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Pell Labeling of Joins of Square of Path Graph

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Abstract: *A graph is composed of p vertices and q edges. A Pell labeling graph is the one with $u \in V(G)$ being distinct. Label $f(u)$ from 0,1,2, … p-1 in a such a way that each edge is labelled with $f^*: E(G) \to N$ such that $f^*(uv) = f(u) + 2f(v)$ are distinct. In this paper we study Square of Path graph P_n^2 and attach an edge to form a join to the square of path graph P_n^2 and prove the join of square of path graph P_n^2 is Pell labelling graph and further study on some interesting results connecting them.*

Keywords : *Square of Path graph P_n^2 ,Pell labelling , Pell labelling graph, Joins of Square of path graph P_n^2*

I. INTRODUCTION

A finite graph has finite vertices and finite edges. Gallian [1] has provided an interesting survey on graph labeling.Rosa[2] has initiated the study on labeling. Pell labelling of graph was introduced by J. Shiama[3] and have proved that paths, cycles, stars, double stars, coconut tree, bi star are Pell labelling graphs. Motivated towards the Pell Labeling graph and study of joins of graphs [4][5][6][7] we in this paper have identified the Square of path graph P_n^2 and proceeded further to study on some important results. For graph preliminaries we consider Gross.J and .Yellen.J, Handbook of graph theory [8].

II. PRELIMINARIES

Definition 2.1: A Pell labelling graph is a bijection function $f: V(G) \to \{0,1,2...p-1\}$ such that for each edge there is an induced distinct edge labelling $f^*: E(G) \to N$ such that $f^*(uv) = f(u) + 2f(v)$.

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Definition 2.2: A path graph P_n consists of vertices $u_1, u_2, ...u_n$ the square of path P_n^2 graph consists n vertices $u_1, u_2, ...u_n$ and 2n-3 edges

Definition .2.3 : Attaching a square of path P_n^2 graph with another square of path P_n^2 graph by an edge is called 1-join square of path P_n^2 graph.

Note: For 1-join square of path P_n^2 graph the number of vertices is 2n and the number of edges is 4n-5

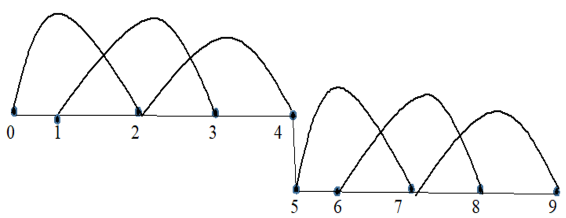

Fig.1- 1-join of square of path P_5^2

The above is a 1-join of square of path P_5^2 graph with 5 vertices and 7 edges attached by an edge to one square path graph P_5^2 with another square path graph P_5^2. Similarly we can construct M-Join of square of path P_n^2 graph.

Note: We here exclusively study the case of M-Join of square of Path P_n^2 graph attached by edges $e_1, e_2, ..., e_n$ to the same order of square of path P_n^2 graph.

III. MAIN RESULTS

Theorem .3.1: 1- Join square of path P_n^2 is a Pell labelling graph.

Proof: Let G=1-Join of square of path P_n^2 graph

Let us prove that G is a Pell labeling graph

Let us prove the theorem by labelling the vertices of the graph G.

We have the number of vertices in square of path P_n^2 graph is n and the number of edges is 3n-2. Now adding one edge between the square of path P_n^2 graph with another square of path P_n^2 graph we have the number of vertices in 1-Join

