Rgδ-Continuous Functions and Their Properties in Topological Spaces

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Abstract: ***A continuity form of rgδ is offered in this commodity. The design of rgδ-irresolute functions and strongly rgδ-continuous functions are also discussed and its properties are investigated.***

Keywords: rgδ-continuous, rgδ-Irresolute maps, strongly rgδ-continuous

I. INTRODUCTION

Many researchers have introduced the concepts of generalized continuous functions, gc-irresolute maps on topological spaces.[[1], [2], [5], [6], [7]]. Some authors continuing their studies on generalization of continuous maps. Irresolute functions are studied by Crossley and Hilderbrand [3] Sheik John [4] introduced and studied the concepts of strongly w-continuous maps. In this write- up, we initiated the perception of rgδ-continuous functions.

II. MAIN RESULTS

In this section the new function called rgδ continuous functions is defined and obtained some results by comparing it with some other continuous functions. Irresolute functions and strongly rgδ-continuous functions are also discussed in the subsequent sections. Definitions of rw, rwg, sg and gpr-continuous extracted from [8], [9], [10], [11], [12] respectively.

Definition 2.1 A function e: (R, ψ) → (N, η) is said to be rgδ-continuous if the inverse image of every closed set in N is rgδ-closed set in R.

Remark 2.2 [In [8], By Theorem 2.2] Every closed set is rgδ-closed but not conversely.

Theorem 2.3 If e: (R, ψ) → (N, η) is continuous then e is rgδ-continuous.

Proof: Let G⊂N, where G is closed. e is continuous

$\Rightarrow e^{-1}(G)$ is closed. By Remark 2.2., $e^{-1}(G)$ is rgδ-closed.

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Hence e is rgδ-continuous.

Remark 2.4 The converse part fails in Theorem 2.3.

Example 2.5 Let M = N = {η,ψ,ω} with ξ= { M, φ,{η},{ψ}, {η, ψ}, {ψ, ω}} and v= { N, φ, {η}, { ω} , { η, ω}, { ψ, ω }}. Let q : (M, ξ) → (N, υ) be defined by q(η) = ψ, q(ψ) = ω, q(ω) = η. Then f is rgδ continuous but not continuous as the inverse image of η in N is ω, which is not in M.

Theorem 2.6 Let h: (P, σ) → (Q, η) is rgδ-continuous if and only if every open in Q is rgδ-open in P

Proof. Let a function h: (P, σ). → (Q, η) is rgδ-continuous and let open set H in Q. Then H^c is a closed set in Q. Since h is rgδ-continuous, $h^{-1}(H^c)$ is rgδ-closed in P. As $h^{-1}(H^c)$ =P, $h^{-1}(H)$ is rgδ-open in P .

Converse Part: Let $h^{-1}(H)$ is rgδ-open in P . Let G be closed set in Q. Then G^c is open Q. Therefore $h^{-1}(G^c)$=P. Therefore $h^{-1}(G)$ is rgδ-open. i.e, $h^{-1}(G)$ is closed in X. Thus h is rgδ-continuous.

Theorem 2.7 If j: (K, υ) → (L, ω) is rw-continuous then j is rgδ-continuous.

Proof. Consider j is rw-continuous, which implies rw- closed set S⊂L. $j^{-1}(S)$ is rw-closed in K. Since every rw- closed set is rgδ-closed set, $j^{-1}(S)$ is rgδ-closed in K. Hence j is rgδ-continuous.

Remark 2.8 The reverse part of theorem 2.7 need not true.

Example 2.9 Let K = { 11, 22,33, 44, 55 } with v = { K, φ, {11},{ 22} , {33} , {11, 22} , {11, 33} ,{ 22, 33}, {11, 22, 33} }. L={ 11, 22,33,44,55,66} Ψ = { L, φ, {11} , {22 }, {33} , {11, 22} , {11, 33} , {22, 33} , {11, 22, 33 } }. Define j: (K, υ) → (L, ψ) by j(11) = 44, j(22) = 11, j(33)= 22, j(44) = 44, j(55) = 44. Then j is rgδ- continuous. Here the closed {22, 33} in (L, ψ) is 33 not rw in K.

Theorem 2.10 If p : Q→R is g continuous implies then p is rgδ-continuous.

Proof. Let p: Q → R is g-continuous. Let K ⊂ R. $p^{-1}(K)$ is g-closed in R. Every g-closed set is rgδ-closed set in R,

p^{-1}(K) is rgδ-closed in R. So p is rgδ-continuous.

Remark 2.11 The contra part fails in Theorem 2.10.

Example 2.12 Lt Q=R={ i, j, k } with τ = { Q, φ,{ i } , { j}, {i, j} } and σ = { R, φ, { i } ,{ j, k}}. Let p: Q → R is identity one. Then p is rgδ-continuous but not g-continuous

Theorem 2.13 Let s : (T, μ) → (U, ρ) is rgδ-continuous, then s is rwg-continuous.

