and quadrilateral snakes are coprime irregular edge-weighting of a graph as follows. Now let us see the definition of coprime irregular edge-weighting of graphs as follows.

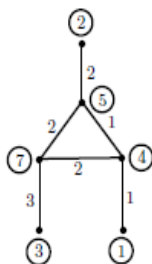


Figure 1.1: A graph G and its coprime irregular labeling

Definition 2.1. An l -edge-weighting of a graph $G = (V, E)$ is a map $\phi: E(G) \rightarrow \{1, 2, 3, \dots, l\}$, where l is a positive integer. For a vertex v of G , the weight $S_\phi(v)$ is the sum of edge-weights appearing on the edges incident at v under the edge-weighting ϕ . An l -edge-weighting ϕ of G is coprime

irregular if $\gcd(S_\phi(u), S_\phi(v)) = 1$ for every pair of adjacent vertices u and v in G . A graph G is said to be coprime irregular if G admits a coprime irregular edge-weighting.

Example 2.2. A graph G given in Figure 1.1 is coprime irregular.

Theorem 2.3. Path P_n on $n \geq 3$ vertices is coprime irregular for all n .

Proof. Let $P_n = (v_1, v_2, v_3, \dots, v_n)$. We now prove this theorem by considering the following cases.

Case (i) n is even.

Define an edge-weighting ϕ of P_n as follows. For all $1 \leq i \leq n-1$, let $\phi(v_i v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \text{ or } 2 \pmod{4} \\ 2 & \text{otherwise} \end{cases}$

Then $S_\phi(v_1) = 1, S_\phi(v_n) = 1$ or 2 according as $n \equiv 2 \pmod{4}$ or $n \equiv 0 \pmod{4}$; and for each $2 \leq i \leq n-1$, we have

$$S_\phi(v_i) = \phi(v_{i-1} v_i) + \phi(v_i v_{i+1}) = \begin{cases} 4 & \text{if } i \equiv 0 \pmod{4} \\ 2 & \text{if } i \equiv 2 \pmod{4} \\ 3 & \text{otherwise.} \end{cases}$$

It is not difficult to see that any two adjacent vertices of P_n whose weights are coprime under ϕ and thus P_n admits a coprime irregular edge-weighting.

Case (ii) n is odd

Now, define for each $1 \leq i \leq n-1$,

$$\phi(v_i v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 2 \text{ or } 3 \pmod{4} \\ 2 & \text{otherwise} \end{cases}$$

Then $S_\phi(v_1) = 2, S_\phi(v_n) = 1$ or 2 according as $n \equiv 3 \pmod{4}$ or $n \equiv 1 \pmod{4}$; and for all $i = 2, 3, \dots, n-1$, we have $S_\phi(v_i) = \phi(v_{i-1} v_i) + \phi(v_i v_{i+1})$

$$= \begin{cases} 2 & \text{if } i \equiv 3 \pmod{4} \\ 3 & \text{if } i \equiv 0 \text{ or } 2 \pmod{4} \\ 4 & \text{otherwise.} \end{cases}$$

Obviously, the weights of any two adjacent vertices of P_n are coprime and thus P_n is coprime irregular.

Theorem 2.4. For all $n \geq 4$, the cycle C_n is coprime irregular.

Proof. We prove this result in the following two cases.

Case (i) n is odd

Considering the edge-weighting ϕ of C_n defined as follows.

For all $i = 1, 2, 3, \dots, n-1$, let

$$\phi(v_i v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 1 \pmod{4} \\ 2 & \text{if } i \equiv 2 \pmod{4} \\ 3 & \text{if } i \equiv 3 \pmod{4} \\ 4 & \text{if } i \equiv 0 \pmod{4} \end{cases}$$

and let $\phi(v_n v_1) = 1$ or 3 according as $n \equiv 1 \pmod{4}$ or $n \equiv 3 \pmod{4}$. Then for any two adjacent vertices of C_n , their weights are coprime. Thus ϕ is coprime irregular edge-weighting of C_n and hence C_n is coprime irregular.

