Rgδ-Continuous Functions and Their Properties in Topological Spaces

S. Rajakumar

Abstract: A continuity form of rgδ is offered in this commodity. The design of rgδ-irresolute functions and strongly rgδ-continuous functions are also discussed and its properties are investigated.

Keywords: rgδ-continuous, rgδ-Irresolute maps, strongly rgδ-continuous

I. INTRODUCTION

Many researchers have introduced the concepts of generalized continuous functions, gc-irresolute maps on topological spaces.[1, 2, 5, 6, 7]. Some authors continuing their studies on generalization of continuous maps. Irresolute functions are studied by Crossley and Hilderbrand [3] Sheik John [4] introduced and studied the concepts of strongly w-continuous maps. In this write-up, we initiated the perception of rgδ-continuous functions.

II. MAIN RESULTS

In this section the new function called rgδ continuous functions is defined and obtained some results by comparing it with some other continuous functions. Irresolute functions and strongly rgδ-continuous functions are also discussed in the subsequent sections. Definitions of rw, rwg, sg and gpr-continuous extracted from [8], [9], [10], [11], [12] respectively.

Definition 2.1 A function e: (R, ψ) → (N, η) is said to be rgδ-continuous if the inverse image of every closed set in N is rgδ-closed set in R.

Remark 2.2 In [8], By Theorem 2.2 Every closed set is rgδ-closed but not conversely.

Theorem 2.3 If e: (R, ψ) → (N, η) is continuous then e is rgδ-continuous.

Proof: Let GcN, where G is closed. e is continuous
⇒ e⁻¹(G) is closed. By Remark 2.2, e⁻¹(G) is rgδ-closed.

Hence e is rgδ-continuous.

Remark 2.4 The converse part fails in Theorem 2.3.

Example 2.5 Let M = N = {η, ψ, ω} with ξ = {M, η, ψ, ω} and ν = {N, φ, {ο}, {ε}, {ω}, {η, ζ}, {ψ}, {ω}}. Let q: (M, ξ) → (N, ν) be defined by q(ε) = η, q(ψ) = ζ, q(ω) = ζ. Then f is rgδ continuous but not continuous as the inverse image of η in N is ω, which is not in M.

Theorem 2.6 Let h: (P, σ) → (Q, η) is rgδ-continuous if and only if every open in Q is rgδ-open in P

Proof. Let a function h: (P, σ) → (Q, η) is rgδ-continuous and let open set H in Q. Then H is a closed set in Q. Since h is rgδ-continuous, h⁻¹(H) is rgδ-closed in P. As h⁻¹(H) = P, h⁻¹(H) is rgδ-open in P.

Converse Part: Let h⁻¹(H) is rgδ-open in P. Let G be closed set in Q. Then G is open Q. Therefore h⁻¹(G) = P. Therefore h⁻¹(G) is rgδ-open. i.e, h⁻¹(G) is closed in X. Thus h is rgδ-continuous.

Theorem 2.7 If j: (K, υ) → (L, ω) is rw-continuous then j is rgδ-continuous.

Proof. Consider j is rw-continuous, which implies rw- closed set S∈L, j⁻¹(S) is rw-closed in K. Since every rw- closed set is rgδ-closed set, j⁻¹(S) is rgδ-closed in K. Hence j is rgδ-continuous.

Remark 2.8 The reverse part of theorem 2.7 need not true.

Example 2.9 Let K = {11, 22, 33, 44, 55} with v = {K, φ, {11}, {22}, {33}, {11, 22}, {11, 33}, {22, 33}, {11, 22, 33}}. Let L = {11, 22, 33, 44, 55, 66} with Ψ = {L, φ, {11}, {22}, {33}, {11, 22}, {11, 33}, {22, 33}, {11, 22, 33}}. Define j: (K, υ) → (L, ψ) by j(11) = 44, j(22) = 11, j(33) = 22, j(44) = 44, j(55) = 44. Then j is rgδ-continuous. Here the closed {22, 33} in (L, ψ) is 33 not rw in K.

