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A Analysis on Queuing System with Setup Time in Revamp Process

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***Abstract**: In this paper, we study about a M/G/1 Queuing model with single stage of service. Service interrupts during the time of service. The server does not get into the repair process immediately. It gets into a Set up time stage for the prior work to be done. On completion of set up stage service, the server will get into the repair process consisting of two stages, in which first stage is compulsory and the second stage of service is optional. For the model defined, we get the steady state results in closed form in terms of the probability generating functions and all the other execution performance measures of the model defined.*

***Keywords**: Setup time, compulsory service, Optional service, Revamp process*

I. INTRODUCTION

Queuing theory was developed to provide models to predict the behavior of systems that attempt to provide service for randomly arising demands. Queuing system of bulk arrival model with optional service examined by [1]. "Reference [2] investigated Poisson input queuing system with startup time". A $M^X/G/1$ queuing system with a setup time and a vacation period studied by [3]. "Reference [6] inspected classification of queuing models for a workstation with interruptions". Queuing model of optional type of service with service stoppage and revamp process in web hosting queuing well analyzed by [7]. "Reference [10] studied revamping quality of service of video streaming over wireless LAN". "Reference [4] studied a batch arrival Poisson queue with a random setup time". "Reference [5] examined a batch service queuing system with multiple vacations". "Reference [9] investigated on the optimal control two queues with server setup times and its analysis". "Reference [8] analyzed queue in Multichannel V2I Communications".

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II. MATHEMATICAL DESCRIPTION OF THE QUEUEING MODEL

Customers arrival follows Poisson procedure, Let $\lambda_g dt$ be the first order probability that a customer's arrives at the system during a short duration of time $(t, t+dt)$, where $\lambda_g > 0$ the mean landing rate of the customer is.

The administration time pursues general distribution.

Service follows distribution function as $J_1^*(x)$ and density function $l_1^*(x)$. Let $\gamma_g(x)dx$ be the conditional density function. Hence we have

$$\gamma_g(x) = \frac{l_1^*(x)}{1-J_1^*(x)},\ l_1^*(x) = \gamma_g(x)e^{-\int_0^s \gamma_g(s)ds} \quad \text{(a)}$$

For set up time,

$$\gamma_m(x) = \frac{l_2^*(x)}{1-J_2^*(x)} \quad ,l_2^*(x) = \gamma_m(x)e^{-\int_0^s \gamma_m(s)ds} \quad \text{(b)}$$

For compulsory stage 1 service,

$$\gamma_{n_a}(x) = \frac{l_3^*(x)}{1-J_3^*(x)} \quad ,l_3^*(x) = \gamma_{n_a}(x)e^{-\int_0^s \gamma_{n_a}(s)ds} \quad \text{(c)}$$

For optional service stage 2,

$$\gamma_{n_b}(x) = \frac{l_4^*(x)}{1-J_4^*(x)} \quad ,l_4^*(x) = \gamma_{n_b}(x)e^{-\int_0^s \gamma_{n_b}(s)ds} \quad \text{(d)}$$

III. GOVERNING EQUATIONS OF THE MODEL

Steady State Conditions Overseeing the Framework

$$\frac{\partial}{\partial x}G_n(x) + (\lambda_g + \gamma_g(\mathrm{x}) + \psi)G_n(x) = \lambda_g G_{n-1}(x). \quad (1)$$

$$\frac{\partial}{\partial x}G_0(x) + (\lambda_g + \gamma_g(\mathrm{x}) + \psi)G_0(x) = 0 \quad . \quad (2)$$

$$\frac{\partial}{\partial x}M_n(x) + \left(\lambda_g + \gamma_m(\mathrm{x})\right)M_n(x) = \lambda_g M_{n-1}(x) . \quad (3)$$

$$\frac{\partial}{\partial x}M_0(x) + \left(\lambda_g + \gamma_m(\mathrm{x})\right)M_0(x) = 0. \quad (4)$$

$$\frac{\partial}{\partial x}R_n^{(a)}(x) + (\lambda_g + \gamma_{n_a}(\mathrm{x}))R_n^{(a)}(x) = \lambda_g R_{n-1}^{(a)}(x). \quad (5)$$

$$\frac{\partial}{\partial x}R_0^{(a)}(x) + (\lambda_g + \gamma_{n_a}(\mathrm{x}))R_0^{(a)}(x) = 0. \quad (6)$$