Case (ii) n is even

Assign $\phi(v_1 v_2) = 1, \phi(v_2 v_3) = 2, \phi(v_3 v_4) = 3$ and $\phi(v_4 v_5) = 4$ and for all the remaining vertices,

Revised Manuscript Received on December 16, 2019.

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$$\varnothing(v_i v_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 0 \pmod{4} \\ 2 & \text{if } i \equiv 1 \pmod{4} \\ 3 & \text{if } i \equiv 2 \pmod{4} \\ 4 & \text{if } i \equiv 3 \pmod{4} \end{cases}$$

One can easily verify that the weights of any two adjacent vertices of C_n are coprime and so \varnothing is a coprime irregular edge-weighting of C_n . Hence C_n is coprime irregular.

Definition 2.5. A triangular snake TS_n is obtained from a path $(u_1, u_2, \dots, u_{n-1}, u_n)$ by joining u_i and u_{i+1} to a new vertex v_i for $i = 1, 2, \dots, n$. A quadrilateral snake QS_n is obtained from the path $(u_1, u_2, \dots, u_{n-1}, u_n)$ by introducing n copies of K_2 , say $v_1 w_1, v_2 w_2, \dots, v_n w_n$ and joining v_i to u_i and w_i to u_{i+1} for $i = 1, 2, \dots, n$.

Theorem 2.6. The triangular snake graph TS_n is coprime irregular for all n .

Proof. Suppose n is even. Now we define an edge-weighting \varnothing as follows. For all $1 \leq i \leq n$, let $\varnothing(u_i v_i) = 1$, $\varnothing(v_i u_{i+1}) = 4$ and $\varnothing(u_i u_{i+1}) = \begin{cases} 2 & \text{if } i \equiv 0 \text{ or } 1 \pmod{4} \\ 4 & \text{otherwise} \end{cases}$.

For $n = 8$, the graph TS_8 and edge-weighting \varnothing are shown in Figure 2.

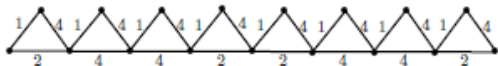


Figure 2

Certainly, by the definition of \varnothing , we have $S_\varnothing(u_1) = 3$, $S_\varnothing(u_{n+1}) = 6$ (or) 8 according as $n \equiv 0 \pmod{4}$ and $n \equiv 2 \pmod{4}$; and for all $i = 2, 3, 4, \dots, n$, we have

$$S_\varnothing(u_i) = \varnothing(u_i v_{i-1}) + \varnothing(u_i v_i) + \varnothing(u_i u_{i+1}) + \varnothing(u_i u_{i-1}) = \begin{cases} 9 & \text{if } i \equiv 1 \pmod{4} \\ 11 & \text{if } i \equiv 0 \text{ or } 2 \pmod{4} \\ 13 & \text{if } i \equiv 3 \pmod{4} \end{cases}$$

Clearly, the weights of any two adjacent vertices of TS_n are coprime and hence \varnothing is a coprime irregular edge-weighting of TS_n .

On the other hand, let we assume that n is odd. Consider the edge-weighting \varnothing of TS_n defined as follows.

For all $i = 1, 2, \dots, n$, $\varnothing(u_i v_i) = \varnothing(v_i u_{i+1}) = 1$ and $\varnothing(u_i u_{i+1}) = \begin{cases} 1 & \text{if } i \equiv 2 \pmod{4} \\ 2 & \text{if } i \equiv 1 \pmod{4} \\ 3 & \text{if } i \equiv 0 \pmod{4} \\ 4 & \text{otherwise} \end{cases}$.

For $n = 9$, the graph TS_9 and its edge-weighting given in the following Figure 3.



