

Non Markovian Queuing System with Restricted Admissibility Method

S. Maragathasundari, R. S. Somasundaram, S. Radha

Abstract: This article take a glance at a cluster area single server channel Queuing system, where the server gives two sorts of vacations viz., beginning one a short vacation and the long vacation is permitted as a second vacation. Long vacation is given in two stages. First stage is compulsory and if in case of need, the server goes for a optional second stage of long vacation. In addition, the concept of restricted admissibility of customers to the system is applied during the time of optional second stage of long vacation. For the above outlined covering issue, the beneficial variable method and probability approach are utilized to determine the Probability generating capacity of the line measure and the normal length of the line. In like way the other execution degrees of the model are settled utilizing Little's law. At last, the model is guarded by procedures for numerical redirection and the model is top notch by the graphical examination.

Keywords: Single arrival, Compulsory long vacation, Optional service, Restricted admissibility

I. INTRODUCTION

“Reference [6] derived the steady state queue size distribution at a random point of time as well as at a departure epoch”. “Reference [4] made a study on extended vacation and service interruption”. “Reference [9] examined balking and re-service in a vacation queue with batch arrival and two types of heterogeneous service”. “Reference [5] analyzed mobile adhoc networks problem- A queuing approach”. “Reference [7] analyzed multiple server vacation”. “Reference [8] investigated queue with breakdown and renegeing”. A study on the analysis of performance measure of bulk input queue with N type of additional optional service, service interruption and deterministic vacation examined by [11]. “Reference [3] investigated a queuing model on FIFO basis”. “Reference [1] investigated the Queuing process”. “Reference [13] examined some new results for departure process in the $M^X/G/1$ queuing system with a single vacation and exhaustive service”. “Reference [2] has been studied restricted admissibility on Queues”.

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II. MATHEMATICAL DESCRIPTION OF THE QUEUEING MODEL

The arithmetical interpretation of the Queuing framework has the capacity to be described by the resulting hypothesis:

Clients touch base at the framework in clumps of variable size in a compound procedure follows Poisson distribution. The administration time pursues general (arbitrary) circulation. First stage of giving service follows distribution function as $\bar{F}_1(x)$ and density function $\bar{f}_1(x)$. Let $\gamma_r(x)dx$ be the conditional density function. Hence we have

$$\gamma_r(x) = \frac{\bar{f}_1(x)}{1-\bar{F}_1(x)}, \quad \bar{f}_1(x) = \gamma_r(x)e^{-\int_0^x \gamma_r(s)ds} \quad (a)$$

For short vacation,

$$\gamma_s(x) = \frac{\bar{f}_2(x)}{1-\bar{F}_2(x)}, \quad \bar{f}_2(x) = \gamma_s(x)e^{-\int_0^x \gamma_s(s)ds} \quad (b)$$

For compulsory long vacation,

$$\gamma_{l_1}(x) = \frac{\bar{f}_3(x)}{1-\bar{F}_3(x)}, \quad \bar{f}_3(x) = \gamma_{l_1}(x)e^{-\int_0^x \gamma_{l_1}(s)ds} \quad (c)$$

For optional long vacation,

$$\gamma_{l_2}(x) = \frac{\bar{f}_4(x)}{1-\bar{F}_4(x)}, \quad \bar{f}_4(x) = \gamma_{l_2}(x)e^{-\int_0^x \gamma_{l_2}(s)ds} \quad (d)$$

Restricted admissibility occurs with mean rate $\beta > 0$.