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Of square of path P_n^2 graph is 2n and the number of edges in 1-Join of square of path graph is 4n-5 .
Now let us label the vertices of 1-join of square of path P_n^2 graph as follows We know that the first square of path P_n^2 has n vertices and second square of path P_n^2 has n vertices and totally 2n vertices is to be labelled.
Let us denote the Vertex Set of first square of path P_n^2 graph as $V=\{u_1,u_2,u_3,u_4,...u_n\}$ and the vertex set of second square of path P_n^2 graph as $V^1=\{u^1{}_1,u^1{}_2,u^1{}_3,u^1{}_4...u^1{}_n\}$. The edge set of first square of path P_n^2 graph is $E=\{e_1,e_2,e_3,e_4,...e_{n-1}\}\cup$ $\{e_{ij},1\le i\le n-2,n-2\le j\le n\}$
and the edge of the second square of path P_n^2 Graph is $E^1=\{e_1{}^1,e_2{}^1,e_3{}^1,e_4{}^1,...e^1{}_{n-1}\}\cup$ $\{e^1{}_{ij},1\le i\le n-2,n-2\le j\le n\}$. Now by adding one edge between the first and second square of path P_n^2 graph we have the vertex set of 1- Join square of path P_n^2 graph as $V(G)=\{u_1,u_2,u_3,u_4,...u_n\}\cup\{u^1{}_1,u^1{}_2,u^1{}_3,u^1{}_4...u^1{}_n\}$ and the edge set as $E(G)=\{e_1,e_2,e_3,e_4,...e_{n-1}\}\cup$ $\{e_1{}^1,e_2{}^1,e_3{}^1,e_4{}^1,...e^1{}_{n-1}\}\cup$ $\{e_{ij},e^1{}_{ij},1\le i\le n-2,n-2\le j\le n\}\cup\{e\}$.
The edges that are connecting between the first and second square of path P_n^2 graph is known as

$e_i=u_iu_{i+1}$ for $1\le i\le n-1$

$e_i^1=u_i^1u_{i+1}{}^1$ for $1\le i\le n-1$

$e_{ij}=u_iu_j$ for $1\le i\le n-2$ and for $n-2\le j\le n$

$e_{ij}{}^1=u_i^1u_j{}^1$ for $1\le i\le n-2$ and for $n-2\le j\le n$

$e=e_{n-1}e_1^1$

Now let us label the vertices of first square of path P_n^2 graph in correspondence to the vertices of second square of path P_n^2 graph as follows

$f(u_i)=i$ for $0\le i\le n-1$

$f(u^1{}_j)=(n-1)+j$ for $1\le j\le n$

Computing the induced edge labelling we have

$f^*(e_i)=f^*(u_iu_{i+1})=f(u_i)+2f(u_{i+1})$

$f^*(e_i^1)=f^*(u^1{}_iu^1{}_{i+1})=f(u^1{}_i)+2f(u^1{}_{i+1})$

$f^*(e_{ij})=f^*(u_iu_j)=f(u_i)+2f(u_j)$

$f^*(e_{ij}{}^1)=f^*(u_i^1u_j{}^1)=f(u^1{}_i)+2f(u^1{}_j)$

$f^*(e)=f^*(u_{n-1}u_1^1)=f(u_{n-1})+2f(u^1{}_1)$

The induced edge labelling is distinct Hence the proof.

We now give a general algorithm for assigning labels for the vertices of M-Join of square of path P_n^2 graph

Algorithm.3.2
Begin
for j=0 to n-1
$f(u_{j+1})=j$
for i = 1 to M
for j= 0 to n-1
$f(u^i{}_{j+1})=i(n)+j$
End

Note.1: Following the algorithm to label the vertices of M-Join of square of path P_n^2 graph we can obtain the induced edge labelling of the graph and hence can prove in general for any M-Join of square of path P_n^2 graph is Pell labelling graph.
Note.2: : We know from basic algebra that if three numbers a,b,c are in arithmetic progression then $b=\dfrac{a+c}{2}$ or otherwise $b-a=c-b$. Let us use this technique in the subsequent theorem to prove that the induced edge labelling of M-Join of square of path P_n^2 graph forms an arithmetic progression.