Figure 3

Then $S_\varnothing(u_1) = 3$, $S_\varnothing(u_{n+1}) = 3$ (or) 5 according as $n \equiv 1 \pmod{4}$ or $n \equiv 3 \pmod{4}$; and for all $i = 1, 2, \dots, n$, we have

$$S_\varnothing(u_i) = \varnothing(u_i v_{i-1}) + \varnothing(u_i v_i) + \varnothing(u_i u_{i+1}) + \varnothing(u_i u_{i-1}) = \begin{cases} 5 & \text{if } i \equiv 2 \pmod{4} \\ 9 & \text{if } i \equiv 0 \pmod{4} \\ 7 & \text{otherwise} \end{cases}$$

Obviously, the weights of any two adjacent vertices of TS_n is a coprime irregular.

Theorem 2.7. For all n , the quadrilateral snake graph QS_n is coprime irregular.

Proof. Consider the following cases.

Case 1. n is even.

Consider the edge-weighting \varnothing of quadrilateral snake defined as follows. For all $i = 1, 2, \dots, n$,

$$\varnothing(u_i v_i) = \varnothing(v_i w_i) = 2, \varnothing(w_i u_{i+1}) = 3 \text{ and } \varnothing(u_i u_{i-1}) = \begin{cases} 1 & \text{if } i \equiv 2 \pmod{4} \\ 3 & \text{if } i \equiv 0 \text{ or } 1 \pmod{4} \\ 5 & \text{otherwise} \end{cases}$$

For $n = 6$, the graph QS_6 and its edge-weighting given in Figure 4.

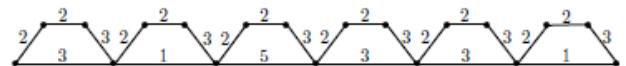


Figure 4

Then $S_\varnothing(u_1) = 5$, $S_\varnothing(u_{n+1}) = 4$ or 6 according as $n \equiv 3 \pmod{4}$ or $n \equiv 1 \pmod{4}$ and for all $i = 1, 2, \dots, n$, we have $S_\varnothing(u_i) = \varnothing(u_i w_{i-1}) + \varnothing(u_i v_i) + \varnothing(u_i u_{i+1}) + \varnothing(u_i u_{i-1}) = \begin{cases} 9 & \text{if } i \equiv 2 \pmod{4} \\ 11 & \text{if } i \equiv 1 \text{ or } 3 \pmod{4} \\ 13 & \text{otherwise} \end{cases}$.

Clearly, the weights of any two adjacent vertices are coprime and so \varnothing is a coprime irregular edge-weighting of QS_n . Therefore the graph QS_n is coprime irregular.

Case 2. n is odd

Consider the edge-weighting \varnothing of the quadrilateral snake defined as follows.

For all $i = 1, 2, \dots, n$,

$$\varnothing(u_i v_i) = \varnothing(v_i w_i) = 3, \varnothing(u_{i+1} w_i) = 4 \text{ and } \varnothing(u_i u_{i-1}) = \begin{cases} 1 & \text{if } i \equiv 2 \pmod{4} \\ 2 & \text{if } i \equiv 1 \pmod{4} \\ 3 & \text{if } i \equiv 0 \pmod{4} \\ 4 & \text{if } i \equiv 3 \pmod{4} \end{cases}$$

For $n = 5$, the graph QS_5 and its edge-weighting illustrated in Figure 5.

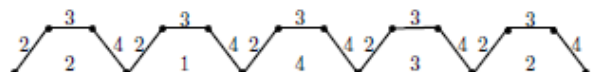


Figure 5

Then $S_{\phi}(u_1) = 4$, $S_{\phi}(u_{n+1}) = 6$ or 8 according as $n \equiv 2 \pmod{4}$ or $n \equiv 0 \pmod{4}$ and for all $i=2, 3, \dots, n$, we have

$$S_{\phi}(u_i) = \phi(u_i w_{i-1}) + \phi(u_i v_i) + \phi(u_i u_{i+1}) + \phi(u_i u_{i-1})$$

$$S_{\phi}(u_i) = \begin{cases} 9 & \text{if } i \equiv 2 \pmod{4} \\ 11 & \text{if } i \equiv 1 \text{ or } 3 \pmod{4} \\ 13 & \text{otherwise.} \end{cases}$$

Certainly from the above, the weights of any two adjacent vertices are coprime and so ϕ is a coprime irregular edge-weighting of QS_n . Hence QS_n is coprime irregular.

