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Towards Improving Experimental Learning-Exploring Bernoulli's Equation

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Abstract*: *This study illustrates through the verification of Bernoulli's equation (as a case study experiment) about how analysis of the gap in experimental results when compared to its theoretical value based on the fundamental principles underlying a hypothesis should become a part of the experimental learning approach which is being adopted in teaching a course. For this experiment, even after making a thorough analysis, it is found difficult to exactly answer the gap in the experimental and theoretical values. Particularly, it is seen that the unaccounted loss is about 10%. Various assumptions adopted in arriving at the conclusions are also elaborated to get a clear picture on carrying out the analysis. This investigation effectuated deep interest among student in the course and also engrossed them to discover the reason for any unaccountable losses in the experiment other than the well identified losses.

Keywords: Bernoulli's equation, pipe flow, civil engineering, fluid mechanics, Head losses.

I. INTRODUCTION

In the recent past, increased emphasis is being laid on experimental learning and project based learning compared to the conventional chalk and talk method for imparting deeper understanding in a course. This purpose is properly fulfilled when the results from the experiments are analysed with respect to the expected/standard results from the theoretical philosophical principles associated with a given experiment. Often this is not much given attention to the gap in the results obtained experimentally and theoretically. If it is properly analyzed, we can enrich the knowledge and in some extreme cases, can also lead to a change in the existing hypothesis or an altogether new hypothesis. Hence, it is preferred to gain deeper insight into experimental errors and find the truth behind the errors which will improve the experimental learning. In this study, an attempt is made to illustrate this recommendation through the experiment on verification of Bernoulli's equation [1] as a case study. If such investigation done with the involvement of students, they will get more interest in the subject [2]. There are many researches already done in Bernoulli's principles with various objectives [3],[4],[5], and [6]. This work investigates the assumptions on which Bernoulli's formula is formulated. The Bernoulli's equation can be verified in the lab by creating flow through a pipe system connecting two tanks. The real flow will have some head loss components which are identified and included in the Bernoulli's equation. Some of the readings or calculated observations based on the experiment carried out in the lab do not clearly agree with the Bernoulli's theory and principle. This work illustrates as to why the deviation in readings or calculation is observed and what are its causes. The work investigates the components of losses and their estimations and tries to identify the reasons for any unaccountable losses other than those which are well identified. The next section briefly discusses the principles of Bernoulli's theorem, followed by the methodology adopted and a discussion on the results obtained.

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II. METHODOLOGY

A. Theory

The Bernoulli's principle is based on the some assumptions which are the fluid is ideal. i.e viscosity is zero, the flow is incompressible, the flow is steady and uniform, and the flow is along a streamline. Bernoulli's equation as applied along a streamline for a real flow can be expressed as

$$\frac{P}{\gamma} + \frac{v^2}{2g} + z \quad = C \qquad (1)$$

Where,

$\frac{P}{\gamma}$= Piezometric head(m)

$\frac{v^2}{2g}$= velocity head(m)

Z= datum head(m)

Total head is the summation of Pressure head, Velocity head and Datum head. Bernoulli's equation, when applied between two points for real fluid is given as

$$\frac{P_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_l \qquad (2)$$

Where, h_l is the loss of energy. The Bernoulli's theorem says that for a perfect incompressible fluid flowing in a continuous stream the total head or total energy of each particle remains same along the

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Fig. 1. Bernoulli's Apparatus

Streamline, if no energy is gained or lost. As, real fluid flows will offer resistance to the flow there are always some losses in the flow and hence in the application of Bernoulli's equation losses have to be taken into consideration. In case of real fluids the velocity is not uniform over the cross section and consequently the velocity head expressed in terms of mean or average velocity is not correct.

The setup (Fig. 1) has a base frame upon which the sump tank is adjoined. The sump tank has water pump set up and a drain cock. Two delivery tanks are placed at the each end, both parallel to each other and separated by a pipe of varying cross section. The diameter of the pipe is varying and therefore flow is non uniform flow. There are seven piezometers fitting in the pipe. Each piezometer is separated by 69mm distance from each other, but the distance between the third to fourth and fourth to fifth piezometer is 68mm. This is due to sudden contraction of the pipe. The setup up has a collecting tank (40cm*40cm) at the tail which has a scale attached to it to measure the 'x' cm rise of water in the tube.