III. GOVERNING EQUATIONS OF THE MODEL

$$\frac{\partial}{\partial x} R_n(x) + (\lambda^r \gamma_r(x))R_n(x) = \lambda^r R_{n-1}(x) \quad (1)$$

$$\frac{\partial}{\partial x} R_0(x) + (\lambda^r \gamma_r(x))R_0(x) = 0 \quad (2)$$

$$\frac{\partial}{\partial x} S_n(x) + (\lambda^r + \gamma_s(x))S_n(x) = \lambda^r S_{n-1}(x) \quad (3)$$

$$\frac{\partial}{\partial x} S_0(x) + (\lambda^r + \gamma_s(x))S_0(x) = 0 \quad (4)$$

$$\frac{\partial}{\partial x} L_n^{(1)}(x) + (\lambda^r + \gamma_{l_1}(x))L_n^{(1)}(x) = \lambda^r L_{n-1}^{(1)}(x) \quad (5)$$

$$\frac{\partial}{\partial x} L_0^{(1)}(x) + (\lambda^r + \gamma_{l_1}(x))L_0^{(1)}(x) = 0 \quad (6)$$

$$\frac{\partial}{\partial x} L_n^{(2)}(x) + (\lambda^r + \gamma_{l_2}(x))L_n^{(2)}(x) = \lambda^r \beta L_{n-1}^{(2)}(x) + \lambda^r (1-\beta)L_n^{(2)}(x) \quad (7)$$

$$\frac{\partial}{\partial x} L_0^{(2)}(x) + (\lambda^r + \gamma_{l_2}(x))L_0^{(2)}(x) = \lambda^r (1-\beta)L_0^{(2)}(x) \quad (8)$$

$$\lambda k =$$

$$(1-c) \int_0^\infty R_0(x) \gamma_r(x) dx + (1-p) \int_0^\infty S_0(x) \gamma_s(x) dx + (1-q) \int_0^\infty L_0^{(1)}(x) \gamma_{l_1}(x) dx + \int_0^\infty L_0^{(2)}(x) \gamma_{l_2}(x) dx \quad (9)$$

IV. BOUNDARY CONDITIONS

The following boundary conditions are used to solve the above equations:

$$R_n(0) = (1 - c) \int_0^\infty R_{n-1}(x) \gamma_r(x) dx + (1 - p) \int_0^\infty S_{n+1}(x) \gamma_s(x) dx + (1 - q) \int_0^\infty L_{n+1}^{(1)}(x) \gamma_{l_1}(x) dx + \int_0^\infty L_{n+2}^{(2)}(x) \gamma_{l_2}(x) dx + \lambda k \tag{10}$$

$$S_n(0) = c \int_0^\infty R_n(x) \gamma_r(x) dx \tag{11}$$

$$L_n^{(1)}(0) = p \int_0^\infty S_n(x) \gamma_s(x) dx \tag{12}$$

$$L_n^{(2)} = q \int_0^\infty L_n^{(1)}(x) \gamma_{l_1}(x) dx \tag{13}$$

$\sum_{n=1}^\infty (1) Xz^n + (2)$ gives

$$\frac{\partial}{\partial x} R_d(x, z) + (\lambda^r - \lambda^r z + \gamma_r(x)) R_d(x, z) = 0 \tag{14}$$

Similarly,

$$\frac{\partial}{\partial x} S_d(x, z) + (\lambda^r - \lambda^r z + \gamma_s(x)) S_d(x, z) = 0 \tag{15}$$

$$\frac{\partial}{\partial x} L_d^{(1)}(x, z) + (\lambda^r - \lambda^r z + \gamma_{l_1}(x)) L_d^{(1)}(x, z) = 0 \tag{16}$$

$$\frac{\partial}{\partial x} L_d^{(2)}(x, z) + (\lambda^r \beta - \lambda^r \beta z + \gamma_{l_2}(x)) L_d^{(2)}(x, z) = 0 \tag{17}$$

Applying the same for boundary conditions, we get

$$zR_d(0, z) = (1 - c) \int_0^\infty R_d(x, z) \gamma_r(x) dx + (1 - p) \int_0^\infty S_d(x, z) \gamma_s(x) dx + (1 - q) \int_0^\infty L_d^{(1)}(x, z) \gamma_{l_1}(x) dx + \int_0^\infty L_d^{(2)}(x, z) \gamma_{l_2}(x) dx + \lambda z k - \lambda k \tag{18}$$

$$S_d(0, z) = c \int_0^\infty R_d(x, z) \gamma_r(x) dx \tag{19}$$

$$L_d^{(1)}(0, z) = p \int_0^\infty S_d(x, z) \gamma_s(x) dx \tag{20}$$

$$L_d^{(2)}(0, z) = q \int_0^\infty L_d^{(1)}(x, z) \gamma_{l_1}(x) dx \tag{21}$$