Theorem 2.10 If p: Q → R is g continuous implies then p is rgδ-continuous.

Proof. Let p: Q → R is g-continuous. Let K ⊂ R. p⁻¹(K) is g-closed in R. Every g-closed set is rgδ-closed set in R,
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\[ p^{-1}(K) \text{ is rgδ-closed in } R. \text{ So } p \text{ is rgδ-continuous.} \]

**Remark 2.11** The contra part fails in Theorem 2.10.

**Example 2.12** Let \( Q = \mathbb{R} = \{ i, j, k \} \) with \( \tau = \{ Q, \varphi, \{ i \}, \{ j \} \} \) and \( \sigma = \{ R, \varphi, \{ i \}, \{ j, k \} \} \). Let \( p : Q \to R \) be identity one. Then \( p \) is rgδ-continuous but not g-continuous.

**Theorem 2.13** Let \( s : (T, \mu) \to (U, \rho) \) is rgδ-continuous, then \( s \) is rwg-continuous.

**Proof.** A function \( s : (T, \mu) \to (U, \rho) \) is rgδ-continuous. Then a \( K \) be in \( U, \) \( s^{-1}(K) \) is in \( T. \) Here every rgδ-closed set is rwg-closed set, \( s^{-1}(K) \) is rwg-closed in \( T. \) Thus \( s \) is rwg-continuous.

**Remark 2.14** In Theorem 2.13, the converse part is not satisfied.

**Example 2.15** Let \( T = \{ l, m, n, o \} \) with \( \mu = \{ T, \varphi, \{ l \}, \{ m \}, \{ m, l \} \} \) and \( U = \{ g, h, i, j \} \) with \( \rho = \{ T, \varphi, \{ g \}, \{ h \}, \{ g, h \}, \{ g, h, i \} \}. \) Define a function \( r : (T, \mu) \to (U, \rho) \) by \( r(l) = g, r(m) = I, \) \( r(n) = r(o) = g. \) Therefore \( r \) is rwg-continuous. Hence \( p \in T, \) \( \{ l, \} \notin T. \) Hence \( \{ l, \} \notin T \) is not rgδ-closed set in \( T. \)

**Remark 2.16** sg-continuous and rgδ-continuous are independent.

**Example 2.17** Let \( T = \{ u, v, w, x \} \) with \( \varphi = \{ T, \varphi, \{ u \}, \{ v \}, \{ u, v \}, \{ u, v, w \}, \varphi \} \) \( \gamma = \{ V, \varphi, \{ v \}, \{ v, \psi \}, \{ v, w, \}, \psi \} \) and \( W = \{ v, \psi, \lambda \} \) with \( \beta = \{ \{ W, \varphi, \{ v, \psi, \lambda \} \} \} \).

**Example 2.18** Let \( V = \{ u, \psi, \lambda, \} \) with \( \gamma = \{ V, \varphi, \{ u \}, \{ \psi \}, \{ u, \psi \}, \{ v, \psi, \lambda \} \} \) and \( W = \{ v, \psi, \lambda \} \) with \( \beta = \{ W, \varphi, \{ v, \psi, \lambda \} \}. \) Define a function \( u : (V, \gamma) \to (W, \gamma) \) by \( u(\alpha) = u(\beta) = \alpha. \) Thus \( u^{-1}(W) \) is sg-closed. Hence \( u \) is sg-continuous.

**Example 2.19** Let \( C = \{ \xi, \kappa, \zeta, \omega \} \) with \( \psi = \{ C, \varphi, \{ \xi \}, \{ \kappa \}, \{ \xi, \kappa \}, \{ \xi, \kappa, \zeta \} \} \) and \( D = \{ j, k, l \} \) with \( \rho = \{ D, \varphi, j \} \).

