Queueing Analysis on Multiple Vacation Policies and Reneging

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Abstract: We study a non markovian queue which renders service to the customers. After the completion of service the server ought to go for the compulsory vacation stage by stage in succession. We consider one of the customer’s behaviors reneging to occur when the server’s vacation is extended. Using Supplementary variable technique the system performance measures is derived.

Keywords: Stages of service, Optional extended vacation, Reneging, Extended Vacation

I. INTRODUCTION


II. MATHEMATICAL DESCRIPTION OF THE QUEUEING MODEL

The arithmetical interpretation of the Queuing framework has the capacity to be described by the resulting hypothesis:

Customer’s arrival follows Poisson procedure. There is one server giving administration. The organization time seeks after general (arbitrary) course with first basic dispersion function H(x) and thickness work h(x). Let \( \theta_1(x) \) dx be the prohibitive chance of organization finish of the principle period of organization in the midst of the interval (x,x+dx), given that the snuck past time is x, so that

\[
\theta_1(x) = -\frac{h(x)}{1-H(x)}
\]

For First stage of compulsory vacation ,

\[
\theta_2(x) = \frac{u(x)}{1-u(x)} , \quad u(x) = e^{-\int_0^x \theta_1(x) \, dx}
\]

For Second Stage of compulsory vacation,

\[
\theta_3(x) = \frac{m(x)}{1-M(x)} , \quad m(x) = \theta_3(x)e^{-\int_0^x \theta_2(x) \, dx}
\]

For Optional Extended Vacation,

\[
\theta_4(x) = \frac{f(x)}{1-F(x)} , \quad f(x) = \theta_4(x)e^{-\int_0^x \theta_3(x) \, dx}
\]

Reneging occur during extended vacation with probability \( r \).

III. PROBABILITY GENERATING FUNCTION

\[
K_n(x,z) = \sum_{n=1}^{\infty} z^n K_n(x) ; \quad K_n(x) = \sum_{n=1}^{\infty} z^n K_n , \quad |z| \leq 1
\]

\[
C_n^{(1)}(x,z) = \sum_{n=1}^{\infty} z^n C_n^{(1)}(x) ; \quad C_n^{(1)}(x) = \sum_{n=1}^{\infty} z^n C_n^{(1)}
\]

\[
E_n^{(1)}(x,z) = \sum_{n=1}^{\infty} z^n E_n^{(1)}(x) ; \quad E_n^{(1)}(x) = \sum_{n=1}^{\infty} z^n E_n
\]

IV. STEADY STATE CONDITIONS OVERSEEING THE FRAMEWORK

\[
\frac{\partial}{\partial x} K_n(x) + \frac{\partial}{\partial x} (\rho^2 + \theta_1(x)) K_n(x) = \lambda^P K_{n-1}(x) \tag{1}
\]

\[
\frac{\partial}{\partial x} K_0(x) + \frac{\partial}{\partial x} (\rho^2 + \theta_1(x)) K_0(x) = 0 \tag{2}
\]

\[
\frac{\partial}{\partial x} C_n^{(1)}(x) + \frac{\partial}{\partial x} (\rho^2 + \theta_2(x)) C_n^{(1)}(x) = \lambda^P C_{n-1}^{(1)}(x) \tag{3}
\]

\[
\frac{\partial}{\partial x} C_0^{(1)}(x) + \frac{\partial}{\partial x} (\rho^2 + \theta_2(x)) C_0^{(1)}(x) = 0 \tag{4}
\]

\[
\frac{\partial}{\partial x} C_n^{(2)}(x) + \frac{\partial}{\partial x} (\rho^2 + \theta_3(x)) C_n^{(2)}(x) = \lambda^P C_{n-1}^{(2)}(x) \tag{5}
\]

\[
\frac{\partial}{\partial x} C_0^{(2)}(x) + \frac{\partial}{\partial x} (\rho^2 + \theta_3(x)) C_0^{(2)}(x) = 0 \tag{6}
\]

\[
\frac{\partial}{\partial x} E_n^{(1)}(x) + \frac{\partial}{\partial x} (\rho^2 + \theta_4(x) + \varepsilon) E_n^{(1)}(x) = \lambda^P E_{n-1}^{(1)}(x) + \xi \tag{7}
\]

where \( x > 0, n \geq 1 \).
V. BOUNDARY CONDITIONS

The above set of equations is to be solved under the following boundary conditions at $x = 0$ and for $x \geq 1$

\[
K_n(0) = \int_0^\infty K_{n+1}(x) \theta_1(x) dx + (1 - r) \int_0^\infty C^{(2)}_{n-1}(x) \theta_3(x) dx + \int_0^\infty K_n(x) \theta_4(x) dx
\]