B. The procedure adopted for the conduction of the experiments:

- Inlet valve of apparatus is opened and water is allowed to flow through the conduit (pipe).
- Outlet value is adjusted such that constant head is maintained in the supply tank. The outlet valve must be adjusted till the head at both the end of the delivery tank have same readings. i.e If h_1 is the head on the left side delivery tank and h_2 is the head on the right side delivery tank, then readings must be further proceeded only when both the heads are stable on their respective readings and not fluctuating.
- If any air bubbles are present in the piezometer, the bubbles must be removed off.
- Once the heads are measured as stable heads, the piezometers are then observed and they are read only when they attain a stable level. The pressure heads of various section of conduit are measured from piezometer readings.i.e all piezometers are read one by one and pressure head are noted down.

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- The valve is adjusted and water is collected in the collecting tank. Now time 't' taken for selected 'x' cm rise in the collecting tank is measured from the scale by using stop watch. This time 't' is basically measured to find the discharge of water in the collecting tank. Discharge calculation is done to find velocity.

C. *Methodology for Head loss estimation:*

There are mainly two types of Head Loss associated with the pipe flow and they are discussed below:

- *Major loss:*

It is also known as friction loss and is estimated using Darcy-Weisbach Equation

$$h_f = \frac{f\,l\,V^2}{2g\,D} \quad (3)$$

where, f = friction factor

l = length between the piezometers (m)

V = velocity (m/s)

d = diameter of the pipe (m)

g= acceleration due to gravity(m/s^2)

Friction factor 'f' can be found out using Moody's chart or alternatively using the equation such as:

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left[\frac{k}{D}\frac{21.25}{R_e^{0.9}}\right] \quad (4)$$

Where k is the roughness value of the material of the pipe

k/D is the relative roughness value.

R_e is the Reynold's number

The roughness value is taken as 0.0025mm by referring as the material as fibre plastic. The material quality may vary and therefore the roughness value may also vary.

- *Minor loss:*

Apart from the major loses there are also the associated minor losses which are discussed below:

a) Entry loss

$$h_e = K\,\frac{v^2}{2g} \quad (5)$$

Where, h_e is the entry loss.

V is the velocity of the flow (m/s)

g is acceleration due to gravity(m/s^2)

Here, K= 0.55 is used for calculation of entry loss, as the water is flowing from tank to pipe.

b) Contraction and Expansion losses

$$h_{ex} = K\,\frac{v^2}{2g} \quad (6)$$

Where, h_{cx} is the contraction or expansion loss.

As the diameter of the pipe is not uniform throughout its length, a sudden contraction is seen near to the fourth piezometer and therefore K is taken as 0.45 for calculation part.

The size of the collecting tank is 40 cm x 40cm; therefore first the area of the collecting tank is calculated. Using the area of the collecting tank and the time measured for x cm rise , actual discharge of water is calculated .

For calculation of velocity the diameter of the pipe is required. As the setup has varying cross section throughout its length, the velocity also changes consequently. The Reynolds number R_e is found to be greater than 20000 which is clearly in the turbulent zone.

III. RESULTS AND DISCUSSIONS

Summary of calculation details are shown in table 1. Table 2 gives proportion of the losses in the total loss.

The following observation can be made from the tabulations:

1) The diameter of the pipe is not uniform that is the setup has varying diameter and therefore velocity is not uniform throughout the pipe. The calculation carried out using average diameter is not found to be appropriate. A diameter value lesser than average diameter is found to be a better representative of the diameter which is arrived based on trial and error. Calculation of discharge may also vary as leakage of water from the collecting tank was observed during measuring rise of water from tank.
2) Pressure head decreases while velocity head increases in the contraction section of the pipe; whereas the reverse is observed in the expansion section.
3) It is observed from the Table1 that, the reading 3, 4 and 5 is not fully reliable as the energy in these three cases is greater than the head reading. This is because of the head losses, as the K values are purely assumed in calculation part. No perfect constant value is known for sudden contraction and expansion.
4) There may be some losses occurring due to the piezometric fitting in the pipe, which are not accounted in this experiment. The major contributors to head losses are loss due to friction and losses due to contraction and expansion. Since the pipe length is very small and since the flow is subjected to both expansion and contraction in this small section, these losses are more significant than even the friction loss (major losses).
5) The percentage of velocity head, pressure head and all the losses are calculated in order