**Theorem 2.20** If \( d : (C, \psi) \to (D, \rho) \) is rgδ-continuous, then \( d \) is gpr-continuous.

**Proof.** Given \( d : (C, \psi) \to (D, \rho) \) is rgδ-continuous. Let \( N \subseteq D \Rightarrow \) rgδ-continuous. Then \( d^{-1}(N) \) is rgδ-closed in \( C. \) Therefore \( d^{-1}(N) \) is gpr-closed in \( C. \) Thus \( d \) is gpr-continuous.

**Remark 2.21** The reverse part is not true in theorem 2.20.

**Example 2.22** Let \( C = \{ \xi, \kappa, \zeta, \omega \} \) with \( \psi = \{ \{ C, \varphi, \{ \xi \}, \{ \kappa \}, \{ \xi, \kappa \}, \{ \xi, \kappa, \zeta \}, \{ \xi, \kappa, \zeta, \omega \} \} \) and \( D = \{ j, k, l \}. \) Define \( d : (C, \psi) \to (D, \rho) \) by \( d(\xi) = k, d(\kappa) = d(\zeta) = j. \) Thus \( d \) is gpr-continuous not rgδ-continuous.

**III. RGA-IRRESOLUTE FUNCTIONS**

**Definition 3.1** A function \( q : (R, \eta) \to (S, \psi) \) is said to be irresolute function if every rgδ-closed \( q^{-1}(S, \psi) \) is rgδ-closed in \( (R, \eta). \)

**Theorem 3.2** A map \( a : (H, \kappa) \to (Z, \zeta) \) is rgδ-irresolute if and only if the \( a^{-1}(Z, \zeta)-open \) is rgδ-open in \((H, \kappa).\)

**Proof** Let \( a : (R, \eta) \to (Z, \zeta) \) be rgδ-irresolute and \( W \) be rgδ-open in \((Z, \zeta). \) \( W^c \) is rgδ-closed in \((Z, \zeta). \) Here \( a \) is rgδ-irresolute, \( a^{-1}(W^c) \) is rgδ-closed in \((H, \kappa).\) Hence \( a^{-1}(W^c) \) is rgδ-open in \((Y, \kappa).\)

**Converse Part:** For every rgδ-open set \( W \) in \((Z, \zeta), a^{-1}(W) \) assume that \( a^{-1}(W) \) is rgδ-open in \((Y, \kappa).\) Let \( U \) be a rgδ-closed set in \((Z, \zeta). \) Now \( U^c \) is rgδ-open in \((Z, \zeta), \) \( a^{-1}(U^c) \) is rgδ-closed in \((Y, \kappa). \) Hence \( a^{-1}(F^c) = (a^{-1}(F))^c, \) we have \( x^c(F) \) is rgδ-closed in \((Y, \kappa). \) Hence \( a \) is rgδ-irresolute.

**Theorem 3.3** A function \( z : (A, \psi) \to (B, \mu) \) is rgδ-irresolute \( \Rightarrow \) rgδ-continuous.

**Proof** Let \( z : (A, \psi) \to (B, \mu) \) be rgδ-irresolute and a closed set \( G \in (B, \mu) \Rightarrow G \) is rgδ-closed \((B, \mu).\) As \( z \) is rgδ-irresolute, \( z^{-1}(G) \) is rgδ-closed in \((A, \psi) \) and therefore \( z \) is rgδ-continuous.
Remark 3.4 $g$-irresolute and $rgδ$ –irresolute function are independent.

Example 3.5 Let $K = L = \{ m, n, o, p \}$ with $\rho = \{ K, \varphi, \{ m \}, \{ n \} \}$ and $\omega = \{ L, \varphi, \{ m \}, \{ m, n \} \}$. The identity map $w : (K, \rho) → (L, \omega)$ is $rgδ$ -irresolute but it is not $g$-irresolute. $r^{-1}(\{ m, p \}) \subset (L, \omega)$ is $\{ m, p \}$ in $(K, \rho)$ which is $rgδ$ closed.