(10)

\[
C^{(1)}_n(0) = \int_0^\infty K_n(x) \theta_1(x) dx
\]

(11)

\[
C^{(2)}_n(0) = \int_0^\infty C^{(1)}_n(x) \theta_2(x) dx
\]

(12)

\[
E_n(0) = r \int_0^\infty C^{(2)}_n(x) \theta_2(x) dx
\]

(13)

Multiply (1) by $z^n$ and sum over $n$ from 1 to $\infty$. Adding to (2), we get

\[
\frac{\partial}{\partial x} K_n(x, z) + (\lambda' + \theta_1(x) - \lambda P z) K_n(x, z) = 0
\]

(14)

\[
\frac{\partial}{\partial x} C^{(2)}_n(x, z) + (\lambda' + \theta_2(x) - \lambda' P z) C^{(2)}_n(x, z) = 0
\]

(15)

\[
\frac{\partial}{\partial x} C^{(3)}_n(x, z) + (\lambda' + \theta_3(x) - \lambda' P z) C^{(3)}_n(x, z) = 0
\]

(16)

\[
\frac{\partial}{\partial x} E_n(x, z) + (\lambda' + \theta_4(x) - \lambda' P z + \xi - \frac{\lambda}{x}) K_n(x, z) = 0
\]

(17)

Next applying the procedure of supplementary variable for the boundary conditions, we have

\[
K_n(0, z) = \int_0^\infty K_n(x, z) \theta_1(x) dx + (1 - r) \int_0^\infty C^{(2)}_{n-1}(x, z) \theta_3(x) dx + \int_0^\infty K_n(x, z) \theta_4(x) dx
\]

(18)

\[
C^{(1)}_n(0, z) = \int_0^\infty K_n(x, z) \theta_1(x) dx
\]

(19)

\[
C^{(2)}_n(0, z) = \int_0^\infty C^{(1)}_n(x, z) \theta_2(x) dx
\]

(20)

\[
E_n(0, z) = r \int_0^\infty C^{(2)}_n(x, z) \theta_2(x) dx
\]

(21)

Integrating (14) from 0 to $\infty$, we get

\[
K_n(x, z) = K_n(0, z) e^{-(\lambda' - \lambda P z)x - \lambda' \theta_1(x) \theta_4 \gamma(x) dx}
\]

(22)

Again integrating the above by parts, we get

\[
K_n(z) = K_n(0, z) [1 - \frac{J_1(A)}{A}]
\]

(23)

where

\[
J_1(A) = \int_0^\infty e^{-(\lambda' - \lambda P z)x - \lambda' \theta_1(x) \theta_4 \gamma(x) dx}
\]

is the Laplace Stieltje's transform of service time.

Again multiplying (22) by $\theta_1(x)$ on both the sides and integrating over $x$, we get

\[
\int_0^\infty K_n(x, z) \theta_1(x) dx = K_n(0, z) J_1(A)
\]

(24)

Similarly for vacation process, we have

\[
C^{(1)}_n(0, z) = \frac{J_1(A)}{A}
\]

(25)

\[
E_n(0, z) = \int_0^\infty C^{(2)}_n(x, z) \theta_2(x) dx = K_n(0, z) J_1(A) J_2(A)
\]

(26)

VI. LIKELIHOOD CREATING CAPACITY OF THE LINE ESTIMATE

To discover the likelihood making point of confinement of the line check paying little personality to the condition of the structure, we let $Y_{\mathcal{Q}}(z)$ be the p.g.f. of the line length

\[
F(z) = K_n(z) + C^{(1)}_n(z) + C^{(2)}_n(z) + E_n(z)
\]

(36)

VII. NORMALIZATION CONDITION

\[
F(1) + S = 1
\]

Gives the idle time $S$ and the Utilization factor.