$$\frac{\partial}{\partial x}R_n^{(b)}(x) + (\lambda_g + \gamma_{n_b}(\mathrm{x}))R_n^{(b)}(x) = \lambda_g R_{n-1}^{(b)}(x) \quad (7)$$

$$\frac{\partial}{\partial x}R_n^{(b)}(x) + (\lambda_g + \gamma_{n_b}(\mathrm{x}))R_n^{(b)}(x) = 0. \quad (8)$$

$$\lambda^g E = \int_0^\infty G_0(x)\gamma_g(\mathrm{x})\mathrm{dx} + (1-l)\int_0^\infty R_n^{(a)}(x)\gamma_{n_a}(\mathrm{x})\mathrm{dx} + \int_0^\infty R_n^{(b)}(x)\gamma_{n_b}(\mathrm{x})\mathrm{dx} \quad (9)$$

IV. BOUNDARY CONDITIONS

The above set of equations is to be solved under the following boundary conditions at $x = 0$ and for $x \geq 1$

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$G_n(0) =$
$(1-l)\int_0^\infty R_{n+1}^{(a)}(x)\gamma_{n_a}(\mathrm{x})\mathrm{dx} + \int_0^\infty R_{n+1}^{(b)}(x)\gamma_{n_b}(\mathrm{x})\mathrm{dx} +$
$\lambda_g E \int_0^\infty G_{n+1}(x)\gamma_g(\mathrm{x})$
. (10)

$M_n(0) = \psi \int_0^\infty G_n(x)dx$. (11)

$R_n^{(a)}(0) = \int_0^\infty M_n(x)\gamma_m(\mathrm{x})\mathrm{dx}$. (12)

$R_n^{(b)}(0) = l\int_0^\infty R_n^{(a)}(x)\gamma_{n_a}(\mathrm{x})\mathrm{dx}$. (13)

multiply (1) by z^n and sum over n from 1 to∞ , adding to (2), we get

$\frac{\partial}{\partial x}G_n(x,z) + (\lambda_g - \lambda_g z + \gamma_g(\mathrm{x}) + \psi)G_n(x,z) = 0.$ (14)

$\int_0^x(14)$Gives,

$G_n(x,z) = G_n(0,z)e^{-(\lambda_g-\lambda_g z+\psi)-\int_0^x \gamma_g(\mathrm{t})\mathrm{dt}}$. (15)

$\int_0^x(15)dx$By parts $\Rightarrow$

$G_n(z) = G_n(0,z)\left[\frac{1-J_1^*(\lambda_g-\lambda_g z+\psi)}{(\lambda_g-\lambda_g z+\psi)}\right].$ (16)

Multiply both sides of (15) by $\gamma_g(\mathrm{x})\mathrm{dx}$ and integrating by parts, we get

$\int_0^\infty G_n(z)\gamma_g(\mathrm{x})\mathrm{dx} = G_n(0,z)J_1^*(\lambda_g - \lambda_g z + \psi)$. (17)

Applying the same process of study for the remaining equations (3) – (8) , we get

$M_n(z) = M_n(0,z)\left[\frac{1-J_2^*(\lambda_g-\lambda_g z)}{(\lambda_g-\lambda_g z)}\right].$ (18)

$\int_0^\infty M_n(x,z)\gamma_m(\mathrm{x})\mathrm{dx} = M_n(0,z)J_2^*(\lambda_g - \lambda_g z).$ (19)

$R_n^{(a)}(z) = R_n^{(a)}(0,z)\left[\frac{1-J_3^*(\lambda_g-\lambda_g z)}{(\lambda_g-\lambda_g z)}\right].$ (20)

$\int_0^\infty R_n^{(a)}(x,z)\gamma_{n_a}(\mathrm{x})\mathrm{dx} = R_n^{(a)}(0,z)J_3^*(\lambda^g - \lambda^g z).$ (21)

$R_n^{(b)}(z) = R_n^{(b)}(0,z)\left[\frac{1-J_4^*(\lambda_g-\lambda_g z)}{(\lambda_g-\lambda_g z)}\right].$ (22)

$\int_0^\infty R_n^{(b)}(x,z)\gamma_{n_b}(\mathrm{x})\mathrm{dx} = R_n^{(b)}(0,z)J_4^*(\lambda_g - \lambda_g z).$ (23)