Proof. A function s: (T, μ) → (U, ρ) is rgδ-continuous. Then a K be in U, s^{-1}(K) is in T . Here every rgδ-closed set is rwg-closed set, s^{-1}(K) is rwg-closed in T. Thus s is rwg-continuous.

Remark 2.14 In Theorem 2.13, the converse part is not satisfied.

Example 2.15 Let T = {l,. m, n, o } with μ = {T, φ, {l}, {m}, {l, m}}. and U = {g,h, i, j } with ρ = {T, φ, {g} , {h}, {g, h}, {g, h, i} }. Define a function r: (T, μ) → (U, ρ) by r(l)=g, r(m)=I, r(n)=I ,r(o)=g. Therefore r is rwg –continuous. Hence p ⊂T, {l,0}⊄T. Hence {l,0}⊄T is not rgδ-closed set in T.

Remark 2.16 sg-continuous and rgδ- continuous are independent

Example 2.17 Let T = { u, v, w,x } with ς ={ T, φ, { u }, { v } , { u, v} , {u , v, w } and Ω = { u,v, w } with ξ={ Ω, φ, { u }, { v } , { u, v } }. Define a function o: (T, ς) → (Ω, ξ) by o(u) = x, o(v) = v, o(w) = f (x) = u. Then o^{-1}(Ω) is rgδ-closed in T . Thus o is rgδ-continuous. As the inverse image of (v, w) in ω is u, v is not sg-closed in T . Therefore o is not sg-continuous.

Example 2.18 Let V = { υ, ψ,λ, κ} with γ = { V, φ, { υ} , { ψ}, { υ, ψ }, {υ , ψ, λ }} and W = { υ , ψ , λ } with β = { W, φ, { v, ψ }}. Define a function u: (V,γ) → (W, κ) by u(υ) = u(β) = α. Thus u^{-1}(W) is sg-closed. Hence u is sg-continuous u(v) = v, u(w) = u(x) = u. Then u^{-1}(W) is rgδ-closed set in T. Thus u is rgδ-continuous. But u is not rgδ-continuous, where the closed set λ in W is not rgδ-closed in V.

Remark 2.19 Implications obtained from 2.1 to 2.18 is given below:

Theorem 2.20 If d : (C,ψ) →(D, ρ) is rgδ-continuous, then d is gpr-continuous.

Proof. Given d: (C, ψ) → (D, ρ) is rgδ-continuous. Let N⊂ D⇒ rgδ-continuous. Then d^{-1}(N) is rgδ-closed in C. Therefore d^{-1}(N) is gpr-closed in C. Thus d is gpr-continuous.

Remark 2.21 The reverse part is not true in theorem 2.20.
Example 2.22 Let C= { ξ , κ, ζ , ω }with ψ= { C, φ, {ξ}, { κ } , { ξ, κ} , { ξ, κ, ζ }}, D = {j. k. l }, ρ = { D, φ, j }. Define d : (C, ψ) → (D, ρ), by d(ξ) = k, d(κ) = d(ζ) = j. Thus d is gpr-continuous not rgδ- continuous. Here d^{-1}(k, l) in D is (ξ, ω) in C, not rgδ-closed in X.

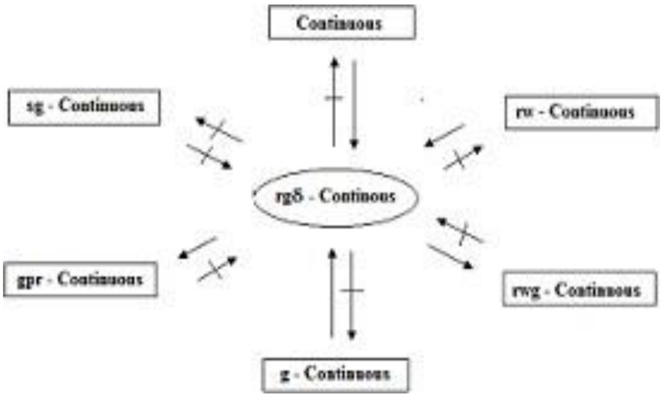

Fig. 1. Implication of rgδ-continuous.

III. RGΔ-IRRESOLUTE FUNCTIONS

Definition 3.1 A function q:(R, η) →(S, ψ) is said to be irresolute function if every rgδ-closed q^{-1}(S, ψ) is rgδ ⊂ (R, η).

Theorem 3.2 A map a : (H, κ) → (Z, ζ) is rgδ-irresolute if and only if the a^{-1}(Z, ζ)-open is rgδ-open in (H, κ).

