A Analysis on Queuing System with Setup Time in Revamp Process

P. Karunakaran, S. Maragathasundari

Abstract: In this paper, we study about a M/G/1 Queuing model with single stage of service. Service interrupts during the time of service. The server does not get into the repair process immediately. It gets into a Set up time stage for the prior work to be done. On completion of set up stage service, the server will get into the repair process consisting of two stages, in which first stage is compulsory and the second stage of service is optional. For the model defined, we get the steady state results in closed form in terms of the probability generating functions and all the other execution performance measures of the model defined.

Keywords: Setup time, compulsory service, Optional service, Revamp process

I. INTRODUCTION


II. MATHEMATICAL DESCRIPTION OF THE QUEUING MODEL

Customers arrival follows Poisson procedure, Let $\lambda_g$ be the first order probability that a customer's arrives at the system during a short duration of time $(t, t + dt)$, where $\lambda_g > 0$ the mean landing rate of the customer is.

The administration time pursues general distribution.

Service follows distribution function as $F_1(x)$ and density function $f_1(x)$. Let $g_i(x)$ be the conditional density function. Hence we have

$$g_1(x) = \frac{d}{dx} G_1(x)$$

For set up time,

$$g_2(x) = \frac{d}{dx} G_2(x)$$

For compulsory stage 1 service,

$$g_3(x) = \frac{d}{dx} G_3(x)$$

For optional service stage 2,

$$g_4(x) = \frac{d}{dx} G_4(x)$$

III. GOVERNING EQUATIONS OF THE MODEL

Steady State Conditions Overseeing the Framework

$$\frac{\partial}{\partial x} G_n(x) + (\lambda_g + \gamma_1) G_n(x) = \lambda_g G_{n-1}(x)$$

$$\frac{\partial}{\partial x} M_n(x) + (\lambda_g + \gamma_1) M_n(x) = \lambda_g M_{n-1}(x)$$

$$\frac{\partial}{\partial x} R_n(x) + (\lambda_g + \gamma_1) R_n(x) = \lambda_g R_{n-1}(x)$$

IV. BOUNDARY CONDITIONS

The above set of equations is to be solved under the following boundary conditions at $x = 0$ and for $x \geq 1$

$$G_n(0) = (1 - l) \int_0^x G_{n+1}(x)g_n(x)dx + \lambda_g \int_0^x G_{n+2}(x)g_n(x)dx$$

$$M_n(0) = \psi \int_0^x M_n(x)g_n(x)dx$$

$$R_n(0) = \int_0^x M_n(x)g_n(x)dx$$

$$R_n(b) = l \int_0^b R_n(x)g_n(x)dx$$
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multiply (1) by \(z^n\) and sum over \(n\) from 1 to \(\infty\), adding to (2), we get
\[
\frac{\partial}{\partial z} G_n(x, z) + (\lambda - \lambda z + \gamma x + \psi) G_n(x, z) = 0.
\]
(14)

\(\int_0^1 G_n(x, z)\,dz\) gives,
\[
G_n(x, z) = G_n(0, z) e^{-(\lambda - \lambda z + \gamma x + \psi) \int_0^1 \gamma y(0)\,dy}.
\]
(15)

\[
L_n = \frac{\int_0^1 G_n(x, z)\,dz}{\int_0^1 G_n(x, z)\,dz} \quad \text{by parts}.
\]
(16)

Multiply both sides of (15) by \(\gamma y(x)/x\) and integrating by parts, we get
\[
\int_0^1 G_n(x, z)\gamma y(x)\,dx = G_n(0, z) f(z, \lambda - \lambda z + \psi) + \psi \int_0^1 \frac{\partial}{\partial z} G_n(x, z)\,dz
\]
(17)

Applying the same process of study for the remaining equations (3) – (8), we get
\[
M_n(z) = M_n(0, z) \left[1 - \frac{1 - f(z, \lambda - \lambda z + \psi)}{(\lambda z - \lambda z^2)}\right].
\]
(18)

\[
R_n^{(s)}(z) = R_n^{(s)}(0, z) \left[1 - \frac{1 - f(z, \lambda - \lambda z + \psi)}{(\lambda z - \lambda z^2)}\right].
\]
(19)

\[
R_n^{(p)}(z) = R_n^{(p)}(0, z) \left[1 - \frac{1 - f(z, \lambda - \lambda z + \psi)}{(\lambda z - \lambda z^2)}\right].
\]
(20)

\[
R_n^{(b)}(z, x)\,dx = R_n^{(b)}(0, z) \left[1 - \frac{1 - f(z, \lambda - \lambda z + \psi)}{(\lambda z - \lambda z^2)}\right].
\]
(21)

\[
R_n^{(b)}(z, x)\,dx = R_n^{(b)}(0, z) \left[1 - \frac{1 - f(z, \lambda - \lambda z + \psi)}{(\lambda z - \lambda z^2)}\right].
\]
(22)

Using Supplementary variable method and using (17), (21), (23) in (10), we get
\[
G_n(0, z) = \frac{\lambda z - \lambda z^2 + \psi}{\lambda z - \lambda z^2 + \psi}.
\]
(24)

Where \(\lambda z - \lambda z^2 + \psi = \psi\).