Using Supplementary variable method and using (17), (21), (23) in (10), we get

$G_n(0,z) = \frac{\lambda_g Ez-\lambda_g E}{z-J_1^*(R)+\left[\frac{1-J_1^*(R)}{R}\right]J_2^*(C)J_3^*(C)[\psi(1-l)+lJ_4^*(C)]}.$ (24)

Where $R = \lambda_g - \lambda_g z + \psi,\quad C = \lambda_g - \lambda_g z.$

V. LIKELIHOOD CREATING CAPACITY OF THE LINE ESTIMATE

Let $\overline{A_k}(z)$ be the p.g.f of the line length such that

$\overline{A_k}(z) = G_n(z) + M_n(z) + R_n^{(a)}(z) + R_n^{(b)}(z).$

$$\overline{A_k}(z) = \frac{\left[\frac{1-J_1^*(R)}{R}\right]E\left\{\begin{matrix}(\lambda_g z-\lambda_g)(1+J_4^*(C)lJ_3^*(C)J_2^*(C)\psi)\\ -\left(\psi(1-J_2^*(C))+\psi(1-J_3^*(C)J_2^*(C))\right)\end{matrix}\right\}}{z-J_1^*(R)+\left[\frac{1-J_1^*(R)}{R}\right]J_3^*(C)J_2^*(C)[\psi(1-l)+lJ_4^*(C)]}. \quad (25)$$

VI. NORMALIZATION CONDITION

$F(1) + E = 1$Gives the idle time S and the Utilization factor.

Idle time $S = \frac{D'(1)}{D'(1)+N'(1)}$. (26)

Utilization factor, $\rho = 1 - E$. (27)

$F(1) = \lim_{z\to1} F(z) = \frac{0}{0}$,indeterminant form.

Hence applying L'Hopital's rule, we get

$F(1) = \frac{D'(1)}{D'(1)+N'(1)}$. (28)

VII. SYSTEM QUEUE PERFORMANCE MEASURES

Let L_q a chance to demonstrate the reliable state typical number of customers in the line. By then

$L_q = \frac{d}{dz}\overline{A_k}(z)|_{z=1} = \frac{d}{dz}\left\{\frac{N(Z)}{D(Z)}\right\}\Big|_{z=1}$

Where $N(Z)$ and $D(Z)$ are the numerator and denominator of (25).

Since $\overline{A_k}(z) = \frac{0}{0}$ at z= 1 , we utilize two fold separation and get

$L_q = \lim_{z\to1}\frac{d}{dz}\overline{A_k}(z) = \frac{D'(1)N''(1)-D''(1)N'(1)}{2(D'(1))^2}$. (29)

$N'(1) = [1 - J_1^*(\psi)]\{\lambda_g(E(J_2) + E(J_3))\}.$ (30)

$N''(1) = -J_1^{*'}(\psi)(E(J_1) + E(J_3))\lambda_g(1+\psi) +$
$(1 - J_1^*(\psi))\{-2(\lambda_g)^2E(J_4)l + [E(J_2^2)(\lambda_g)^2 +$
$(\lambda_g)^2E(J_3^2) + 2E(J_2)E(J_3)(\lambda_g)^2]\}$
(31)

$D'(1) = 1 - \lambda_g J_1^{*'}(\psi) - J_1^{*'}(\psi)[\psi(1-l) + l] +$
$\left[\frac{1-J_1^*(\psi)}{\psi}\right][\psi(1-l) + l]\lambda_g(E(J_2) + E(J_3)) +$
$\left[\frac{1-J_1^*(\psi)}{\psi}\right]\lambda_g lE(J_4)$
(32)

$D''(1) = -J_1^{*''}(\psi)(\lambda_g)^2 + J_1^{*''}(\psi)\lambda_g[\psi(1-l) + l] -$
$J_1^{*'}(\psi)\lambda_g lE(J_4) + \left[\frac{1-J_1^*(\psi)}{\psi}\right][-\lambda_g lE(J_4)]\lambda_g(E(J_2) +$
$E(J_3)) + \left[\frac{1-J_1^*(\psi)}{\psi}\right][\psi(1-l) + l][E(J_3^2) + E(J_2^2) +$
$2E(J_2)E(J_3)](\lambda_g)^2 - J_1^{*'}(\psi)\lambda_g lE(J_4) +$
$\left[\frac{1-J_1^*(\psi)}{\psi}\right]l(\lambda_g)^2[E(J_4^2) + E(J_4)(E(J_2) + E(J_3))]$
(33)

Substituting (30) – (33) in (29), we obtain L_q and all the other measures using Little's formula: $W_q = \frac{L_q}{\lambda}$, $= \frac{L}{\lambda}$, $L = L_q + \rho$.