Theorem.3.3: M-join of square of path P_n^2 Pell graph the induced edge labelling has the following property

(i) $f^*(u_iu_{i+1}),f^*(u_{i+1}u_{i+2})...$ are in arithmetic progression with common difference 3 by fixing $i=1$
$f^*(u^M{}_iu^M{}_{i+1}),f^*(u^M{}_{i+1}u^M{}_{i+2})...$ are in arithmetic progression with common difference 3 where M=1,2,… and M is the number of joins of square of path P_n^2 graph and by fixing $i=1$

(ii) $f^*(u_iu_{i+2}),f^*(u_{i+1}u_{i+3})...$ are in arithmetic progression with common difference 3 by fixing $i=1$.
$f^*(u^M{}_iu^M{}_{i+2}),f^*(u^M{}_{i+1}u^M{}_{i+3})...$ are in arithmetic progression with common difference 3 by fixing $i=1$.

Proof: Consider M- Join of square of path P_n^2 Pell graph.
Now labelling the graph as given in the algorithm we can obtain the induced edge labelling case by case for different joins of square of path P_n^2 graph.
Now let us prove the property (i) as follows
To prove $f^*(u_iu_{i+1}),f^*(u_{i+1}u_{i+2})...$ are in arithmetic progression . let us consider

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$$a = f^*(u_iu_{i+1}) = f(u_i) + 2f(u_{i+1})$$
$$b = f^*(u_{i+1}u_{i+2}) = f(u_{i+1}) + 2f(u_{i+2})$$
$$c = f^*(u_{i+2}u_{i+3}) = f(u_{i+2}) + 2f(u_{i+3})$$

Now let us use the property that a,b,c are in Arithmetic progression namely $b-a=c-b$

On computing

$$b-a = f^*(u_{i+1}u_{i+2}) - f^*(u_iu_{i+1}) = (f(u_{i+1}) + 2f(u_{i+2}))$$
$$-(f(u_i) + 2f(u_{i+1}))$$

Simplifying the terms

$$b-a = 2f(u_{i+2}) - f(u_i) - f(u_{i+1})$$

Similarly computing

$$c-b = f^*(u_{i+2}u_{i+3}) - f^*(u_{i+1}u_{i+2}) = f(u_{i+2}) + 2f(u_{i+3})$$
$$-f(u_{i+1}) - 2f(u_{i+2})$$

Simplifying the terms

$$c-b = 2f(u_{i+3}) - f(u_{i+1}) - f(u_{i+2})$$

Now equating $b-a=c-b$ and simplifying we have

$$2f(u_{i+2}) - f(u_i) = 2f(u_{i+3}) - f(u_{i+2})$$

On further simplifying we find

$$3f(u_{i+2}) = f(u_i) + 2f(u_{i+3})$$

Which is the condition required for the induced edge labelling $f^*(u_iu_{i+1}), f^*(u_{i+1}u_{i+2})\ldots$ to be in arithmetic progression. To verify it let us assume i=1 so as to identify the labels of the vertices in the condition on substituting in the condition we find the terms are $f(u_1)=0;\ f(u_3)=2;\ f(u_4)=3$

Hence on substituting in the condition the labels we find that it is 6 on both sides. Similarly we can assign values for i as 2,3,… and verify that the induced edge labelling are in arithmetic progression.

In similar way we can prove

$f^*(u_iu_{i+2}), f^*(u_{i+1}u_{i+3})\ldots$ are in arithmetic progression Similarly we can prove result (ii) namely $f^*(u_iu_{i+2}), f^*(u_{i+1}u_{i+3})\ldots$ and $f^*(u^M{}_iu^M{}_{i+2}), f^*(u^M{}_{i+1}u^M{}_{i+3})\ldots$ are in arithmetic progression . We find from the labelling techniques that the common difference is 3 for both result (i) and result (ii).

Hence the proof.

Example.1

The following is the 1-Join of square of path P_5^2 graph . we label as in theorem.3.2

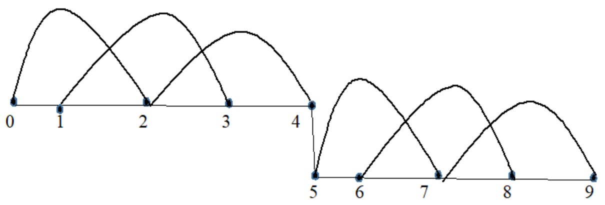

Fig.2- 1-join of square of path P_5^2

For Result .(i) is true i.e $f^*(u_iu_{i+1}), f^*(u_{i+1}u_{i+2})\ldots$ are in arithmetic progression with common difference 3 .