Definition 2.8. The corona G^+ of a graph G is the graph obtained from G by attaching exactly one pendant edge at each of the vertices of G .

Theorem 2.9. Corona of a triangular snake graph is coprime irregular.

Proof. Consider the following cases.

Case 1. n is even.

Define an edge-weighting ϕ of the corona of a triangular snake as follows.

For all $i=1, 2, 3, \dots, n$, define

$$\phi(u_i v_i) = \phi(u_{i+1} v_i) = \phi(v_i v'_i) = \phi(u_i u'_i) = 1, \phi(u_i u_{i+1}) = \begin{cases} 2 & \text{if } i \equiv 0 \pmod{4} \\ 3 & \text{if } i \equiv 1 \pmod{4} \\ 5 & \text{if } i \equiv 2 \pmod{4} \\ 6 & \text{if } i \equiv 3 \pmod{4} \end{cases}$$

and for all $i=1, 2, 3, 4, \dots, n-1$, $\phi(u_i v_i) = \phi(u_{i+1} v_i) = \phi(v_i v'_i) = \phi(u_i u'_i) = 1$.

For the corona of a triangular snake TS_6 as given in Figure 6.

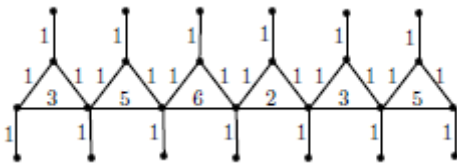


Figure 6

Then $S_{\phi}(v_i) = 3$, $S_{\phi}(v'_i) = 1$, $S_{\phi}(u'_i) = 1$, for all $i=1, 2, 3, 4, \dots, n-1$, $S_{\phi}(u_1) = 5$, $S_{\phi}(u_n) = 4$ or 7 according as $n \equiv 1 \pmod{4}$ or $n \equiv 3 \pmod{4}$ where $k \geq 0$ and for all $i=2, 3, 4, \dots, n-1$, we have

$$S_{\phi}(u_i) = \phi(u_i v_{i-1}) + \phi(u_i v_i) + \phi(u_i u_{i+1}) + \phi(u_i u'_i)$$

$$S_{\phi}(u_i) = \begin{cases} 8 & \text{if } i \equiv 1 \pmod{4} \\ 11 & \text{if } i \equiv 0 \text{ or } 2 \pmod{4} \\ 14 & \text{otherwise.} \end{cases}$$

Clearly, the weights of any two adjacent vertices are relatively prime and hence ϕ is a coprime irregular edge-weighting of corona of a triangular snake. So that the corona of a triangular snake is coprime irregular edge-weighting and hence Case 1 follows.

Case 2. n is odd

Define an edge-weighting ϕ of the corona of a triangular snake as follows. For all $i=1, 2, 3, \dots, n-1$,

$$\phi(u_i v_i) = \phi(u_{i+1} v_i) = 3 \text{ and } \phi(v_i v'_i) = \phi(u_i u'_i) = 1.$$

$$\phi(u_i u_{i+1}) = \begin{cases} 2 & \text{if } i \equiv 1 \text{ or } 2 \pmod{4} \\ 4 & \text{otherwise.} \end{cases}$$

and for all $i=1, 2, 3, \dots, n-1$, $\phi(u_i v_i) = \phi(u_{i+1} v_i) = 3$ and $\phi(v_i v'_i) = \phi(u_i u'_i) = 1$.

For the corona of a triangular snake TS_5 as given in Figure 7.