Example 3.6 Let $Z = Y = \{ \eta, \gamma, \nu \}$ with $Γ = \{ Z, \varphi, \{ m \} \}$ and $ε = \{ Y, \varphi, \{ \eta \}, \{ \gamma, \nu \} \}$. Then the function $κ : (Z, Γ) → (Y, E)$ defined by $κ(ε) = η, κ(γ) = c(γ) = ν$ is $g$-irresolute but it is not $rgδ$-irresolute function. Since $ε^{-1}(\{ γ, ν \}) \in (Y, E) \notin (Z, Γ)$.

IV. STRONGLY RGA-CONTINUOUS

Definition 4.1 A function $r : (E, θ) → (F, ς)$ is said to be strongly $rgδ$-Continuous function if the inverse image of $rgδ$-open in $(F, ς)$ is open in $(E, θ)$.

Theorem 4.2 A map $i : (V, ω) → (W, θ)$ is a strongly $rgδ$-continuous function iff $i^{-1}(H)$ is closed in $(W, θ)$.

Proof. Assume $i : (V, ω) → (W, θ)$ is a strongly $rgδ$-continuous function. Let $H$ be $rgδ$ closed in $(W, θ)$. Now $H^c$ is a $rgδ$-open set in $(W, θ)$. Assume $i :$ is strongly $rgδ$-continuous, $i^{-1}(H^c) = V^c \cap (W, θ)$. Therefore $i^{-1}(H)$ is closed in $(W, θ)$.

Converse Part: $i^{-1}(H)$ is closed in $(W, θ)$. Let $G \subset C$ be a $rgδ$-open in $(W, θ)$, then $G^c$ is a $rgδ$-closed in $(W, θ)$. Since $i^{-1}(G^c) = V^c \cap (W, θ)$, $i^{-1}(H)$ is open in $(V, ω)$. Thus $i$ is strongly $rgδ$-continuous.

Theorem 4.3 A map $n : (G, α) → (H, β)$ is a strongly $rgδ$-continuous function if $n$ is continuous.

Proof. Let $n : (G, α) → (H, β)$ is a strongly $rgδ$-continuous function. Let $J$ be open in $(H, β)$. As $J$ is $rgδ$- open in $(H, β)$. As $n$ is strongly $rgδ$-continuous, $n^{-1}(H)$ is continuous.

Theorem 4.4 Suppose $v : (C, β) → (D, γ)$ is a strongly $rgδ$-continuous function then $v$ is strongly $g$-continuous.

Proof. Let $v$ is strongly $rgδ$-continuous. Let $E \subset D$, where $E$ is $g$-open. Therefore $E$ is $rgδ$-open in $(D, γ)$, $v$ is strongly $rgδ$-continuous. As $v^{-1}(E) \subset (C, β)$, $v$ is strongly $g$-continuous.

Example 4.5 Let $Q = \{ s, t, u, v \}$ and $R = \{ f, x, y, z \}$ $E = \{ R, \varphi, \{ s \}, \{ t \}, \{ s, t \} \}$ and $ω = \{ R, \varphi, \{ f \}, \{ x \}, \{ f, x \} \}$. The map $t : (Q, E) → (R, G)$ defined by $t(s) = f, t(t) = x, and t(u) = y$. $t$ is strongly $g$-continuous $t^{-1}(y) = u \in (Q, s)$.

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REFERENCES


AUTHORS PROFILE

Dr. S. Rajakumar is currently a Assistant Professor in the Department of Mathematics in Kalasalingam Academy of Research and Education (Deemed to be University), Krishnankoil-626126. He obtained his Ph.D., degree in Manonmaniam Sundaranar University, Tirunelveli. His area of research interest is Topology and Bitopological spaces. He has 17+ years of teaching experience. He has a few publications in standard journals.