For we know the labelling of 1-join of square of path P_5^2 graph is as follows

$f(u_1)=0$, $f(u_2)=1$, $f(u_3)=2$, $f(u_4)=3$, $f(u_5)=4$, $f(u^1{}_1)=5$, $f(u^1{}_2)=6$, $f(u^1{}_3)=7$, $f(u^1{}_4)=8$, $f(u^1{}_5)=9$

The induced edge labelling of the graph is given as

$f^*(u_1u_2)=2$; $f^*(u_2u_3)=5$; $f^*(u_3u_4)=8$; $f^*(u_4u_5)=11$ $f^*(u^1{}_1u^1{}_2)=17$; $f^*(u^1{}_2u^1{}_3)=20$; $f^*(u^1{}_3u^1{}_4)=23$; $f^*(u^1{}_4u^1{}_5)=26$

The above induced edge labelling proves the result.(i) in theorem.3.2

Also let us compute the induced edges labelling in order to prove the result .(ii) of theorem.3.2 namely $f^*(u_iu_{i+2}), f^*(u_{i+1}u_{i+3})\ldots$ and for the joins $f^*(u^M{}_iu^M{}_{i+2}), f^*(u^M{}_{i+1}u^M{}_{i+3})\ldots$ as

$f^*(u_1u_3)=4$; $f^*(u_2u_4)=7$; $f^*(u_3u_5)=10$; $f^*(u^1{}_1u^1{}_3)=19$; $f^*(u^1{}_2u^1{}_4)=22$; $f^*(u^1{}_3u^1{}_5)=25$

The above induced edge labelling proves the result .(ii) in theorem.3.2.

Note: Similarly for M-Join of square of path P_n^2 Pell the result is true.

Theorem.3.4: M-Join of square of path P_n^2 Pell graph the following property holds good

$f^*(u_nu^M{}_1) = f^*(u_{n-2}u_n)+4$, for M=1 and , for $M \geq 2$ where M is the number of joins of square of path P_n^2 graph and n is the number of vertices of square of path P_n^2 graph.

Proof : Let us consider M-Join of square of path P_n^2 Pell graph following the labelling techniques provided in Theorem.3.1 and stated in the Algorithm. For proving the result let us choose n=3 and M=1.

Now to Prove $f^*\left(u_n u^M{}_1\right) = f^*\left(u_{n-2} u_n\right) + 4$

Let us substitute n=3 and M=1 and obtain as follows

$f^*\left(u_3 u^1{}_1\right) = f^*\left(u_1 u_3\right) + 4$

Now $f^*\left(u_3 u^1{}_1\right) = f\left(u_3\right) + 2f\left(u^1{}_1\right) = 2 + 2(3) = 8$

Also

$f^*\left(u_1 u_3\right) + 4 = f\left(u_1\right) + 2f\left(u_3\right) + 4 = 0 + 2(2) + 4 = 8$

Hence the result is true for n=3 and M=1. Similarly we can prove for any n and for M=1 in a similar fashion by following the schema of labelling of vertices .

The result $f^*\left(u^{M-1}{}_n u^M{}_1\right) = f^*\left(u^{M-1}{}_{n-2} u^{M-1}{}_n\right) + 4$ can also be proved for $M \geq 2$. Hence the proof.

Theorem.3.5: M-Join of square of path P_n^2 Pell graph the following property holds good

(i) $f^*\left(u_i u_{i+2}\right) = \left(f\left(u_i\right) + f\left(u_{i+1}\right) + f\left(u_{i+2}\right)\right) + 1$,

$f^*\left(u^M{}_i u^M{}_{i+2}\right) = \left(f\left(u^M{}_i\right) + f\left(u^M{}_{i+1}\right) + f\left(u^M{}_{i+2}\right)\right) + 1$,

where M is the number of joins and $i = 1, 2 \ldots n-2$ of the square of path P_n^2 graph.