(14) from 0 to x gives

$$R_d(x, z) = R_d(0, z) e^{-(\lambda^r - \lambda^r z)x - \int_0^x \gamma_r(t) dt} \tag{22}$$

Integrating (22) by parts we get

$$R_d(z) = R_d(0, z) \left(\frac{1 - \bar{F}_1(h)}{h} \right) \tag{23}$$

Where $h = \lambda^r - \lambda^r z$

Multiply (22) by $\gamma_r(x) dx$ and integrating by parts, we get

$$\int_0^\infty R_d(x, z) \gamma_r(x) dx = R_d(0, z) \bar{F}_1(h) \tag{24}$$

Similarly, we have

$$S_d(z) = c R_d(0, z) \bar{F}_1(h) \left(\frac{1 - \bar{F}_2(h)}{h} \right) \tag{25}$$

$$\int_0^\infty S_d(x, z) \gamma_s(x) dx = c R_d(0, z) \bar{F}_1(h) \bar{F}_2(h) \tag{26}$$

$$L_d^{(1)}(z) = p c R_d(0, z) \bar{F}_1(h) \bar{F}_2(h) \left(\frac{1 - \bar{F}_3(h)}{h} \right) \tag{27}$$

$$\int_0^\infty L_d^{(1)}(x, z) \gamma_{l_1}(x) dx = p c R_d(0, z) \bar{F}_1(h) \bar{F}_2(h) \bar{F}_3(h) \tag{28}$$

$$L_d^{(2)}(z) = q p c R_d(0, z) \bar{F}_1(h) \bar{F}_2(h) \bar{F}_3(h) \left(\frac{1 - \bar{F}_4(e)}{h} \right) \tag{29}$$

Where $e = \lambda^r \beta + \lambda^r \beta z$

$$\int_0^\infty L_d^{(2)}(x, z) \gamma_{l_2}(x) dx = q p c R_d(0, z) \bar{F}_1(h) \bar{F}_2(h) \bar{F}_3(h) \bar{F}_4(e) \tag{30}$$

Substituting (24), (26), (28), (30) in (18), we get

$$R_d(0, z) = \frac{\lambda z k - \lambda k}{z - (1-c)\bar{F}_1(h) - (1-p)c\bar{F}_1(h)\bar{F}_2(h) - (1-q)pc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h) - qpc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h)\bar{F}_4(e)} \tag{31}$$

Using (31) in (23), (25), (27), (29) we get

$$R_d(z) = \frac{[\bar{F}_1(h) - 1]}{z - (1-c)\bar{F}_1(h) - (1-p)c\bar{F}_1(h)\bar{F}_2(h) - (1-q)pc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h) - qpc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h)\bar{F}_4(e)} \tag{32}$$

$$S_d(z) = \frac{c\bar{F}_1(h)[\bar{F}_2(h) - 1]}{z - (1-c)\bar{F}_1(h) - (1-p)c\bar{F}_1(h)\bar{F}_2(h) - (1-q)pc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h) - qpc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h)\bar{F}_4(e)} \tag{33}$$

$$L_d^{(1)}(z) = \frac{pc\bar{F}_1(h)\bar{F}_2(h)[\bar{F}_3(h) - 1]}{z - (1-c)\bar{F}_1(h) - (1-p)c\bar{F}_1(h)\bar{F}_2(h) - (1-q)pc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h) - qpc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h)\bar{F}_4(e)} \tag{34}$$

$$L_d^{(2)}(z) = q p c \bar{F}_1(h) \bar{F}_2(h) \bar{F}_3(h) \frac{[\bar{F}_4(e) - 1]}{\beta} \tag{35}$$