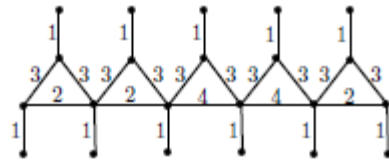


Figure 7

Then $S_{\phi}(v_i) = 7$, $S_{\phi}(v'_i) = 1$, $S_{\phi}(u'_i) = 1$, for all $i=1, 2, 3, 4, \dots, n-1$, $S_{\phi}(u_1) = 6$, $S_{\phi}(u_n) = 6$ or 8 according as $n \equiv 0 \pmod{4}$ or $n \equiv 2 \pmod{4}$ where $k \geq 0$ is an integer and for all $i=2, 3, 4, \dots, n-1$, we have

$$S_{\phi}(u_i) = \phi(u_i v_{i-1}) + \phi(u_i v_i) + \phi(u_i u_{i+1}) + \phi(u_i u'_i) + \phi(u_i u_{i-1})$$

$$S_{\phi}(u_i) = \begin{cases} 11 & \text{if } i \equiv 2 \pmod{4} \\ 13 & \text{if } i \equiv 1 \text{ or } 3 \pmod{4} \\ 15 & \text{otherwise.} \end{cases}$$

Clearly, the weights of any two adjacent vertices are relatively prime and hence ϕ is a coprime irregular edge-weighting of corona of a triangular snake. So that the corona of a triangular snake is coprime irregular edge-weighting. Hence case 2.

Theorem 2.7. Corona of a quadrilateral snake graph is coprime irregular for all n .

Proof. We prove the theorem in the following cases.

Case 1. n is odd

Define an edge-weighting ϕ of the corona of a quadrilateral snake as follows.

For all $i=1, 2, 3, \dots, n-1$, assign

$$\phi(u_i u'_i) = \phi(v_i w'_i) = \phi(v_i v'_i) = 1, \phi(u_i v_i) = 2 \text{ and } \phi(u_{i+1} v'_i) = 3$$

$$\phi(u_i u_{i+1}) = \begin{cases} 2 & \text{if } i \equiv 1 \pmod{6} \\ 3 & \text{if } i \equiv 2 \pmod{6} \\ 4 & \text{if } i \equiv 3 \pmod{6} \\ 5 & \text{if } i \equiv 0 \pmod{6} \\ 6 & \text{if } i \equiv 5 \pmod{6} \\ 7 & \text{if } i \equiv 4 \pmod{6} \end{cases}$$

For all $i=1, 2, 3, \dots, n-1$, $\phi(u_i u'_i) = \phi(v_i w'_i) = \phi(v_i v'_i) = 1$, $\phi(u_i v_i) = 2$ and $\phi(u_{i+1} v'_i) = 3$.

Then $S_{\phi}(v_i) = 4$, $S_{\phi}(v'_i) = 5$, $S_{\phi}(w_i) = S_{\phi}(w'_i) = S_{\phi}(u'_i) = 1$, for all $i=1, 2, 3, 4, \dots, n-1$, $S_{\phi}(u_1) = 5$, $S_{\phi}(u_n) = 7$ (or) 9 (or) 11 according as $n \equiv 3 \pmod{6}$ or $n \equiv 1 \pmod{6}$ or $n \equiv 5 \pmod{6}$.

For all $i=2, 3, 4, \dots, n-1$, we have

$$S_{\phi}(u_i) = \phi(u_i v'_{i-1}) + \phi(u_i v_i) + \phi(u_i u_{i+1}) + \phi(u_i u'_i) + \phi(u_i u_{i-1})$$

$$S_{\phi}(u_i) = \begin{cases} 11 & \text{if } i \equiv 2 \pmod{6} \\ 13 & \text{if } i \equiv 1 \text{ or } 3 \pmod{6} \\ 17 & \text{if } i \equiv 0 \text{ or } 4 \pmod{6} \\ 19 & \text{otherwise.} \end{cases}$$

Clearly, the weights of any two adjacent vertices are coprime and hence ϕ is a coprime irregular edge-weighting of corona of a quadrilateral snake. So that the corona of a quadrilateral snake is coprime irregular edge-weighting.