Proof Let a : (R, η) →(Z, ζ) be rgδ-irresolute and W be rgδ-open in (Z, ζ). W^c is rgδ-closed in (Z, ζ). Here a is rgδ-irresolute, $a^{-1}(W^{c})$ is rgδ-closed in (H, κ). Here $a^{-1}(W^{c}) = (a^{-1}(W))^c$. Hence a^{-1}(W) is rgδ-open in (Y, κ)

Converse Part: For every rgδ-open set W in (Z, ζ) assume that a^{-1}(W) is rgδ-open in (Y, κ). Let U be a rgδ- closed set in (Z, ζ). Now U^c is rgδ-open in (Z, ζ) , $a^{-1}(U^{c})$ is rgδ-closed in (Y, κ). Here $a^{-1}(F^{c}) = (a^{-1}(F))^c$, we have x^{-1}(F) is rgδ-closed in (Y, κ). Hence a is rgδ-irresolute.

Theorem 3.3 A function z: (A, ψ) → (B, μ) is rgδ-irresolute ⇒ rgδ-continuous.

Proof Let z : (A, ψ) →(B, μ) be rgδ-irresolute and a closed set G∈(B, μ). ⇒G is rgδ (B, μ). As z is rgδ-irresolute, z^{-1}(G) is rgδ-closed in (A, ψ) and therefore z is rgδ-continuous.

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Remark 3.4 g-irresolute and rgδ –irresolute function are independent.

Example 3.5 Let $K = L = \{ m, n, o, p\}$ with $\rho = \{ K, \varphi, \{m\}, \{n\}, \{m, n\}\}$ and $\omega = \{ L, \varphi, \{m\}, \{m, n\}\}$. The identity map $w : (K, \rho)\rightarrow(L, \omega)$ is rgδ -irresolute but it is not g-irresolute. $r^{-1}\{m, p\}\subset (L, \omega)$ is $\{ m, p\}$ in (K, ρ) which is rgδ closed.

Example 3.6 Let $Z = Y =\{ \eta, \gamma, \nu\}$ with $\Gamma = \{ Z, \varphi, \{\eta\}\}$ and $\in = \{ Y, \varphi, \{\eta\}, \{\gamma, \nu\}\}$. Then the function $e: (Z, \Gamma) \rightarrow(Y, \mathcal{E})$ defined by $e(\eta)= \eta$, $e(\gamma)= e(\nu)= \nu$ is g-irresolute but it is not rgδ-irresolute function. Since $e^{-1}\{\gamma, \nu\}\in (Y, \mathcal{E})\not\subseteq (Z, \Gamma)$.

IV. STRONGLY RGΔ-CONTINUOUS

Definition 4.1 A function $r: (E, \theta) \rightarrow (F, \varsigma)$ is said to be strongly rgδ-Continuous function if the inverse image of rgδ-open in (F, ς) is open in (E, θ).

Theorem 4.2 A map $i: (V, \omega) \rightarrow (W, \theta)$ is a strongly rgδ-continuous function iff $i^{-1}(H)$ is closed in (W, θ).

Proof. Assume $i: (V, \omega) \rightarrow (W, \theta)$ is a strongly rgδ-continuous function. Let H be rgδ closed in (W, θ). Now H^c is a rgδ-open set in (W, θ). As i- is strongly rgδ-continuous, $i^{-1}(H^c) = V^{-1}(H)$ .Therefore $i^{-1}(H)$ is closed in (W, θ).

Converse Part: $i^{-1}(H)$ is closed in (W, θ). Let $G\subset$ rgδ open (W, θ), then G^c is a rgδ-closed in (W, θ). Since $i^{-1}(G^c) = V^{-1}(G)$ is closed in (V, ω). i.e., $i^{-1}(G)$ is open in (V, ω). Thus i is strongly rgδ-continuous.

Theorem 4.3 A map $n : (G, \alpha) \rightarrow (H, \beta)$ is a strongly rgδ-continuous then n is continuous.

Proof. Let $n : (G, \alpha) \rightarrow(H, \beta)$ is a strongly rgδ-continuous Let J be open in (H, β). As J is rgδ- open in (H, β). As n-is strongly rgδ-continuous, $n^{-1}(H)$. Thus n is continuous.

Theorem 4.4 Suppose $v: (C, \beta) \rightarrow (D, \gamma)$ is a strongly rgδ-continuous then v is strongly g-continuous.

Proof. Let v is strongly rgδ-continuous. Let $E\subset D$, where E is g-open. Therefore E is rgδ-open in (D, γ), v is strongly rgδ-continuous, As $v^{-1}(E)\subset (C, \beta)$, v is strongly g-continuous.

Example 4.5 Let $Q = \{s, t, u, v\}$ and $R = \{ f, x, y, z\}$ $\mathcal{E} = \{ R, \varphi, \{s\}, \{t\}, \{s, t\}\}$ and $\omega = \{R, \varphi, \{f\}, \{x\}, \{f, x\}\}$. The map $t : (Q, \mathcal{E}) \rightarrow (R, \omega)$ defined by $t(s) = f$, $t(t) = x$, and $t(u) = y$. t is strongly g-continuous $t^{-1}(y) = u \in(Q, s)$.

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