V. LIKELIHOOD CREATING CAPACITY OF THE LINE ESTIMATE

Let \(\mathcal{A}_k(z)\) be the p.g.f of the line length such that
\[
\mathcal{A}_k(z) = G_n(z) + M_n(z) + R_n^{(s)}(z) + R_n^{(p)}(z).
\]
(25)

VI. NORMALIZATION CONDITION
\[
F(1) + E = 1 \quad \text{gives the idle time} \quad S \quad \text{and the Utilization factor}.
\]
(26)

Idle time \(S = \frac{\theta'(1)}{\theta'(1) + \psi(1)}\).

Utilization factor, \(\rho = 1 - E\).

\[
F(1) = \lim_{z \to 1} F(z) = \frac{\delta}{\delta z} \quad \text{indeterminant form}.
\]
(27)

Hence applying L’Hôpital’s rule, we get
\[
F(1) = \frac{\delta}{\delta z} \left[\frac{\theta'(1)}{\theta'(1) + \psi(1)}\right]_{z=1}.
\]
(28)

VII. SYSTEM QUEUE PERFORMANCE MEASURES

Let \(L\) be the chance to demonstrate the reliable state typical number of customers in the line. By then
\[
L_q = \frac{d}{dz} \mathcal{A}_k(z)|_{z=1} \quad \text{and} \quad \frac{d}{dz} \left(\frac{N(Z)}{D(Z)}\right)|_{z=1}.
\]
(29)

Where \(N(Z)\) and \(D(Z)\) are the numerator and denominator of (25).

Since \(\mathcal{A}_k(z) = \frac{\delta}{\delta z} \quad \text{at} \quad z = 1\), we utilize two fold separation and get
\[
\mathcal{L}_q = \lim_{z \to 1} \frac{d}{dz} \mathcal{A}_k(z) = \frac{\theta'(1)\theta''(1) - \theta''(1)\psi(1)}{2(\theta'(1)^2)}.
\]
(30)

\[
N'(1) = [1 - f_1(\psi)](\lambda E(f_1) + \psi E(f_2)) + [1 - f_1(\psi)](\psi E(f_1) + (1 + \psi) E(f_2)) + [1 - f_1(\psi)](\lambda E(f_1) + \psi E(f_2) + 2E(f_2)E(f_3)) + [1 - f_1(\psi)](\lambda E(f_1) + \psi E(f_2) + 2E(f_2)E(f_3)) = \mathcal{L}_q^2.
\]
(31)

\[
N''(1) = \frac{d^2}{dz^2} \left(\frac{N'(1)}{N'(1)}\right)\theta'(1) = \frac{d^2}{dz^2} \left(\frac{N'(1)}{N'(1)}\right)(\lambda E(f_1) + \psi E(f_2) + 2E(f_2)E(f_3)) + [1 - f_1(\psi)](\psi E(f_1) + (1 + \psi) E(f_2)) + [1 - f_1(\psi)](\lambda E(f_1) + \psi E(f_2) + 2E(f_2)E(f_3)) = \mathcal{L}_q^2.
\]
(32)

\[
D'(1) = 1 - \lambda f_1(\psi) - f_1''(\psi)\psi \psi(1 - 1) + \lambda E(f_1) + \psi E(f_2) + \psi E(f_3) + (\lambda + \psi) E(f_2)E(f_3) + \mathcal{L}_q^2.
\]
(33)

Substituting (30) – (33) in (29), we obtain \(L_q\) and all the other measures using Little’s formula:
\[
W_q = \frac{L_q}{\lambda}, \quad \rho = \frac{L}{\lambda}, \quad L = L_q + \rho.
\]

VIII. NUMERICAL JUSTIFICATION OF THE MODEL

Here we consider the service time to follow exponential distribution.

The values are collected accordingly:
\[
\begin{array}{c|c|c|c|c|c|c}
\gamma & \lambda & \psi & S & E & \rho \\
\hline
0.7759 & 0.2241 & 0.1294 & 0.3535 & 0.0462 & 0.1263 \\
0.7364 & 0.2636 & 0.1441 & 0.4077 & 0.0515 & 0.1456 \\
0.7058 & 0.2942 & 0.1441 & 0.4465 & 0.0544 & 0.1595 \\
0.6815 & 0.3185 & 0.1441 & 0.4753 & 0.0560 & 0.1698 \\
0.6621 & 0.3379 & 0.1441 & 0.4968 & 0.0568 & 0.1774 \\
\end{array}
\]

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Table I

Effect of change of \(\psi = 1.5, 2.5, 3.5, 4.5, 5.5, 6.5\)

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Further this model can be extended by adding the concept of balking, reneging, feedback service, extended vacation etc. 

**REFERENCES**


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Mr. P. Karunakaran pursued his B.Sc degree from Ayya Nadar Janaki Anmial College, Sivakasi in 2009. He got his M.Sc degree from Ayya Nadar Janaki Anmial College, Sivakasi in 2012. He obtained his M.Phil. degree from Ayya Nadar Janaki Anmial College, Sivakasi in 2013. He had 6 years of teaching experience. Now he is doing research area in Queuing Theory in Kalasalingam Academy of Research and Education, Anand Nagar, Krishnankoil-626126, Tamilnadu, India.

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