VIII. NUMERICAL JUSTIFICATION OF THE MODEL

Here we consider the service time to follow exponential distribution.

The values are collected accordingly:

$\gamma_g = 2.5, \gamma_m = 3, \gamma_{n_a} = 3.5, \gamma_{n_b} = 4, l = 0.6,$

$J_1^*(\psi) = \frac{\gamma_g}{\gamma_g+\psi}, J_1^{*'}(\psi) = \frac{-\gamma_g}{(\gamma_g+\psi)^2}, J_1^{*''}(\psi) = \frac{2\gamma_g}{(\gamma_g+\psi)^3},$

$E(J_2) = \frac{1}{\gamma_m}, E(J_2^2) = \frac{2}{(\gamma_m)^2}, E(J_3) = \frac{1}{\gamma_{n_a}},$

$E(J_3^2) = \frac{2}{(\gamma_{n_a})^2}, E(J_4) = \frac{1}{\gamma_{n_b}}, E(J_4^2) = \frac{2}{(\gamma_{n_b})^2}$

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Table I Effect of change of($\psi = 1.5,2,2.5,3,3.5$)

Q	ρ	L_q	L	W_q	W
0.7759	0.2241	0.1294	0.3535	0.0462	0.1263
0.7364	0.2636	0.1441	0.4077	0.0515	0.1456
0.7058	0.2942	0.1523	0.4465	0.0544	0.1595
0.6815	0.3185	0.1568	0.4753	0.0560	0.1698
0.6621	0.3379	0.1589	0.4968	0.0568	0.1774

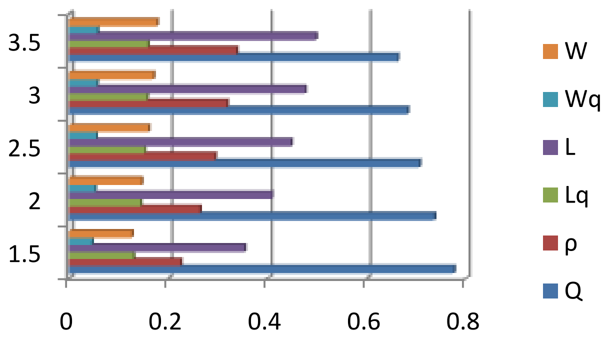

Fig. 1 Effect of change of ψ

From Table I it is clear that if the Service interruption increases it leads to an increase in all the performance measures. Since the Service interruption gets increased the idle time gets decreased and utilization factor is increased. Waiting time of queue and waiting time of server are also increased when the service interruption gets increased. When we include maintenance work we will avoid waiting time of the queue and also waiting time of server.

Table II Effect of change of ($l = 0.2,0.4,0.6,0.8,1$)

Q	ρ	L_q	L	W_q	W
0.7795	0.2205	0.2511	0.4716	0.0897	0.1684
0.7777	0.2223	0.1909	0.4132	0.0682	0.1476
0.7759	0.2241	0.1294	0.3535	0.0462	0.1263
0.7740	0.2260	0.0666	0.2926	0.0238	0.1045
0.7721	0.2279	0.0025	0.2304	0.0009	0.0823

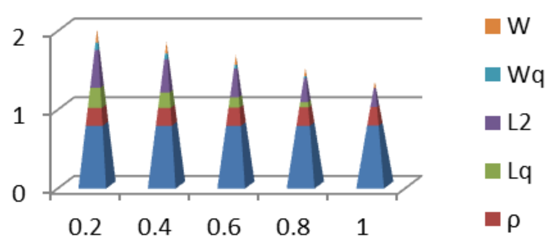

Fig. 2 Effect of change of l

Table 2 indicates that, as the probability of oiptonal service gets increased, the idle time is decreased but utilization factor is increased. Since the optional service increases it leads to decrease in all the performance measures.

IX. CONCLUSION

In this paper, we have examined a single service. When the service interruption occurs the server gets into a setup time with two stages. The first stage of service is compulsory and the second stage is optional. This paper clearly analyses the steady state results and some queuing performance measures. Further this model can be extended by adding the concept of balking, reneging, feedback service, extended vacation etc.

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