V. LIKELIHOOD CREATING CAPACITY OF THE LINE ESTIMATE

Let $M_d(z)$ be the probability generating function of the queue size, It is given by

$$M_d(z) = k [R_d(z) + S_d(z) + L_d^{(1)}(z) + L_d^{(2)}(z)]$$

Adding (32) to (35), we get,

$$M_d(z) = \frac{pc\bar{F}_1(h)\bar{F}_2(h)[\bar{F}_3(h) - 1] + qpc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h)\frac{[\bar{F}_4(e) - 1]}{\beta}}{z - (1-c)\bar{F}_1(h) - (1-p)c\bar{F}_1(h)\bar{F}_2(h) - (1-q)pc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h) - qpc\bar{F}_1(h)\bar{F}_2(h)\bar{F}_3(h)\bar{F}_4(e)} \tag{36}$$

VI. IDLE TIME AND UTILIZATION FACTOR

The normalization condition $M_d(1) + k = 1$

is used to determine k. At $z = 1$ $M_d(z)$ attains 0/0 indeterminate form.

$$\lim_{z \rightarrow 1} M_d(1) = \frac{N'(1)}{D'(1)}$$

Hence using L.H rule we get, Hence the utilization factor ρ is found using $\rho = 1 - k$

To find L_q the length of the Queue and the Queue performance measures.

The Average queue size is obtained by the usage of L H rule in (36)

$$L_q(z) = \lim_{z \rightarrow 1} \frac{D'(z)N''(z) - N'(z)D''(z)}{2(D'(z))^2} \tag{37}$$

$$N'(1) = \lambda^r E(F_1) + c\lambda^r E(F_2) + pc\lambda^r E(F_3) + \lambda^r qpc E(F_4)$$

$$N''(1) = (\lambda^r)^2 E(F_1^2) + c(\lambda^r)^2 [2E(F_1)E(F_2) + E(F_2^2)] + pc(\lambda^r)^2 [2E(F_1)E(F_3) + 2E(F_2)E(F_3) + E(F_3^2)] + qpc(\lambda^r)^2 [2(E(F_1) + E(F_2) + E(F_3))E(F_4) + E(F_4^2)]$$



$$D'(1) = 1 - \lambda^r(1 - c)E(F_1) - (1 - p)c\lambda^r[E(F_1) + E(F_2)] - (1 - q)pc\lambda^r[E(F_1) + E(F_2) + E(F_3)] - qpcc\lambda^r[E(F_1) + E(F_2) + E(F_3) + \beta E(F_4)]$$

$$D''(1) = -(1 - c)(\lambda^r)^2 E(F_1^2) - (1 - P)c(\lambda^r)^2 [E(F_1^2) + E(F_2^2) + 2E(F_1)E(F_2)] - (1 - q)pc(\lambda^r)^2 [E(F_1^2) + E(F_2^2) + E(F_3^2) + 2[E(F_1)E(F_2) + E(F_2)E(F_3) + E(F_1)E(F_3)]] - qpcc(\lambda^r)^2 [E(F_1^2) + E(F_2^2) + E(F_3^2) + \beta^2 E(F_4^2) + 2[E(F_1)E(F_2) + E(F_2)E(F_3) + E(F_3)\beta E(F_4) + \beta E(F_4)E(F_1) + E(F_1)E(F_3) + \beta E(F_2)E(F_4)]]$$

(38)

Substituting (38) in (37) we obtain L_q in closed form.