(ii) $f^*\left(u_i u_{i+2}\right) = f^*\left(u_i u_{i+1}\right) + 2$,

$f^*\left(u^M{}_i u^M{}_{i+2}\right) = f^*\left(u^M{}_i u^M{}_{i+1}\right) + 2$ and

(iii) $f^*\left(u_{i+1} u_{i+2}\right) = f^*\left(u_i u_{i+2}\right) + 1$,

$f^*\left(u^M{}_{i+1} u^M{}_{i+2}\right) = f^*\left(u^M{}_i u^M{}_{i+2}\right) + 1$,

Proof: Consider M-Join of square of path P_n^2 Pell graph with the labelling techniques given in theorem.3.1 and algorithm. Now to prove the property as stated in the theorem. Let us choose i=1

Now let us substitute i=1 in

$f^*\left(u_i u_{i+2}\right) = \left(f\left(u_i\right) + f\left(u_{i+1}\right) + f\left(u_{i+2}\right)\right) + 1$

Which is computed as follows

$f^*\left(u_1 u_3\right) = \left(f\left(u_1\right) + f\left(u_2\right) + f\left(u_3\right)\right) + 1$

We know $f^*\left(u_1 u_3\right) = f\left(u_1\right) + 2f\left(u_3\right)$

On substituting the labels for the corresponding vertices we have

$f^*\left(u_1 u_3\right) = f\left(u_1\right) + 2f\left(u_3\right) = 0 + 2(2) = 4$

Now computing the R.H.S of the result as follows

$f\left(u_1\right) + f\left(u_2\right) + f\left(u_3\right) + 1 = 0 + 1 + 2 + 1 = 4$

Hence we find that the result

$f^*\left(u_i u_{i+2}\right) = \left(f\left(u_i\right) + f\left(u_{i+1}\right) + f\left(u_{i+2}\right)\right) + 1$ is true for i=1.

Similarly it can be proved for any value of i. Hence the result (i) of the theorem.

Now let us prove result (ii) by choosing i=1

To prove $f^*\left(u_i u_{i+2}\right) = f^*\left(u_i u_{i+1}\right) + 2$. Let us substitute i=1 and hence find the following

$f^*\left(u_i u_{i+2}\right) = f^*\left(u_i u_{i+1}\right) + 2$

On substituting the corresponding labels for the vertices we have

$f^*\left(u_1 u_3\right) = f\left(u_1\right) + 2f\left(u_3\right) = 0 + 2(2) = 4$

The R.H. S of the result is computed as follows

$f^*\left(u_1 u_2\right) + 2 = f\left(u_1\right) + 2f\left(u_2\right) + 2$

On substituting the corresponding labels for the vertices we have

$f^*\left(u_1 u_2\right) + 2 = f\left(u_1\right) + 2f\left(u_2\right) + 2 = 0 + 2(1) + 2 = 4$

Hence we find the result $f^*\left(u_i u_{i+2}\right) = f^*\left(u_i u_{i+1}\right) + 2$ is true for i=1. Hence the result (ii) of the theorem.

Now to claim the result (iii) of the theorem namely

$f^*\left(u_{i+1} u_{i+2}\right) = f^*\left(u_i u_{i+2}\right) + 1$

Let us choose i=1 and compute L.H.S and R.H.S as follows

L.H.S is given by

$f^*\left(u_2 u_3\right) = f\left(u_2\right) + 2f\left(u_3\right) = 1 + 2(2) = 5$

R.H.S is given by

$f^*\left(u_1 u_3\right) + 1 = f\left(u_1\right) + 2f\left(u_3\right) + 1 = 0 + 2(2) + 1 = 5$

Hence it is evident that result (iii) is also true for i=1. In general the result (i) (ii) and (iii) is all true for all values of I and for all values of M by proceeding in a similar way. Hence the theorem.

IV. CONCLUSION

We are studying on different graphs in a similar approach to generalize the results

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