Table- I: Identification of head loss components

S. No	Pressure head	Diameter	Cross sectional area 'A'	Time for 'x' cm rise 't'	Discharge 'Q'	'V'	$\frac{v^2}{2g}$	Reynold no.	Friction factor	Energy losses			Head		Total energy
										h_f	h_e	h_c	h_1	h_2	
	m	m	m^2	s	m^3/s	m/s	m			m	m	m	m	m	m
1	0.301	0.041	0.00131	10.5	0.000761	0.577	0.016	23672.67	0.024	0.0006	0.009	0.0076	0.384	0.325	0.335
2	0.286	0.034	0.00090	10.5	0.000761	0.839	0.035	28546.45	0.023	0.0017	0.009	0.016	0.384	0.325	0.357
3	0.243	0.027	0.00057	10.5	0.000761	1.331	0.090	35947.38	0.022	0.004	0.009	0.040	0.384	0.325	0.413
4	0.115	0.022	0.00037	10.5	0.000761	2.005	0.204	44117.24	0.021	0.013	0.009	0.0009	0.384	0.325	0.415
5	0.21	0.027	0.00057	10.5	0.000761	1.331	0.090	35947.38	0.022	0.014	0.009	0.004	0.384	0.325	0.414
6	0.221	0.034	0.00090	10.5	0.000761	0.839	0.035	28546.45	0.023	0.0046	0.009	0.001	0.384	0.325	0.377
7	0.254	0.041	0.00131	10.5	0.000761	0.577	0.016	23672.67	0.024	0.001	0.009	0.0006	0.384	0.325	0.393

Table- II: Comparison of percentage contribution by head loss components

S.No	h_1	$\frac{P}{\gamma}$	$\frac{V^2}{2g}$	h_f	h_e	h_c/h_{ex}	energy	h_1 - energy	$\frac{P}{\gamma}$ (%)	$\frac{V^2}{2g}$ (%)	h_f %	h_e%	h_c/h_{ex} %	total %	Unaccounted losses
1	0.384	0.301	0.016	0.0006	0.00935	0.0076	0.335	0.048	89.6	5.0	0.18	2.7	2.2	100	14.4
2	0.384	0.286	0.035	0.0017	0.00935	0.0161	0.357	0.026	80.0	10.0	0.66	2.6	6.6	100	7.4
3	0.384	0.243	0.090	0.0043	0.00935	0.0406	0.413	0.029	58.7	21.8	1.61	2.2	15.5	100	7.2
4	0.384	0.115	0.204	0.0137	0.00935	0.0009	0.415	0.031	27.6	49.3	4.92	2.2	15.7	100	7.5
5	0.384	0.21	0.090	0.0146	0.00935	0.0046	0.414	0.030	50.6	21.7	8.45	2.2	16.8	100	7.4
6	0.384	0.221	0.035	0.0046	0.00935	0.0013	0.377	0.006	58.5	9.5	10.51	2.4	18.9	100	1.7
7	0.384	0.254	0.016	0.0018	0.00935	0.0006	0.393	0.009	64.4	4.3	10.53	2.3	18.2	100	2.5

to check as to which among them has the major contribution in the deviation of the result.

6) As seen from the Table 2 the losses due to the contraction / expansion of the pipe contribute to almost two-third of the velocity head.

7) It is observed that even after accounting for all the possible losses still there are unaccountable losses. This unaccountable loss is about 14 percentage in the first section of the pipe and is much lessees as the flow passes through the remaining sections. There is almost 7% losses which are unaccounted in the pipe starting from section 2 onwards. This may be due to fact that the fitting of the piezometers in the pipe system will incur some losses. This can be understood to be due to the fact that as the flow enter from the tank to the pipe , turbulence associated with that creates more energy losses and as the flow gets established , these losses gets reduced.

IV. CONCLUSIONS

This study attempted to verify Bernoulli's theorem and identify the loss components in the verification of Bernoulli's experiment. Both major and minor loss components are identified. Minor loss components associated with the convergent and divergent section of the pipe is almost two-third of the velocity head as seen from the components proportion calculations. It is seen that friction loss contributes 10% at the end of the pipe section. Many losses (piezometer fitting loss, turbulence effect when the flow sets up in the pipe) are present, which have not been accounted, may be around 10% on an average.

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