Further, by the usage of Little's law we get the other

execution measures $W_q = \frac{L_q}{\lambda}$, $W = \frac{L}{\lambda}$, $L = L_q + \rho$

VII. NUMERICAL ILLUSTRATION

THE VALUES ARE COLLECTED

ACCORDINGLY:

$$\lambda^r = 3, \beta = 0.6, c = 0.5, p = 0.3, q = 0.4, \gamma_r = 3.5, \gamma_s = 5, \gamma_{i_1} = 4, \gamma_{i_2} = 4.5, E(F_1) = \frac{1}{\gamma_r}, E(F_2) = \frac{1}{\gamma_s}, E(F_3) = \frac{1}{\gamma_{i_1}}, E(F_4) = \frac{1}{\gamma_{i_2}}, E(F_1^2) = \frac{2}{\gamma_r^2}, E(F_2^2) = \frac{2}{\gamma_s^2}, E(F_3^2) = \frac{2}{\gamma_{i_1}^2}, E(F_4^2) = \frac{2}{\gamma_{i_2}^2}$$

Table I
Effect of change of (p = 0.3, 0.4, 0.5, 0.6, 0.7)

| Q | ρ | L_q | L | W_q | W |
|--------|--------|---------|---------|--------|--------|
| 0.2891 | 0.7109 | 16.7738 | 17.4847 | 5.5913 | 5.8282 |
| 0.3322 | 0.6678 | 13.3433 | 14.0111 | 4.4478 | 4.6704 |
| 0.3747 | 0.6203 | 10.9610 | 11.5813 | 3.6537 | 3.8604 |
| 0.4168 | 0.5832 | 9.2299 | 9.8131 | 3.0766 | 3.2710 |
| 0.4585 | 0.5415 | 7.9177 | 8.4592 | 2.6392 | 2.8197 |

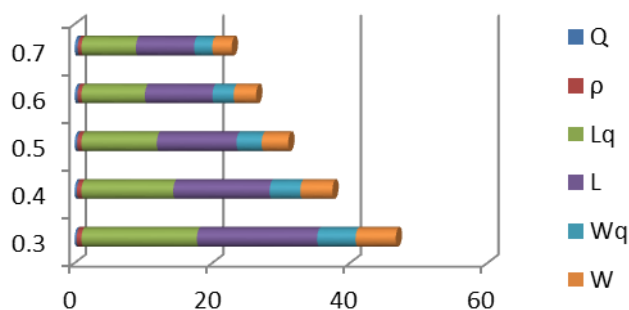


Fig. 1 Effect of change of p

From Table I It is clear that if the probability of taking long vacation increases it leads to decrease in all the performance measures. Since the long vacation gets increased the idle time gets amplified.

Table II
Effect of change of (q = 0.4, 0.5, 0.6, 0.7, 0.8)

| Q | ρ | L_q | L | W_q | W |
|--------|--------|---------|---------|--------|--------|
| 0.2891 | 0.7109 | 16.7738 | 17.4847 | 5.5913 | 5.8282 |
| 0.2939 | 0.7061 | 16.1775 | 16.8836 | 5.3925 | 5.6279 |
| 0.2986 | 0.7014 | 15.6126 | 16.3140 | 5.2042 | 5.4380 |
| 0.3032 | 0.6968 | 15.0766 | 15.7734 | 5.0255 | 5.2578 |
| 0.3079 | 0.6921 | 14.5684 | 15.2605 | 4.8561 | 5.0868 |

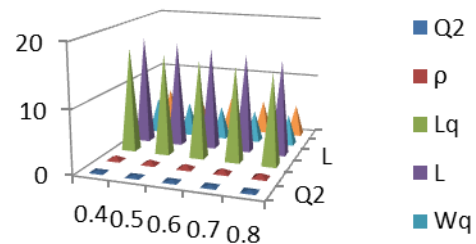


Fig. 2 Effect of change of q

Table 2 indicates that, as the probability of taking optional second stage of long vacation gets increased, its corresponding execution measures gets decreased. In addition, idle time increase whereas the utilization factor gets diminished.

VIII. CONCLUSION

The above model gives an intensive report about the lining issue including a few parameters like Single landing, single administration, short excursion, phases of Compulsory long get-away, restricted acceptability during the season of long get-away. The model is very much examined and its practically equivalent to lining execution measures are resolved. Numerical investigation and pictographic portrayal gives an unmistakable image of the model characterized.

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