Idle time

\[
S = \frac{d'(1)}{d(1) + N'(1)}
\]

(37)

Utilization factor, $\rho = 1 - S$

(38)

\[
F(1) = \lim_{t \to 1} \frac{0}{\theta_4 \text{indeterminate form}}
\]

Hence applying L'Hopital's rule, we get

\[
F(1) = \frac{d'(1)}{d'(1) + N'(1)}
\]

(39)
VII. SYSTEM QUEUE PERFORMANCE MEASURES

Let \( L_q \) a chance to demonstrate the reliable state typical number of customers in the line. By then

\[
L_q = \lim_{z \to 1} \frac{d}{dz} F(z) |_{z=1} = \frac{d}{dz} \left( \frac{N(z)}{D(z)} \right) |_{z=1}
\]

Where \( N(Z) \) and \( D(Z) \) are the numerator and denominator of (36).

Since \( F(z) = \frac{z}{\beta} \) at \( z = 1 \), we utilize two fold separation and get

\[
L_q = \lim_{z \to 1} \frac{d}{dz} Q_q(z) = \frac{\beta^2(1)N'(1) - \beta^2(1)N'(1)}{2(\beta^2(1))^2} \quad (40)
\]

\[
N'(1) = \beta^2[E(J_1) + E(J_2) + E(J_3)] + E(J_4) \quad (41)
\]

\[
N'(1) = (\beta^2)^2 [E(J_1^2) + 2E(J_1)E(J_2) + 2E(J_1)E(J_3) + 2E(J_2)E(J_3) + E(J_3)] \quad (42)
\]

\[
D'(1) = 1 - \beta^2 E(J_1) - (1 - \rho E(J_4)) \beta^2 E(J_1) + E(J_2) + E(J_3) + \rho E(J_4)(-\beta^2 + \xi) \quad (43)
\]

\[
D'(1) = -(\beta^2)^2 E(J_1^2) + 2E(J_1)(-\beta^2 + \xi) E(J_1) + 2E(J_2) + (1 + \rho)(\beta^2)^2 E(J_1) + 2E(J_2)E(J_3) + 2E(J_2)E(J_3) + E(J_3) + (E(J_1) + E(J_2) + E(J_3))(-\beta^2 + \xi) - \beta^2 E(J_3)(-\lambda + \xi)^2 \quad (44)
\]

Substituting (40) – (44) in (40), we obtain \( L_q \) and all the other measures using Little’s formula

\[
W_q = \frac{L_q}{\lambda}, \quad W = \frac{\rho}{\lambda}, \quad L = L_q + \rho.
\]

IX. NUMERICAL JUSTIFICATION OF THE MODEL

Assume that service time follows exponential distribution. Assume that service time follows exponential distribution in particular and based on this condition, the numerical justification is elaborated below:

The values are collected accordingly:

\[
E(J_1) = \frac{1}{\theta_1}, \quad E(J_2) = \frac{1}{\theta_2}, \quad E(J_3) = \frac{1}{\theta_3}, \quad E(J_4) = \frac{1}{\theta_4}, \quad E(J_1^2) = \frac{2}{\theta_1^2}, \quad E(J_2^2) = \frac{2}{\theta_2^2}, \quad E(J_3^2) = \frac{2}{\theta_3^2}, \quad E(J_4^2) = \frac{2}{\theta_4^2}, \quad \rho = 0.5, \xi = 1, \theta_1 = 1.5, \theta_2 = 2, \theta_3 = 2.5, \theta_4 = 3, \lambda = 3.5
\]

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<th>( L )</th>
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In Table 1, keeping the majority of the parameters dependable and developing the input favorable position parameter alone, it prompts an expansion in all the execution measures obviously. Like way in Table 2, the likelihood of association intrusion makes the structure to a little expansion in the execution measures. All the above outcomes are as expected. Graphical structures still stresses the model to a reasonable comprehension.

X. NUMERICAL DISCUSSION

In Table 1, keeping the majority of the parameters dependable and developing the input favorable position parameter alone, it prompts an expansion in all the execution measures obviously. Like way in Table 2, the likelihood of association intrusion makes the structure to a little expansion in the execution measures. All the above outcomes are as expected. Graphical structures still stresses the model to a reasonable comprehension.

XI. CONCLUSION

The model described above particularly cleared up the administration, stages of compulsory vacation and optional extended vacation in a non markovian covering model. Every one of these parameters has an effect over all the execution measures. The outcomes are not surprisingly. As a future work, a remain by server, feedback service, break down, reservice can be introduced in the midst of the period of discrete. Fix procedure can be given in stages. Furthermore balking can be presented.
Queuing Analysis on Multiple Vacation Policies and Reneging

This showcase accept obvious employment in gathering units, correspondence structure, movement crossing focuses, and so forth.

REFERENCES


AUTHORS PROFILE

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